

Computer Graphics (Spring 2008)

COMS 4160, Lecture 7: Curves 2
<http://www.cs.columbia.edu/~cs4160>

To Do

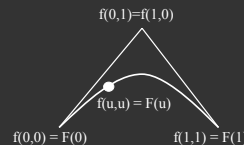
- Start on HW 2 (cannot be done at last moment)
 This (and previous) lecture should have all information need
- Start thinking about partners for HW 3 and HW 4
 - Remember though, that HW2 is done individually
 - Your submission of HW 2 must include partner for HW 3

Outline of Unit

- Bezier curves (last time)
- deCasteljau algorithm, explicit, matrix (last time)
- Polar form labeling (blossoms)
- B-spline curves
- Not well covered in textbooks (especially as taught here). Main reference will be lecture notes. If you do want a printed ref, handouts from CAGD, Seidel

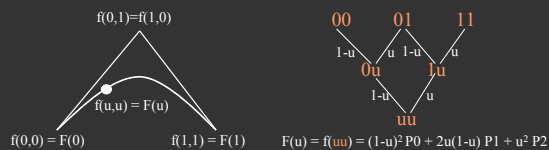
Idea of Blossoms/Polar Forms

- (Optional) Labeling trick for control points and intermediate deCasteljau points that makes thing intuitive
- E.g. quadratic Bezier curve $F(u)$
 - Define auxiliary function $f(u_1, u_2)$ [number of args = degree]
 - Points on curve simply have $u_1 = u_2$ so that $F(u) = f(u, u)$
 - And we can label control points and deCasteljau points not on curve with appropriate values of (u_1, u_2)



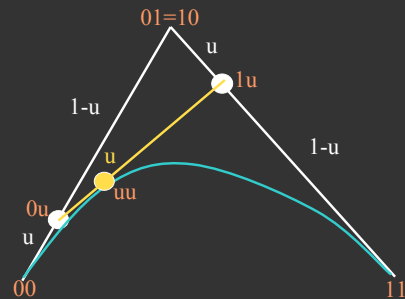
Idea of Blossoms/Polar Forms

- Points on curve simply have $u_1 = u_2$ so that $F(u) = f(u, u)$
- f is symmetric $f(0,1) = f(1,0)$
- Only interpolate linearly between points with one arg different
 - $f(0,u) = (1-u)f(0,0) + uf(0,1)$ Here, interpolate $f(0,0)$ and $f(0,1) = f(1,0)$

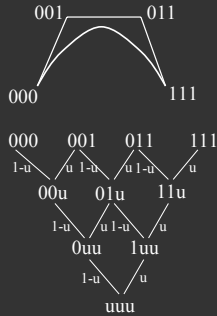


$$F(u) = f(uu) = (1-u)^2 P_0 + 2u(1-u) P_1 + u^2 P_2$$

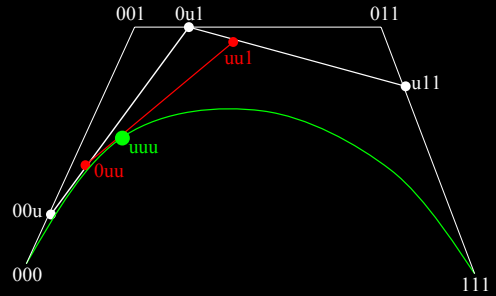
Geometric interpretation: Quadratic



Polar Forms: Cubic Bezier Curve



Geometric Interpretation: Cubic

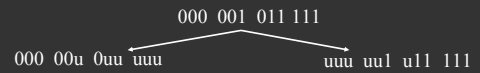


Why Polar Forms?

- Simple mnemonic: which points to interpolate and how in deCasteljau algorithm
- Easy to see how to subdivide Bezier curve (next) which is useful for drawing recursively
- Generalizes to arbitrary spline curves (just label control points correctly instead of 00 01 11 for Bezier)
- Easy for many analyses (beyond scope of course)

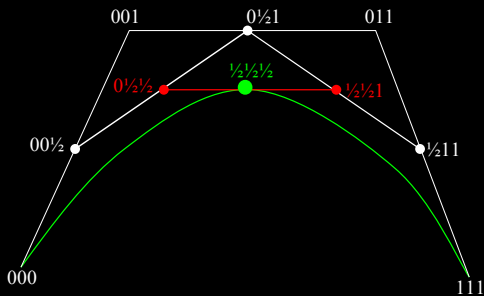
Subdividing Bezier Curves

- Drawing: Subdivide into halves ($u = \frac{1}{2}$) Demo: [hw2.exe](#)
- Recursively draw each piece
 - At some tolerance, draw control polygon
 - Trivial for Bezier curves (from deCasteljau algorithm); hence widely used for drawing

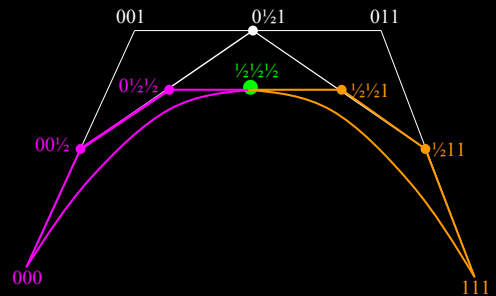


- Why specific labels/ control points on left/right?
- How do they follow from deCasteljau?

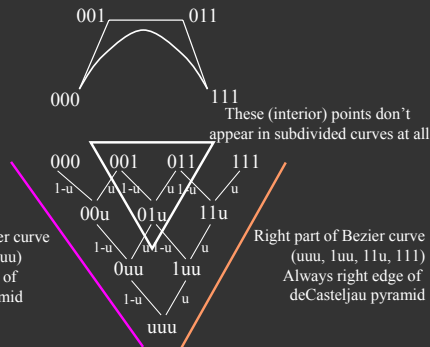
Geometrically



Geometrically



Subdivision in deCasteljau diagram



Summary for HW 2

- Bezier2 (Bezier discussed last time)
- Given arbitrary degree Bezier curve, recursively subdivide for some levels, then draw control polygon [hw2.exe](#)
- Generate deCasteljau diagram; recursively call a routine with left edge and right edge of this diagram
- You are given some code structure; you essentially just need to compute appropriate control points for left, right

DeCasteljau: Recursive Subdivision

Input: Control points C_i with $0 \leq i \leq n$ where n is the degree.
Output: L_i, R_i for left and right control points in recursion.

```

1 for (level = n ; level >= 0 ; level --) {
2   if (level == n) { // Initial control points
3     for (i = 0 ; i <= n ; i++) p_i^level = C_i ; continue ; }
4   for (i = 0 ; i <= level ; i++)
5     p_i^level = 1/2 * (p_i^level+1 + p_{i+1}^level+1) ;
6 }
7 for (i = 0 ; i <= n ; i++) L_i = p_0^i ; R_i = p_i^i ;

```

- DeCasteljau (from last lecture) for midpoint
- Followed by recursive calls using left, right parts

Outline of Unit

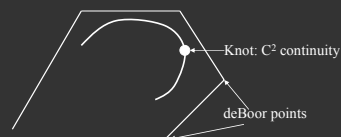
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Bezier: Disadvantages

- Single piece, no local control (move a control point, whole curve changes) [hw2.exe](#)
- Complex shapes: can be very high degree, difficult
- In practice, combine many Bezier curve segments
 - But only position continuous at join since Bezier curves interpolate end-points (which match at segment boundaries)
 - Unpleasant derivative (slope) discontinuities at end-points
 - Can you see why this is an issue?

B-Splines

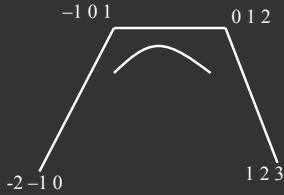
- Cubic B-splines have C^2 continuity, local control
- 4 segments / control point, 4 control points/segment
- Knots where two segments join: Knotvector
- Knotvector uniform/non-uniform (we only consider uniform cubic B-splines, not general NURBS)



Demo: [hw2.exe](#)

Polar Forms: Cubic Bspline Curve

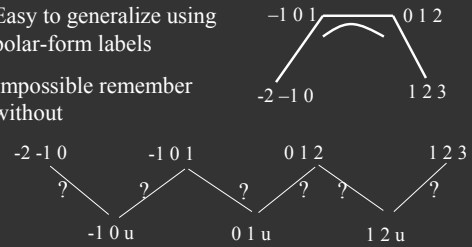
- Labeling little different from in Bezier curve
- No interpolation of end-points like in Bezier
- Advantage of polar forms: easy to generalize



Uniform knot vector:
-2, -1, 0, 1, 2, 3
Labels correspond to this

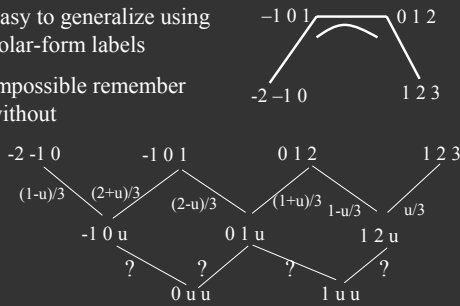
deCasteljau: Cubic B-Splines

- Easy to generalize using polar-form labels
- Impossible remember without



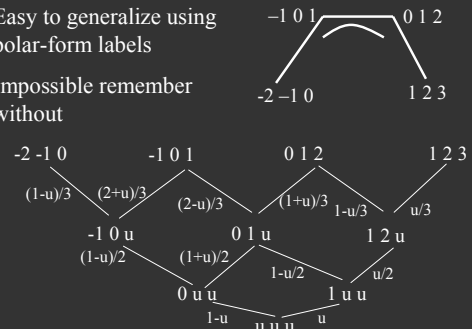
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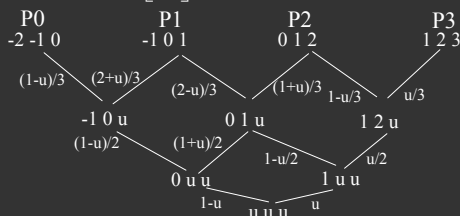
deCasteljau: Cubic B-Splines

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Explicit Formula (derive as exercise)

$$F(u) = [u^3 \ u^2 \ u \ 1] M \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} \quad M = \frac{1}{6} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$



Summary of HW 2

- BSpline Demo [hw2.exe](#)
- Arbitrary number of control points / segments
 - Do nothing till 4 control points (see demo)
 - Number of segments = # cpts - 3
- Segment A will have control pts A, A+1, A+2, A+3
- Evaluate Bspline for each segment using 4 control points (at some number of locations, connect lines)
- Use either deCasteljau algorithm (like Bezier) or explicit form [matrix formula on previous slide]
- Questions?