Admin: Rocco OH: Fri 5-7 pm (usual place)

Last time? Applic. of Occam $H_m$:

- Greedy set cover heur:
  - CHF for $K$-sparse disj: \( m \) ex. \( \mathcal{H}_m = \{ \text{all } K\text{-sparse disj} \} \)
  - \( \mathcal{H}_m \text{ - sparse disj} \}

Today: last topic in KV Ch 1 + 2.

Proper vs Improper PAC learning.

\( \mathcal{H} = \mathcal{C} \)

There are natural classes \( \mathcal{C} \) s.t.

- can eff PAC learn \( \mathcal{C} \) w/ improper alg

- (assuming a standard hardness assump. from \( \mathcal{C} \)),
  no eff alg can properly PAC learn \( \mathcal{C} \).

\( \mathcal{C} = 3\text{-term ONF} \)

Next: Chap 3 KV

Back up. Recall: Have an eff proper PAC alg for
Q: what about $C = \text{all 3-term ONFs over } \{0,1\}^n$?

$(f = T_1 \lor T_2 \lor T_3)$, each $T_i$ is a conj.)

Recall: de Morgan's law:

$ab \land cd \equiv (a \lor c) \land (a \lor d) \land (b \lor c) \land (b \lor d)$

This gives us that

FACT: Let $f$ be a 3-term ONF. Then $f$ is equiv. to a 3-CNF. (AND of clauses, each of OR, length $\leq 3$)

$ab\lor cd\lor uv\land gh \equiv (a \lor c) \land (a \lor d) \land (b \lor c) \land (b \lor d)$

Claim: $C = \text{all 3-term ONF over } \{0,1\}^n$ is eff. PAC learnable using $\mathcal{H} = \text{all 3-CNFs}$.

PF: Reduce to learning conj over $\{0,1\}^n$, $N = O(n^3)$. There are $N = O(n^3)$ many length-3 clauses over $\{0,1\}^n$: $\leq 8n^3$
\[(x_1 \lor x_2 \lor x_3), \ldots, (\overline{x_1} \lor x_4 \lor x_{22}) \ldots\]

Introduce new vars \(y_1, \ldots, y_n\), one for each clause.

Given an \(x = (x_1, \ldots, x_n) \in \{0,1\}^n\), transform to

\(y = (y_1, \ldots, y_n) \in \{0,1\}^n\). Target \(c(x) = c(y)\), \(c\) is a conjunctor.

Run PAC alg for conj over \(\{0,1\}^n\), get hyp \(h(y)\)
which is a conj over \(y_1, \ldots, y_n\), this is a 3-CNF over \(x_1, \ldots, x_n\).

Time, s.c. of \(O(1) \cdot \frac{(N + 1/n^{1/3})}{\epsilon} \cdot 0(n^3)\).

Efficient.

\(\phi\over \{0,1\}^n\) weird dist. induced by \(\phi\over \{0,1\}^n\), handles any \(\Theta^n\). \(\Box\)

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Not a coin. that alg non-proper.

Main result:

**Thm:** If there is an eff alg to PAC learn \(\mathcal{E} = \{3\text{-term ONF over } \{0,1\}\}^n\) using \(\mathcal{H}\in\mathcal{E}\) (proper alg).

then there's an eff randomized alg. to solve \(\text{(poly(n)-time on } n\text{-node graphs)}\)

**GRAPH-3-COLORABILITY** problem. (so \(\text{NP \subseteq RP}\))

(can toss coins \(\forall \) graphs \(G\) gives right ans.\ w\ probability \(> 99\%\) \(\forall G\) \(> 1/2^n\).

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**3COL:** Input: graph \(G\) (undirected)
Qi: Can you color nodes of $G$ s.t. only use 3 colors, & no edge has both endpoints same color?

3COL is NP-complete.
- widely believed no poly$(n)$-time alg.
  $\$1M. Best alg: $2^{\Theta(n)}$ on $n$-node graphs.

**Proof of Thm**

Hi level arg: we’ll give a mapping $f$

- $G$: graph
- $n$: node

$G$: graph $G$

![](image)

- $f$: eff. computable poly$(n)$
- $S$: set of 5

**All graphs $G$**:

$\left( G \text{ is 3-colorable} \right) \iff \left( S \text{ is consistent w/ some 3-term DNF} \right)$

If we have such an $f$. Let’s prove Thm.
We have: \textbf{eff $\text{PAC alg}$ for 3-term ONEs of}

We give: \underline{eff rand alg for 3-col.}

Here it is: Input is $n$-node $G$, $|S| \leq \text{poly}(n)$

1. Run \textbf{f} on $G$ to get $S = S^+ \cup S^-$.
 Define $D$ to be unif dist. over these $|S| + \text{ex.}$
 Define $\varepsilon = \frac{1}{2|S|}$. Define $\delta = \frac{1}{100}$

2. Runs proper PAC alg on $E \times (D)$ w/ $\varepsilon, \delta$ as params. \textbf{[poly(n) time: ]}
 Yields 3-term $\text{ONF hyp h. (proper) }$

3. Evaluate $h$ on each ex. in $S$. \textbf{(eff. eval!!)}
 If $h$ cons w/ $S$, output "Yes, $G$ is 3-colorable"
 O/w, "No, $G$ not 3-col"

Works? Yes.

• $S$ ps $G$ is 3-colorable. Then $S$ cons w/ some 3-term $\text{ONF}$. So $\text{PAC alg}$ on $D$ must w/ prob. $\geq 0.99$ output an $\varepsilon$-acc hyp. $\varepsilon = \frac{1}{2|S|}$. So hyp must be cons w/ all of $S$.
 And alg will output.

• Next time: $G$ not 3-col \textbf{alg. outputs "no"}