CSEE 4823 Advanced Logic Design
Handout: Lecture #2
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Combinational Logic:
Basic Definitions +
2-Level Logic Minimization

Review: Basic Definitions

Literal: a variable (x) or its complement (x')

Product: an "AND" of literals (e.g. xy'z, a'bcd')

Cube: a product (another equivalent name)

Minterm: a product including a literal for every input of the function

Example: If a function has 3 inputs, A/B/C, then A'BC' is a minterm, but A'C is not.

A minterm is also an input vector or combination (i.e. corresponds to a single row in the truth table)

ON-set minterm: minterm where the function is 1

OFF-set minterm: minterm where the function is 0

DC-set minterm: minterm where the function is DC (-)

Implicant: a cube/product which contains no OFF-set minterm (i.e. 0 value)

Prime Implicant (PI, prime): a maximal implicant (i.e. it is contained in no larger implicant)

Essential Prime Implicant (essential): a prime which contains at least one ON-set minterm (i.e. 1 value) which is not contained by any other prime

Sum-of-products (SOP, disjunctive normal form): a sum of products ("AND-OR" 2-level circuit)

Cover: a set of primes (SOP) containing all the ON-set minterms (1 points) of a function

Complete Sum: a cover containing all possible prime implicants of the function

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Review: 2-Level Logic Minimization Problem

The 2-Level Logic Minimization Problem: given a Boolean function f

(i) Find a minimum-cost set of prime implicants which "covers" (i.e. contains) all ON-set minterms -- (… and possibly some DC-set minterms)

Or, equivalently:

(ii) Find a minimum-cost cover F of function f

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2-Level Logic Minimization: Example

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	0	0	1	1
10	0	0	0	0

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2-Level Logic Minimization: Example

Solution #1: All Primes = 5 Products (AND gates)

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	0	0	1	1
10	0	0	0	0

"Complete Sum" = cover containing all prime implicants

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2-Level Logic Minimization: Example

Solution #2: Subset of Primes = 4 Products (AND gates)

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	0	0	1	1
10	0	0	0	0

Locally sub-optimal solution

"Redundant Cover" = can remove a product and still have legal cover

2-Level Logic Minimization: Example

Solution #3: Subset of Primes = 4 Products (AND gates)

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	0	0	1	1
10	0	0	0	0

Locally optimal solution

"Irredundant Cover" (but still globally sub-optimal!)
= cannot remove any product and still have legal cover

2-Level Logic Minimization: Example

Solution #4: Subset of Primes = 3 Products (AND gates)

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	0	0	1	1
10	0	0	0	0

Globally optimal solution

OPTIMAL SOLUTION (also irredundant)

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Exact 2-Level Logic Minimization: Quine-McCluskey (QM) Method

Quine-McCluskey Method: Examples

Example #1: f(A,B,C,D) = m(0,4,5,11,15) + d(2,6,9)

[m = ON-set minterms, d = DC-set minterms]

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	1	0	-
11	0	0	1	1
10	-	-	0	0

Quine-McCluskey Method: Examples

Example #1 (cont.)

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	1	0	-
11	0	0	1	1
10	-	-	0	0

P1 P2 P3 P4

Generate all prime implicants

Quine-McCluskey Method: Examples

Example #1 (cont.)

CD \ AB	00	01	11	10
00	1 ⊛	1	0	0
01	0	1 ⊛	0	-
11	0	0	1 ⊛	1
10	-	-	0	0

P1 P2 P3 P4

⊛ = distinguished minterm

Prime Implicant Table

prime implicants

ON-set minterms	P1	P2	P3	P4
⊛ 0	X			
4	X	X		
⊛ 5		X		
11			X	X
⊛ 15			X	

◯ = essential prime

Approach: remove & save essentials {p1, p2, p3}, and delete intersecting rows … *empty table: nothing left to cover*.

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Quine-McCluskey Method: Examples

Example #2: f(A,B,C) = m(0,1,2,6) + d(5)

[m = ON-set minterms, d = DC-set minterms]

BC \ A	0	1
00	1	0
01	1	-
11	0	0
10	1	1

More complex example: illustrates "table reduction step" using column dominance

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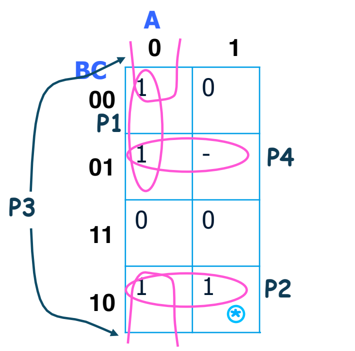
Quine-McCluskey Method: Examples

Example #2: f(A,B,C) = m(0,1,2,6) + d(5)

[m = ON-set minterms, d = DC-set minterms]



⊛ = distinguished minterm

Prime Implicant Table

prime implicants

ON-set minterms	P1	P2	P3	P4
0	X		X	
1	X			X
2		X	X	
⊛ 6		X		

◯ = essential prime

Initial PI Table

Quine-McCluskey Method: Examples

Example #2: f(A,B,C) = m(0,1,2,6) + d(5)

[m = ON-set minterms, d = DC-set minterms]

prime implicants

ON-set minterms	P1	P2	P3	P4
0	X		X	
1	X			X
2		X	X	
⊛ 6		X		

◯ = essential prime

Initial PI Table

prime implicants

ON-set minterms	P1	P3	P4
0	X	X	
1	X		X

Reduced PI Table (a)

Approach: remove & save essential p2, and delete intersecting rows.

Quine-McCluskey Method: Examples

Example #2: f(A,B,C) = m(0,1,2,6) + d(5)

[m = ON-set minterms, d = DC-set minterms]

prime implicants

ON-set minterms	P1	P3	P4
0	X	X	
1	X		X

Reduced PI Table (a)

prime implicants

	P1
0	X
1	X

Reduced PI Table (b)

"Column Dominance":
- column p1 'column-dominates' column p3
- column p1 'column-dominates' column p4

…*delete dominated columns {p3,p4}*

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Quine-McCluskey Method: Examples

Example #2: f(A,B,C) = m(0,1,2,6) + d(5)

[m = ON-set minterms, d = DC-set minterms]

prime implicants

	P1
0	X
1	X

◎ = secondary essential prime

Reduced PI Table (b)

"Secondary Essential Primes":
- column p1 *has now become 'essential'*

Approach: remove & save secondary essential p1, and delete intersecting rows.
… *empty table: nothing left to cover*.

Final solution: {p1,p2}

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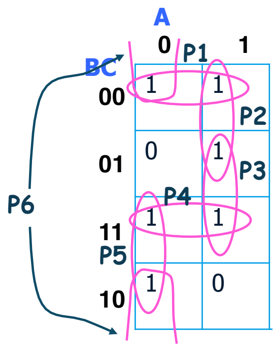
Quine-McCluskey Method: Examples

Example #3: f(A,B,C) = m(0,2,3,4,5,7)

[m = ON-set minterms, d = DC-set minterms]



More complex example: illustrates (i) no reduction possible, and (ii) resulting "cyclic core"

FOR EXACT SOLUTION: can use Petrick's Method (or more advanced techniques)
SEE QUINE-MCCLUSKEY HANDOUT

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