An example of the Reasonable Person Principle: A reasonable person expects to lose credit for failing to explain an answer. Neglect to supply explanations at your own risk.
1. (16) The **diameter** of a graph $G$ is the maximum distance between any two vertices, taken over all pairs in the graph.

1a (4). Find the diameter of the cartesian product $P_m \times P_n$ of two path graphs.

1b (4). Find the diameter of the cartesian product $C_m \times C_n$ of two cycle graphs.

1c (8). Calculate the diameter of the Möbius ladder $ML_n$. 


2. (28) The graph $C(i, j) \mod n$ has the numbers 0, ..., n-1 as its vertices. There is an edge between each pair of numbers whose difference is $\pm i$ or $\pm j \mod n$.

2a (12). In each of the graphs $C(1,3) \mod 16$, $C(1,5) \mod 16$, and $C(1,7) \mod 16$, list the vertices at distance 2 from vertex 0.

2b (4). Use (2a) to prove that $C(1,3) \mod 16$ and $C(1,7) \mod 16$ are non-isomorphic.
2c (12). Show that the graphs $C(1,3) \mod 16$ and $C(1,5) \mod 16$ are isomorphic. (Construct a vertex bijection that preserves adjacency.)
3. (20) Prove that the number of spanning trees in the wheel $W_5$ equals 45. (Hint: partition into cases.)
4. (24) Suppose that graph $G$ has degree sequence $d_1, L, d_n$.

4a (8). How many edges does the line graph $L(G)$ have?

4b (8). Draw a simple connected graph $G$ whose line graph $L(G)$ has the degree sequence 1122233.

4c (8). Draw a simple connected graph $H$ whose line graph $L(H)$ has the degree sequence 1122233, such that $H$ is NOT isomorphic to the graph $G$ of part (b).
5 (12). Decide whether the following graph is Hamiltonian.