# Question 1

$$
\begin{aligned}
f_1(y_{-1}, x_1 \dots x_N, j, y) &= 1 \text{ if } j = 1 \text{ and } y_j \in \{A, B\} \\
&= 0 \text{ otherwise}
\end{aligned}
$$

$$
\begin{aligned}
f_2(y_{-1}, x_1 \dots x_N, j, y) &= 1 \text{ if } j \geq 2 \text{ and} \\
& \quad \text{NOT}(y_{-1}, y \in \{A, B\} \text{ or } y_{-1}, y \in \{C, D\}) \\
&= 0 \text{ otherwise}
\end{aligned}
$$

$$
\begin{aligned}
f_3(y_{-1}, x_1 \dots x_N, j, y) &= 1 \text{ if odd}(x_j) \text{ and } y_j \in \{A, C\} \\
&= 0 \text{ otherwise}
\end{aligned}
$$

$$
\begin{aligned}
f_4(y_{-1}, x_1 \dots x_N, j, y) &= 1 \text{ if even}(x_j) \text{ and } y_j \in \{B, D\} \\
&= 0 \text{ otherwise}
\end{aligned}
$$

Here odd$(x)$ returns true if $x$ has an odd number of letters.
even$(x)$ returns true if $x$ has an even number of letters.
NOT$(TRUE) = FALSE$ and NOT$(FALSE) = TRUE$.

# Question 2a

Many solutions are possible; one is

$$v = \langle -1, 0, 0, 0 \rangle$$

# Question 2b

With $v = \langle 1, 1, -1, -1 \rangle$, we have the following scores for vectors (a), (b) and (c):

(a) $-1$

(b) $1$

(c) $-2$

The update is then

$$v = v + \langle 0, 0, 1, 1 \rangle - \langle 1, 1, 1, 0 \rangle$$

giving $v = \langle 0, 0, -1, 0 \rangle$.

# Question 2c

We've seen from Question 2b that after the first update, $v = \langle 0, 0, -1, 0 \rangle$. With this value for $v$ we have the following scores for vectors (a), (b) and (c):

(a) $-1$
(b) $-1$
(c) $-1$

Under the tie-breaking rule, (a) is taken as the $\arg\max$. The parameters are updated as follows:

$$v = v + \langle 0, 0, 1, 1 \rangle - \langle 1, 0, 1, 1 \rangle$$

giving $v = \langle -1, 0, -1, 0 \rangle$.
With this value of $v$ it can be verified that the $\arg\max$ returns (c), the correct solution, and the algorithm has converged.

# Question 3

The example $z_i$ chosen at each update to $v$ still satisfies

$$u \cdot f(x_i, y_i) - u \cdot f(x_i, z_i) \geq \delta$$

and

$$||f(x_i, y_i) - f(x_i, z_i)||_2^2 \leq R^2$$

and

$$v^k \cdot (f(x_i, y_i) - f(x_i, z_i)) \leq 0$$

hence the original perceptron proof goes through without any modifications required.