CSEE 4823 Advanced Logic Design
Handout: Lecture #2
9/8/16

Prof. Steven M. Nowick

nowick@cs.columbia.edu

Department of Computer Science (and Elect. Eng.)
Columbia University
New York, NY, USA

Combinational Logic:
Basic Definitions +
2-Level Logic Minimization

Review: Basic Definitions

Literal: a <u>variable</u> (x) or <u>its complement</u> (x')

Product: an "AND" of literals (e.g. xy'z, a'bcd')

Cube: a product (another equivalent name)

Minterm: a product including a literal for <u>every</u> input of the function

Example: If a function has 3 inputs, A/B/C, then A'BC' is a minterm, but A'C is not.

A minterm is also an input vector or combination (i.e. corresponds to a single row in the truth table)

ON-set minterm: minterm where the function is 1

OFF-set minterm: minterm where the function is 0

DC-set minterm: minterm where the function is DC (-)

Implicant: a cube/product which contains <u>no OFF-set minterm </u>(i.e. 0 value)

Prime Implicant (PI, prime): a maximal implicant (i.e. it is contained in no larger implicant)

Essential Prime Implicant (essential): a prime which contains at least one ON-set minterm (i.e. 1 value) which is <u>not</u> contained by any other prime

Sum-of-products (SOP, disjunctive normal form):
a sum of products ("AND-OR" 2-level circuit)

Cover: a set of primes (SOP) <u>containing all the ON-set minterms</u> (1 points) of a function

Complete Sum: a cover containing all possible prime implicants of the function

#3

Review: 2-Level Logic Minimization Problem

The 2-Level Logic Minimization Problem: given a Boolean function f

(i) Find a <u>minimum-cost set of prime implicants</u> which "covers" (i.e. contains)

all ON-set minterms -- (... and possibly some DC-set minterms)

Or, equivalently:

(ii) Find a <u>minimum-cost cover F</u> of function f

#4

2

2-Level Logic Minimization: Example

```
        AB
        00    01    11    10
CD
   00   1     1     0     0

   01   0     1     1     0

   11   0     0     1     1

   10   0     0     0     0
```

2-Level Logic Minimization: Example

Solution #1: All Primes = 5 Products (AND gates)

```
        AB
        00    01    11    10
CD
   00   1     1     0     0

   01   0     1     1     0

   11   0     0     1     1

   10   0     0     0     0
```

"Complete Sum" = cover containing all prime implicants

2-Level Logic Minimization: Example

Solution #2: Subset of Primes = 4 Products (AND gates)

AB

CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	0	0	1	1
10	0	0	0	0

Locally **sub-optimal** solution

"Redundant Cover" = can remove a product and still have legal cover

---

2-Level Logic Minimization: Example

Solution #3: Subset of Primes = 4 Products (AND gates)

AB

CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	0	0	1	1
10	0	0	0	0

Locally **optimal** solution

"Irredundant Cover" (but still globally sub-optimal!)
= cannot remove any product and still have legal cover

2-Level Logic Minimization: Example

Solution #4: Subset of Primes = 3 Products (AND gates)

AB

CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	0	0	1	1
10	0	0	0	0

Globally optimal solution

OPTIMAL SOLUTION (also irredundant)

Exact 2-Level Logic Minimization:
Quine-McCluskey (QM) Method

Quine-McCluskey Method: Examples

Example #1: f(A,B,C,D) = m(0,4,5,11,15) + d(2,6,9)

[m = ON-set minterms,   d = DC-set minterms]

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	1	0	-
11	0	0	1	1
10	-	-	0	0

Quine-McCluskey Method: Examples

Example #1 (cont.)

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	1	0	-
11	0	0	1	1
10	-	-	0	0

P1, P2, P3, P4

Generate all prime implicants

6

Quine-McCluskey Method: Examples

Example #1 (cont.)

Prime Implicant Table

K-map (AB / CD):

CD \ AB	00	01	11	10
00	1 ✪	1	0	0
01	0	1 ✪	0	-
11	0	0	1 ✪	1
10	-	-	0	0

P1, P2, P3, P4 labeled

prime implicants

ON-set minterms	P1	P2	P3	P4
✪ 0	X			
4	X	X		
✪ 5		X		
11			X	X
✪ 15			X	

◯ = essential prime

✪ = distinguished minterm

Approach: remove & save essentials {p1, p2, p3}, and delete intersecting rows … _empty table: nothing left to cover_.

#13

Quine-McCluskey Method: Examples

Example #2: f(A,B,C) = m(0,1,2,6) + d(5)

[m = ON-set minterms, d = DC-set minterms]

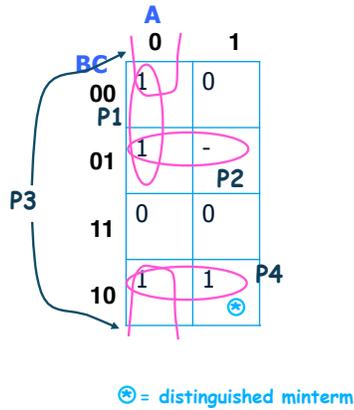| BC \ A	0	1
00	1	0
01	1	-
11	0	0
10	1	1

More complex example: illustrates "table reduction step" using column dominance

#14

7

Quine-McCluskey Method: Examples

Example #2: f(A,B,C) = m(0,1,2,6) + d(5)

[m = ON-set minterms, d = DC-set minterms]

Prime Implicant Table

ON-set minterms	prime implicants			
	P1	(P2)	P3	P4
0	X		X	
1	X			X
2		X	X	
⊛ 6		X		

◯ = essential prime

Initial PI Table

⊛ = distinguished minterm

#15

Quine-McCluskey Method: Examples

Example #2: f(A,B,C) = m(0,1,2,6) + d(5)

[m = ON-set minterms, d = DC-set minterms]

ON-set minterms	prime implicants			
	P1	(P2)	P3	P4
0	X		X	
1	X			X
2		X	X	
⊛ 6		X		

◯ = essential prime

Initial PI Table

ON-set minterms	prime implicants		
	P1	P3	P4
0	X	X	
1	X		X

Reduced PI Table (a)

Approach: remove & save essential p2, and delete intersecting rows.

#16

8

Quine-McCluskey Method: Examples

Example #2: f(A,B,C) = m(0,1,2,6) + d(5)

[m = ON-set minterms, d = DC-set minterms]

prime implicants

	P1	P3	P4
0	X	X	
1	X		X

ON-set minterms

Reduced PI Table (a)

prime implicants

	P1
0	X
1	X

Reduced PI Table (b)

"Column Dominance":
  - column p1 'column-dominates' column p3
  - column p1 'column-dominates' column p4
…*delete dominated columns {p3,p4}*

#17

Quine-McCluskey Method: Examples

Example #2: f(A,B,C) = m(0,1,2,6) + d(5)

[m = ON-set minterms, d = DC-set minterms]

prime implicants

(P1)

	P1
0	X
1	X

◎ = secondary essential prime

Reduced PI Table (b)

"Secondary Essential Primes":
  - column p1 *has now become 'essential'*

Approach: remove & <u>save secondary essential p1</u>, and delete intersecting rows.
… *empty table: nothing left to cover*.

Final solution: {p1,p2}

#18

9

Quine-McCluskey Method: Examples

Example #3: f(A,B,C) = m(0,2,3,4,5,7)

[m = ON-set minterms, d = DC-set minterms]

More complex example: illustrates (i) no reduction possible, and (ii) resulting "cyclic core"

FOR EXACT SOLUTION: can use Petrick's Method (or more advanced techniques)
SEE QUINE-MCCLUSKEY HANDOUT