

## Computer Graphics (Spring 2008)

COMS 4160, Lecture 22: Global Illumination  
<http://www.cs.columbia.edu/~cs4160>

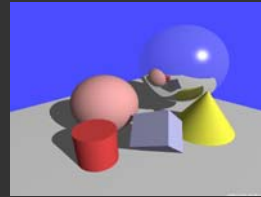
## Illumination Models

So far considered mainly local illumination

- Light directly from light sources to surface

Global Illumination: multiple bounces

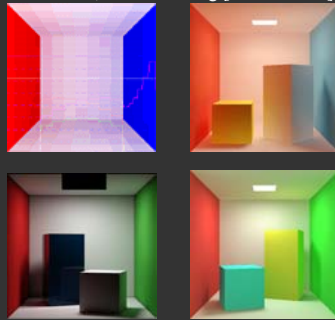
- Already ray tracing: reflections/refractions



Some images courtesy Henrik Wann Jensen

## Global Illumination

Diffuse interreflection, color bleeding [Cornell Box]



## Global Illumination

Caustics: Focusing through specular surface



Major research effort in 80s, 90s till today

## Overview of lecture

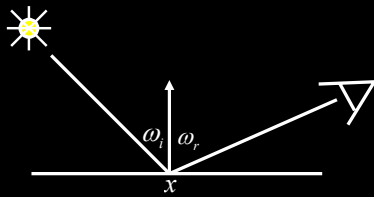
- *Theory* for all methods (ray trace, radiosity)
- We derive *Rendering Equation* [Kajiya 86]
  - Major theoretical development in field
  - Unifying framework for all global illumination
- Discuss existing approaches as special cases

Fairly theoretical lecture (but important). Not well covered in any of the textbooks. Closest are 2.6.2 in Cohen and Wallace handout (but uses slightly different notation, argument [swaps  $x$ ,  $x'$  among other things]) and 19.2 in Shirley (different notation, omits emission, but has a reasonably good intuitive discussion that we somewhat follow).

## Outline

- *Reflectance Equation (review)*
- *Global Illumination*
- *Rendering Equation*
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
- Surface Parameterization (Standard Form)

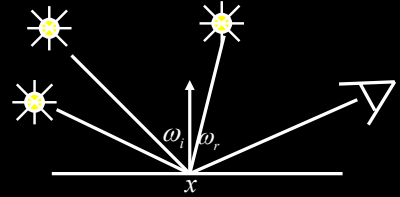
### Reflectance Equation (review)



$$L_r(x, \omega_r) = L_e(x, \omega_r) + L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i \cdot n)$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle
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### Reflectance Equation (review)

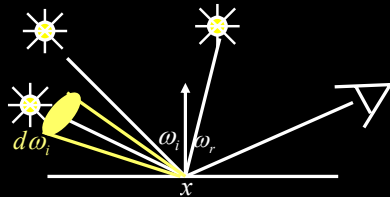


Sum over all light sources

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \sum L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i \cdot n)$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle
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### Reflectance Equation (review)



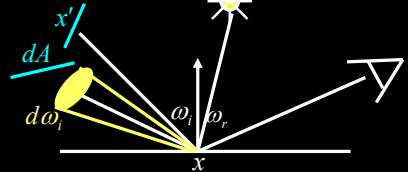
Replace sum with integral

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_i(x, \omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle
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### Global Illumination

Surfaces (interreflection)



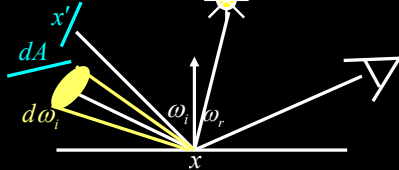
$$\omega_i \sim x' - x$$

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light (from surface)	BRDF	Cosine of Incident angle
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### Rendering Equation

Surfaces (interreflection)



$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

### Rendering Equation (Kajiya 86)



Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

## Outline

- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
- *As a general Integral Equation and Operator*
- *Approximations (Ray Tracing, Radiosity)*
- Surface Parameterization (Standard Form)

The material in this part of the lecture is fairly advanced and not covered in any of the texts. The slides should be fairly complete. This section is fairly short, and I hope some of you will get some insight into solutions for general global illumination

## Rendering Equation as Integral Equation

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_r) f(x, \omega_r, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v) K(u, v) dv$$

Kernel of equation

## Linear Operator Equation

$$l(u) = e(u) + \int l(v) K(u, v) dv$$

Kernel of equation  
Light Transport Operator

$$L = E + KL$$

Can be discretized to a simple matrix equation [or system of simultaneous linear equations] (L, E are vectors, K is the light transport matrix)

## Solution Techniques

All global illumination methods try to solve (approximations of) the rendering equation

- Too hard for analytic solution: numerical methods
- General theory of solving integral equations

Radiosity (next lecture; usually diffuse surfaces)

- General class numerical *finite element* methods (divide surfaces in scene into a finite set elements or patches)

## Ray Tracing and extensions

- General class numerical *Monte Carlo* methods
- Approximate set of all paths of light in scene

$$L = E + KL$$

$$IL - KL = E$$

$$(I - K)L = E$$

$$L = (I - K)^{-1} E$$

Binomial Theorem

$$L = (I + K + K^2 + K^3 + \dots) E$$

$$L = E + KE + K^2E + K^3E + \dots$$

## Ray Tracing

$$L = E + KE + K^2E + K^3E + \dots$$

Emission directly From light sources

Direct Illumination on surfaces

Global Illumination (One bounce indirect) [Mirrors, Refraction]

(Two bounce indirect) [Caustics etc]

## Ray Tracing

$$L = E + KE + K^2E + K^3E + \dots$$

↓

Emission directly  
From light sources

↓

OpenGL Shading

↓

Direct Illumination  
on surfaces

↓

Global Illumination  
(One bounce indirect)  
[Mirrors, Refraction]

↓

(Two bounce indirect)  
[Caustics etc]

## Outline

- Reflectance Equation (review)
- Global Illumination
- Rendering Equation
- As a general Integral Equation and Operator
- Approximations (Ray Tracing, Radiosity)
- *Surface Parameterization (Standard Form)*

Page 461 of Shirley is reasonably close to this part of lecture, although it uses different notation. See also pages 38 and 39 in handout, which may have a clearer explanation of the ideas.

## Rendering Equation

Surfaces (interreflection)

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

## Change of Variables

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$d\omega_i = \frac{dA' \cos \theta_o}{|x - x'|^2}$$

## Change of Variables

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\text{all } x' \text{ visible to } x} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \frac{\cos \theta_i \cos \theta_o}{|x - x'|^2} dA'$$

$$d\omega_i = \frac{dA' \cos \theta_o}{|x - x'|^2}$$

$$G(x, x') = G(x', x) = \frac{\cos \theta_i \cos \theta_o}{|x - x'|^2}$$

## Rendering Equation: Standard Form

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Integral over angles sometimes insufficient. Write integral in terms of surface radiance only (change of variables)

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\text{all surfaces } x'} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \frac{\cos \theta_i \cos \theta_o}{|x - x'|^2} dA'$$

Domain integral awkward. Introduce binary visibility fn V

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\text{all surfaces } x'} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) G(x, x') V(x, x') dA'$$

Same as equation 2.52 Cohen Wallace. It swaps primed And unprimed, omits angular args of BRDF, - sign.  
Same as equation above 19.3 in Shirley, except he has no emission, slightly diff. notation

$$d\omega_i = \frac{dA' \cos \theta_o}{|x - x'|^2}$$

$$G(x, x') = G(x', x) = \frac{\cos \theta_i \cos \theta_o}{|x - x'|^2}$$

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## Overview

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- *Theory* for all methods (ray trace, radiosity)
- We derive *Rendering Equation* [Kajiya 86]
  - Major theoretical development in field
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- Discuss existing approaches as special cases