Computer Graphics (Fall 2008)
COMS 4160, Lecture 22: Global Illumination
http://www.cs.columbia.edu/~cs4160

Illumination Models
So far considered mainly local illumination
- Light directly from light sources to surface
Global Illumination: multiple bounces
- Already ray tracing: reflections/refractions

Overview of lecture
- Theory for all methods (ray trace, radiosity)
- We derive Rendering Equation [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases

Fairly theoretical lecture (but important). Not well covered in any of the textbooks. Closest are 2.6.2 in Cohen and Wallace handout (but uses slightly different notation, argument [swaps x, x’ among other things]) and 19.2 in Shirley (different notation, omits emission, but has a reasonably good intuitive discussion that we somewhat follow).

Outline
- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
- Surface Parameterization (Standard Form)
Outline

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The material in this part of the lecture is fairly advanced and not covered in any of the texts. The slides should be fairly complete. This section is fairly short, and I hope some of you will get some insight into solutions for general global illumination.

Rendering Equation as Integral Equation

\[ L_v(x, \omega_v) = L_v(x, \omega_v) + \int L_v(x', -\omega_v) f(x', \omega_v, \omega_v) \cos \theta_d \omega_d \]

Reflected Light (Output Image)
Emission
Reflected Light
BRDF
Cosine of Incident angle

UNKNOWN
KNOWN
UNKNOWN
KNOWN
KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

\[ I(u) = e(u) + \int I(v) K(u, v)dv \]

Kernel of equation

Solution Techniques

All global illumination methods try to solve (approximations of) the rendering equation

- Too hard for analytic solution: numerical
  General theory of solving integral equations

Radiosity (next lecture; usually diffuse surfaces)

- General class numerical finite element methods
  (divide surfaces in scene into a finite set elements or patches)
  - Set up linear system (matrix) of simultaneous equations
  - Solve iteratively

Ray Tracing and extensions

- General class numerical Monte Carlo methods
- Approximate set of all paths of light in scene

\[ L = E + KL \]

Can be discretized to a simple matrix equation [or system of simultaneous linear equations]
(L, E are vectors, K is the light transport matrix)

Ray Tracing

\[ L = E + KE + K^2E + K^3E + ... \]

Emission directly
From light sources
Direct Illumination on surfaces
Global Illumination (One bounce indirect) [Mirrors, Refraction]
(Two bounce indirect) [Caustics etc]
Ray Tracing

\[ L = E + KE + K^2E + K^3E + \ldots \]

- Emission directly from light sources
- Direct illumination on surfaces
- Global illumination (One bounce indirect) [Mirrors, Refraction]
- (Two bounce indirect) [Caustics etc]

Outline

- Reflectance Equation (review)
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OpenGL Shading

Outline

Reflectance Equation (review)

Global Illumination

Rendering Equation

As a general Integral Equation and Operator

Approximations (Ray Tracing, Radiosity)

Surface Parameterization (Standard Form)

Page 461 of Shirley is reasonably close to this part of lecture, although it uses different notation. See also pages 38 and 39 in handout, which may have a clearer explanation of the ideas.

Rendering Equation

Dominant surfaces (interreflection)

\[ L_i(x, \omega_i) = L_i(x, \omega_i) + \int_\Omega L_i(x', -\omega_i) f(x, \omega_i, \omega_o) \cos \theta_i d\omega_o \]

- Reflected Light (Output Image)
- Emission
- Reflected Light
- BRDF
- Cone of Incident angle

Change of Variables

\[ L_i(x, \omega_i) = L_i(x, \omega_i) + \int_\Omega L_i(x', -\omega_i) f(x, \omega_i, \omega_o) \cos \theta_i d\omega_o \]

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

\[ d\omega_i = \frac{dA'}{|x - x'|^2} \]

Rendering Equation: Standard Form

\[ L_i(x, \omega_i) = L_i(x, \omega_i) + \int_\Omega L_i(x', -\omega_i) f(x, \omega_i, \omega_o) \cos \theta_i d\omega_o \]

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Change of Variables

\[ L_i(x, \omega_i) = L_i(x, \omega_i) + \int_\Omega L_i(x', -\omega_i) f(x, \omega_i, \omega_o) \cos \theta_i d\omega_o \]

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

\[ d\omega_i = \frac{dA'}{|x - x'|^2} \]

Same as equation 19.3 in Shirley, except he has no emission, slightly different notation.
Overview

- *Theory* for all methods (ray trace, radiosity)
- We derive *Rendering Equation* [Kajiya 86]
  - Major theoretical development in field
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