Question 1

$$
\begin{aligned}
g_1(\alpha \to \beta) &= 1 \text{ if } \alpha \to \beta = \texttt{S -> NP VP}, 0 \text{ otherwise} \\
g_2(\alpha \to \beta) &= 1 \text{ if } \alpha \to \beta = \texttt{N -> dog}, 0 \text{ otherwise} \\
g_3(\alpha \to \beta) &= 1 \text{ if } \alpha \to \beta = \texttt{NP -> NP NP}, 0 \text{ otherwise}
\end{aligned}
$$

Question 2a

Input: a sentence $s = x_1 \ldots x_n$, a PCFG $G = (N, \Sigma, S, R, q)$.
Initialization:
For all $i \in \{1 \ldots n\}$, for all $X \in N$,

$$\pi(i, i, X) = \begin{cases} 0 & \text{if } X \rightarrow x_i \in R \\ -\infty & \text{otherwise} \end{cases}$$

Algorithm:

- For $l = 1 \ldots (n-1)$
  - For $i = 1 \ldots (n-l)$
    - Set $j = i + l$
    - For all $X \in N$, calculate

$$\pi(i, j, X) = \max_{\substack{X \rightarrow Y Z \in R, \\ k \in \{i \ldots (j-1)\}}} \left(v \cdot g(s, X \rightarrow Y Z, i, k, j) + \pi(i, k, Y) + \pi(k+1, j, Z) \right)$$

Output: Return $\pi(1, n, S)$

Question 2b

- Set $v = 0$
- For $t = 1 \dots T, i = 1 \dots n$,
  - $z^{(i)} = \arg\max_{y \in \mathcal{T}(s^{(i)})} \text{score}(y; v)$
  - If $z^{(i)} \neq y^{(i)}$

    $$v = v + f(s^{(i)}, y^{(i)}) - f(s^{(i)}, z^{(i)})$$

    where for any (s, y) where s is a sentence and y is a parse tree,

    $$\begin{aligned} & f(s, y) \\ = & \sum_{X \to Y\ Z, i, k, j} \delta(y, X \to Y\ Z, i, k, j) g(s, X \to Y\ Z, i, k, j) \end{aligned}$$

Question 3a

Set $v_j = 1$ for all $j = 1 \ldots 9$.
It can be verified that this gives the correct tagging for each example.

Question 3b

The model contains features that given a history $\langle x_1 \ldots x_n, i, y_{-1} \rangle$ only consider the current word x_i and the previous tag y_{-1}. Thus we have a model that actually makes the independence assumption

$$\prod_{j=1}^{n} p(y_j | x_1 \ldots x_n, y_1 \ldots y_{j-1}) = \prod_{j=1}^{n} p(y_j | x_j, y_{j-1})$$

We require

$$p(A|a) \times p(B|b, A) \times p(C|c, B) = 1$$

and also

$$p(A|a) \times p(D|b, A) \times p(E|e, D) = 1$$

But $p(B|b, A) + p(D|b, A) = 1$, so this is clearly not possible.