Your Name (2 pts for LEGIBLY PRINTING your name on this line)

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Total 100

SUGGESTION: Do the EASIEST problems first!

HINT: Some of the solution methods involve pre-college math as well as new methods from this class.

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1 An example of the Reasonable Person Principle: A reasonable student expects to lose a lot of credit for neglecting to EXPLAIN an answer. Omit explanations at your own risk.
1 (16 pts). Consider the following recurrence.

\[ g_0 = 1; \quad g_1 = 1; \quad g_2 = 2; \]
\[ g_n = g_{n-1} + g_{n-2} + g_{n-3} \]

Prove for all \( n \geq 0 \) that \( g_n \geq 1.8^{n-1} \).
2 (16 pts). Prove that \( n \)¢ postage for any \( n \geq 14 \) can be formed from stamps of the denominations 5¢, 7¢, and 11¢.
3 (16 pts). Prove the following inequality for $1 \leq k < n$.

\[
\binom{n}{k}^2 - \binom{n-1}{k-1}^2 \geq \binom{n-1}{k}^2 + \binom{n-1}{k-1}\binom{n-1}{k}
\]
4 (16 pts). Count the number of solutions of the equation

\[ x + y + z = 24 \]

with integers \( x \geq 2, \ y \geq 3, \) and \( z \geq 4 \). Hint: this is equivalent to counting the number of ways to choose 24 pieces of fruit from large supplies of apples, bananas, and coconuts with at least 2 apples, at least 3 bananas, and at least 4 coconuts.
5 (18 pts). Five fair dice are rolled.

5a (4). What is the probability of no fours?

5b (4). What is the probability of exactly one four.

5c (6). What is the probability of exactly two fours?

5d(4). What is the probability of at least two fours?
6 (16 pts). Solve the following recurrence.

\[ b_0 = 0; \quad b_1 = 1; \]
\[ b_n = 8b_{n-1} - 15b_{n-2} \]