GRAPH THEORY W4203 FINAL EXAM

open book

SOLUTIONS

____
your name

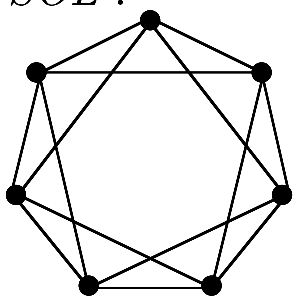
PROBLEM	POSSIBLE	SCORE[1]
1	55	
2	45	
3	20	
4	30	
TOTAL	150	

NB. Partial credit is awarded for meritorious work, even if there are minor mistakes or gaps.

[1] An example of the Reasonable Person Principle: **A reasonable person expects to lose almost all of the credit for failing to explain an answer.** Neglect to supply explanations at your own risk.

1a. (10) Draw the graph $circ(7:1,2)$, and write the cycle index polynomial for its vertex automorphism group.

$SOL:$



$$\frac{1}{14}\left\{{t_1}^7 + 6t_7 + 7t_1{t_2}^3\right\}$$

1b. (5) Calculate the number of non-equivalent vertex-(≤2)-colorings.

$$SOL: \frac{1}{14}\left\{2^7 + 6\cdot 2 + 7\cdot 2\cdot 2^3\right\} = \frac{252}{14} = 18$$

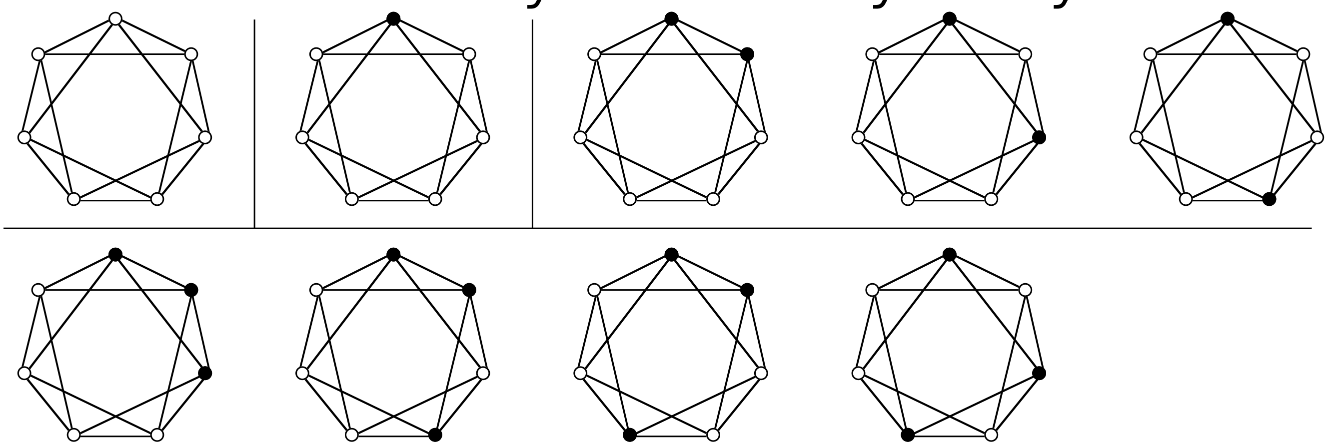
1c. (15) Calculate the Polya inventory for the vertex-(≤2)-colorings.

$SOL:$

	w^7	w^6b	w^5b^2	w^4b^3	w^3b^4	w^2b^5	wb^6	b^7
${t_1}^7$	1	7	21	35				
$6t_7$	6	0	0	0				
$7t_1{t_2}^3$	7	7	21	21				
	14	14	42	56	56	42	14	14
$\div 14$	1	1	3	4	4	3	1	1

1d. (9) For each graph-coloring type counted in parts (b) and (c), draw a representative graph. (You can use symmetry to reduce the number of cases.)

$SOL:$ double this list by black-white symmetry



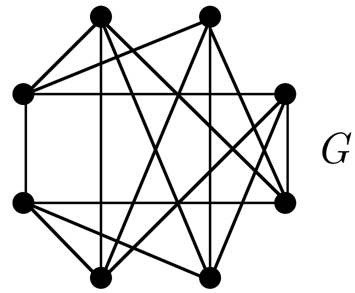
1e. (12) Write the cycle index polynomial for its edge automorphism group.

$$SOL: \frac{1}{14}\left\{t_1^{\,14} + 6t_7^{\,2} + 7t_1^{\,2}t_2^{\,6}\right\}$$

1f. (4) Calculate the number of non-equivalent edge-(≤2)-colorings.

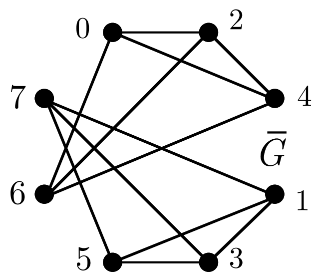
$$SOL: \frac{1}{14}\left\{2^{14} + 6\cdot 2^2 + 7\cdot 2^2\cdot 2^6\right\} = \frac{18200}{14} = 1300$$

2. (45 pts) This problem is concerned with designing a 1-vertex voltage graph specifying the graph G.



2a. (5) Draw the edge complement $\bar{G}$.

$SOL:$

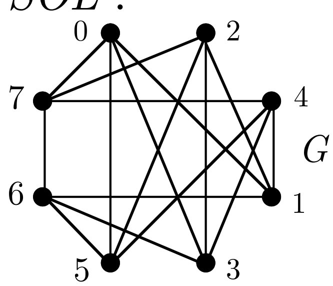


2b. (10) Determine a circulant graph isomorphic to $\bar{G}$, and label the vertices of your drawing of $\bar{G}$ accordingly.

$SOL: circ(8:2,4)$

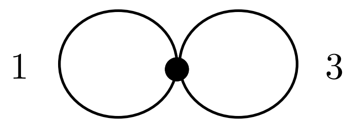
2c. (5) Transfer your vertex labelling of $\bar{G}$ to the given drawing of G.

$SOL:$



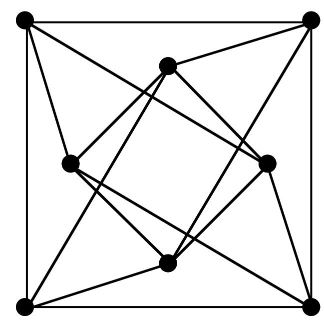
2d. (10) Draw a 1-vertex voltage graph for G that is consistent with your answers to (b) and (c).

$SOL:$ It follows that $G \cong circ(8:1,3)$.



Z_8

2e. (10) Decide whether the following graph is isomorphic to G. (Give a proof.)



$SOL:$ Yes, its edge-complement is $2K_4$. Of course, this implies that $G \cong K_{4,4}$.

3a. (15) Write the face-boundary walks of the imbedding specified by the following rotation system.

$u.$	$a+$	$g+$	$f-$
$v.$	$a-$	$b+$	$h+$
$w.$	$b-$	$c+$	$i+$
$x.$	$c-$	$d+$	$g-$
$y.$	$d-$	$e+$	$h-$
$z.$	$e-$	$f+$	$i-$

$SOL:$ (a+ b+ c+ d+ e+ f+)
(a- g+ c- i+ e- h-)
(b- h+ d- g- f- i-)

3b. (5) In what surface is the graph imbedded?
$SOL:$ V-E+F = 6-9+3 = 0. Therefore, the torus S_1.

4a. (15) Show algebraically that the minimum genus of the graph $circ(11:1,3)$ is at least 1.

$SOL:$ V=11, F=22

Girth ≥ 4, since the equation $x+y+z=0 \bmod 11$ has no solutions with $x,y,z \in \left\{\pm 1, \pm 3\right\}$. Thus, $F \leq \frac{44}{4} = 11$.

Thus, 2-2g = V-E+F ≤ 11-22+11 = 0, and g≥1.

4b. (15) Design an imbedded voltage graph that specifies an imbedding of $circ(11:1,3)$ in the torus.

$SOL:$