PROOFS OF PROXIMITY FOR DISTRIBUTION TESTING

(Distribution testing – now with proofs!)

Tom Gur (UC Berkeley)

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FOCS 2017 Workshop: Frontiers in Distribution Testing

Joint work with **Alessandro Chiesa** (UC Berkeley)

PROOFS OF PROXIMITY?

WHAT AND WHY?

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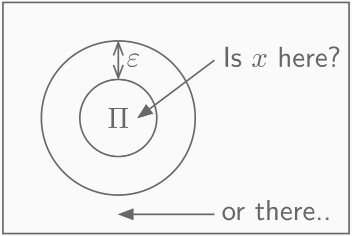
Application: delegation of computation

For example, consider interactive proofs of proximity [RVW13]



THE STANDARD SETTING: FUNCTIONS

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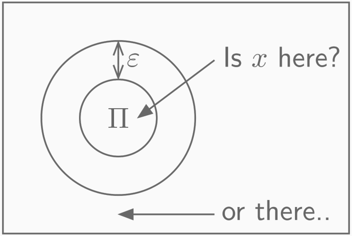

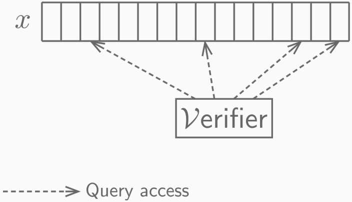

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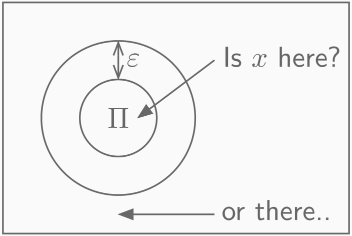

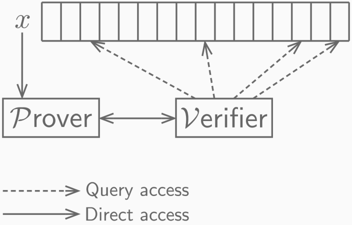

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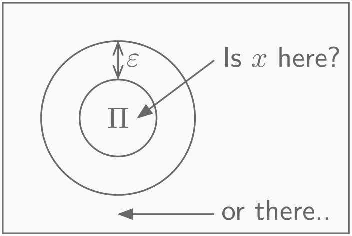

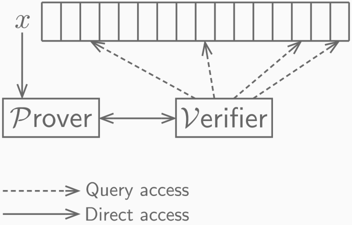

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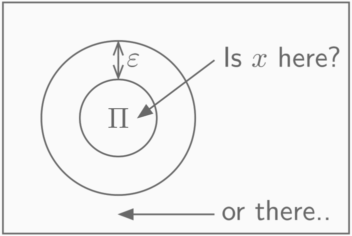

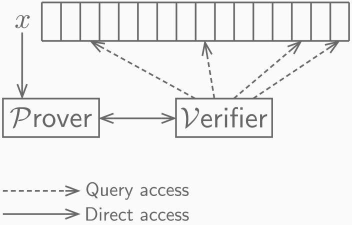

- If $x \in \Pi$, $\exists$ prover strategy P such that $\langle P(x), V^x \rangle(\varepsilon) = 1$
- If x is ε-far from Π, $\forall$ prover strategy $\langle P^*, V^x \rangle(\varepsilon) = 0$ w.h.p.

AN EASY EXAMPLE

How can proofs help testing algorithms?

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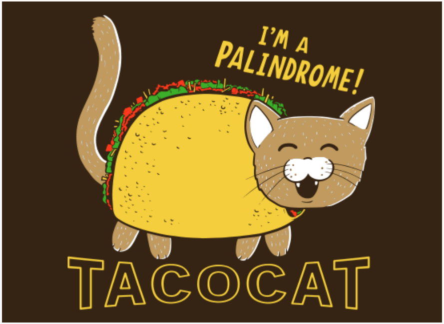
Tell us **where** one palindrome ends and the other starts! [FGL14]



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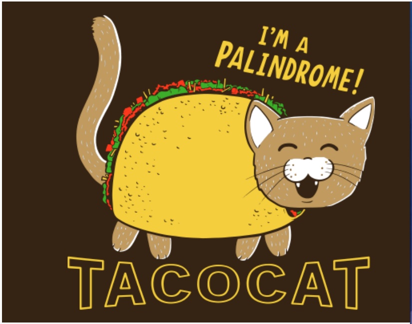

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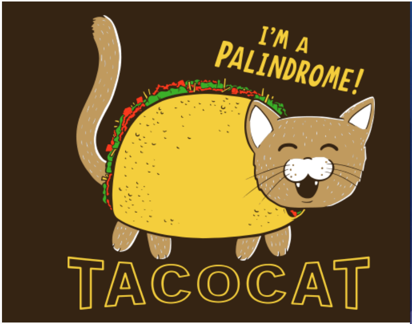

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“Concatenated palindromes” requires $\Omega(\sqrt{n})$ queries [AKNS01]

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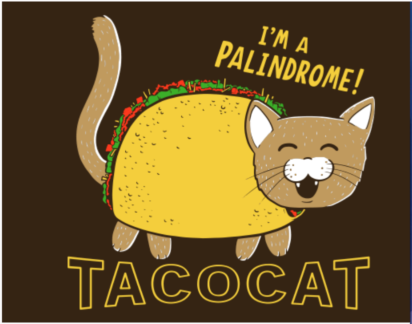

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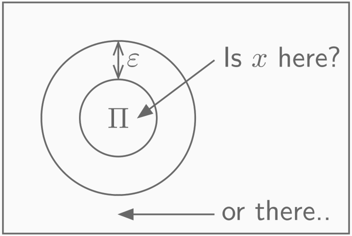
However, a tiny proof of length $\log(n)$ reduces the queries to $O(1)$

NOW FOR DISTRIBUTIONS!



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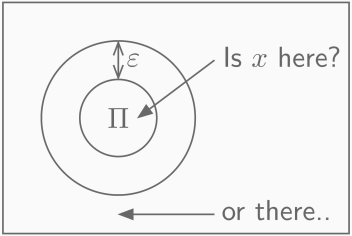

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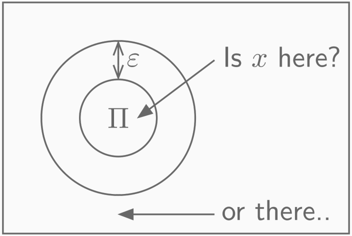

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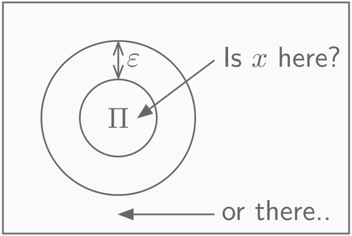

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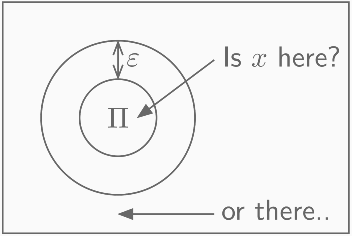

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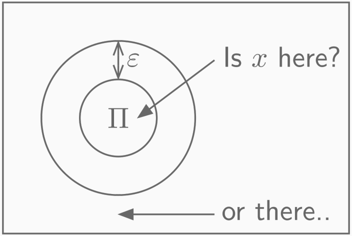

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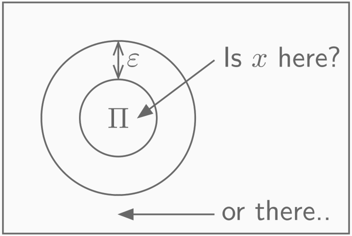

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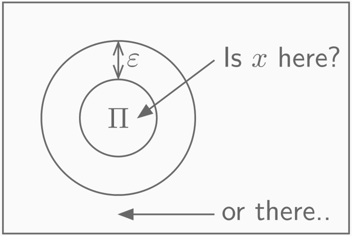

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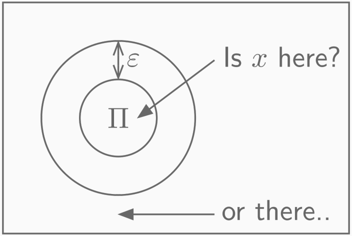

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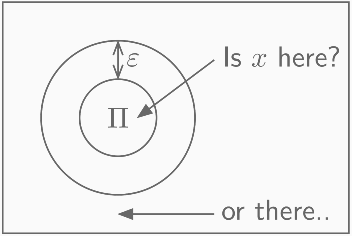

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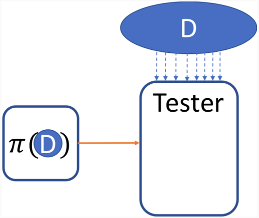
Sample access to D

Decide with high probability:

$$\text{Is } D \in \Pi, \text{ or } \delta_{\mathrm{TV}}(D, \Pi) > \varepsilon?$$

NP distribution testers



Deterministic algorithm T with

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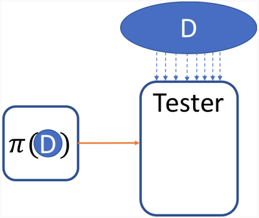

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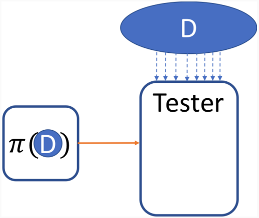

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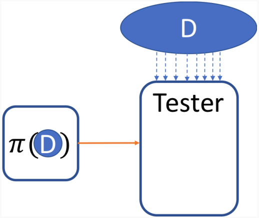

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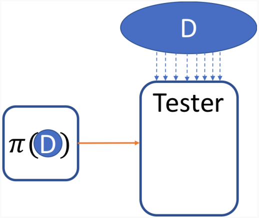

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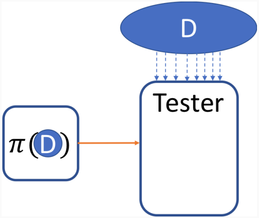

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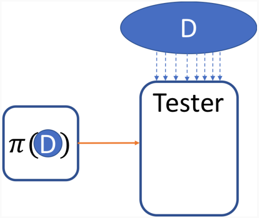

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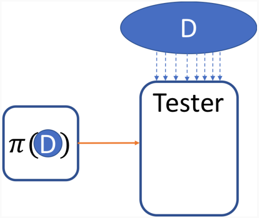

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IP distribution testers

MA distribution testers that interact with a prover

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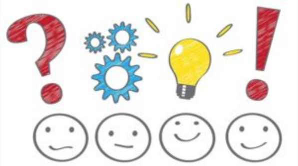

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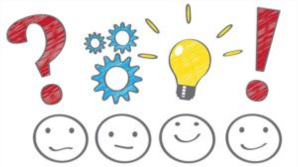

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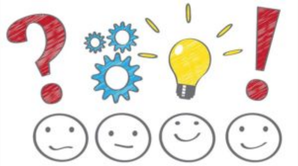

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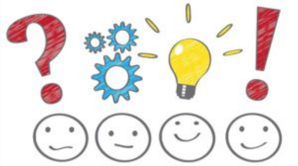

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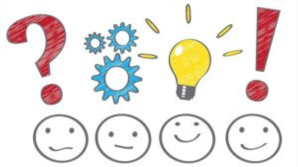

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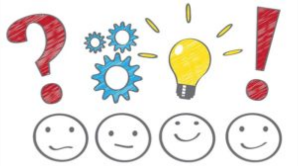

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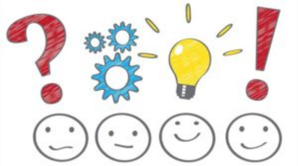

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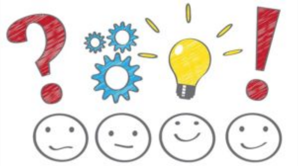

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- What can and cannot be achieved with each proof system?

FUNCTIONS VS DISTRIBUTIONS

		Testing Distributions this work	**Testing Functions** [RVW13; GR16; FGL14; GR17]
non-interactive proofs	**Proofs of linear length**	**Different!**	reduce sample complexity of *any* property to $O(1)$
	MA proofs of proximity vs. standard testers	**Different!**	exponentially stronger
	Probabilistic (MA) vs. deterministic (NP) verification	**Different!**	NP proofs of proximity are extremely weak
	Hardest property for non-interactive proofs	**Different!**	non-explicit (random property); linear length proof is required to outperform standard testers
interactive proofs	**Private vs. public coin protocols**	**Different!**	almost equivalent
	AM round hierarchy coin protocols	**Different!**	there is a property for which the AM complexity is $\approx \mathsf{n}^{1/\mathsf{r}}$ for r-round protocols

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...unless a prover is giving us support!

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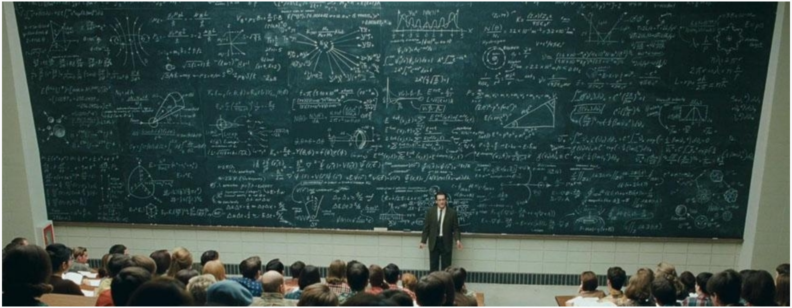
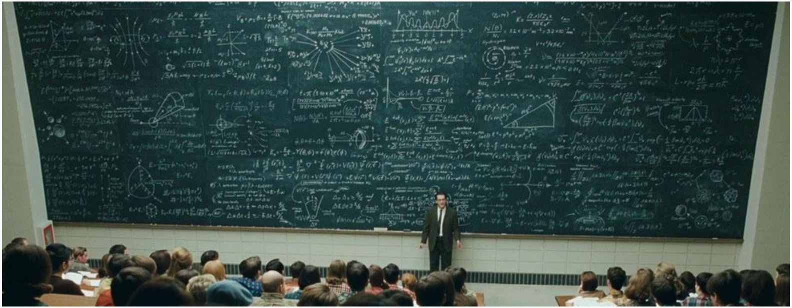
Caveat: this requires a long proof ($O(n \log n)$ bits)

ON LONG PROOFS – FUNCTIONS





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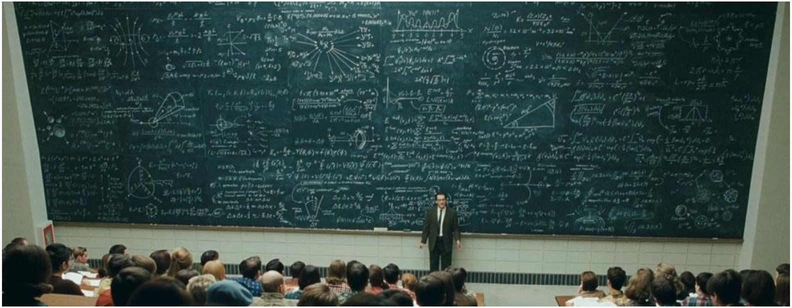

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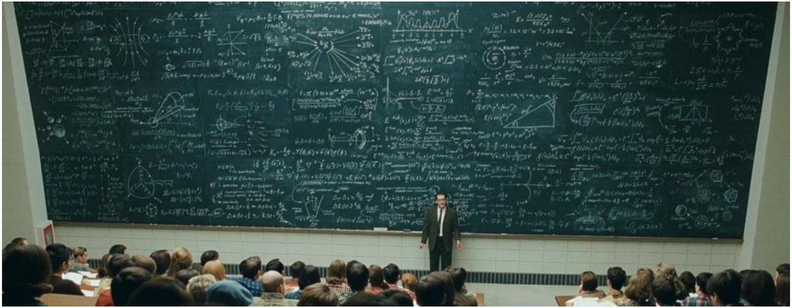

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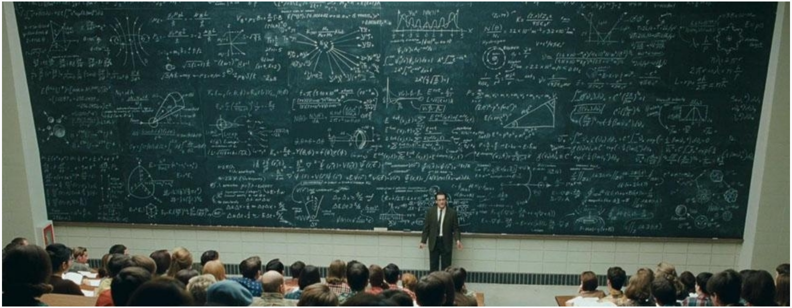

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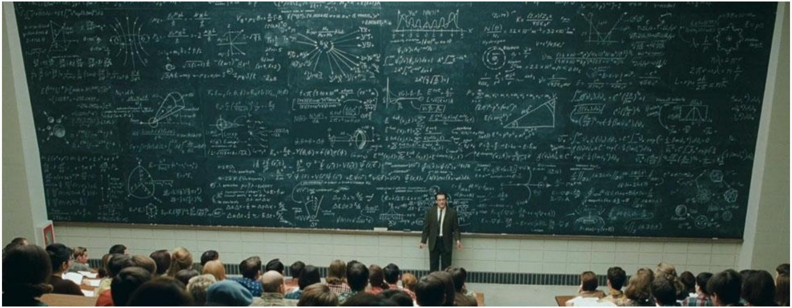

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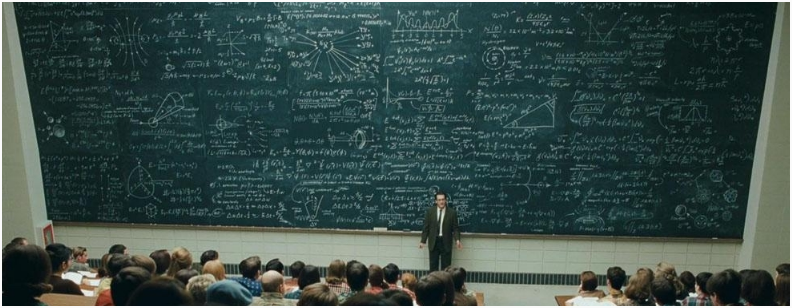

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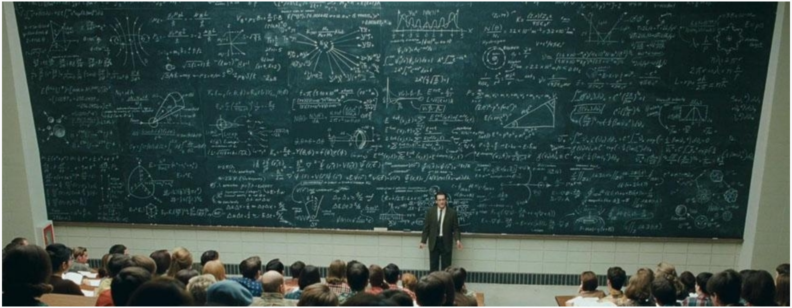

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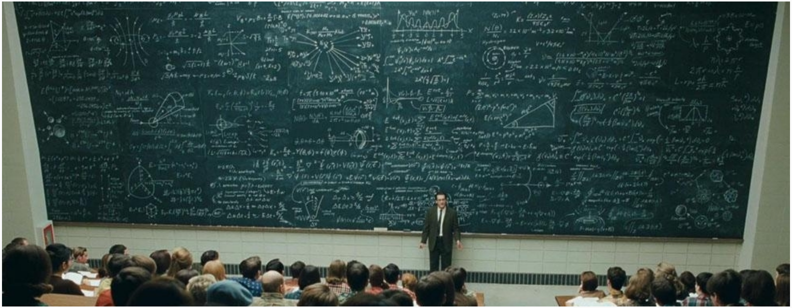

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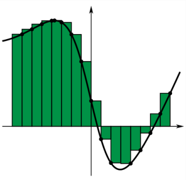

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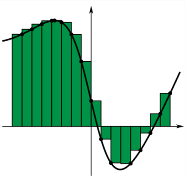

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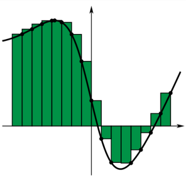

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We can use a tolerant tester to make sure it rules the same

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But can we do better? Not much... (not without interaction)

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For every Π and MA distribution tester for Π with proof length p and sample complexity s, it holds that $p \cdot s = \Omega(\mathrm{SAMP}(\Pi))$

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To this end, we invoke the tester $O(p)$ times, increasing the sample complexity to $O(p \cdot s)$.

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- This lemma allows us to “lift” standard lower bounds to MA lower bounds
- Dramatically different behavior than in the functional setting (there MA is exponentially stronger than standard testers)

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Every MA distribution tester with sample complexity s can be emulated by an NP distribution tester with the same proof length and sample complexity $O(s + \log n)$.

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Main technical difficulty: prove a randomness reduction lemma

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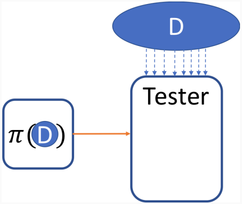
INTERACTION: SKY IS THE LIMIT...

Before (MA/NP)



REPLACING PROOF BY PROVER

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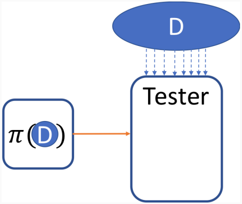

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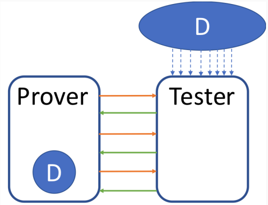
Now (IP)



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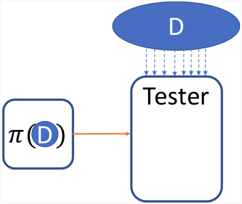

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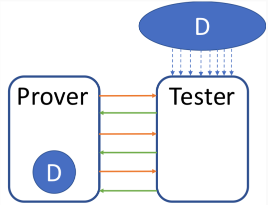

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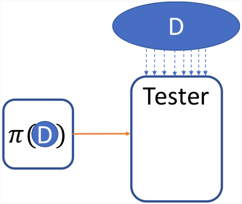

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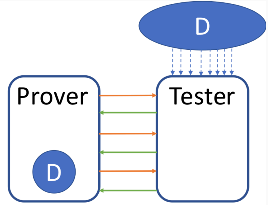

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Using 1 round of interaction!

HOW CAN INTERACTION HELP?

Consider the isolated elements property:

$$\Pi_{\text{Isolated}} = \{D \in \Delta([n]) \ : \ \forall i \in [n] \ i \notin \mathrm{supp}(D) \text{ or } (i+1) \notin \mathrm{supp}(D)\}$$

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- By the lifting lemma, every MA distribution tester has $\text{proof} \cdot \text{sample} = \Omega(\sqrt{n})$

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- We show that Π_{Isolated} requires $\Omega(\sqrt{n})$ samples for standard testers (via reduction from SMP communication complexity [BCG17])
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For (private-coin) IP, we can do exponentially better!

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Prover has to guess their preimage!

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non-interactive proofs	**Proofs of linear length**	reduce sample complexity of *any* property to $O(\sqrt{\mathsf{n}})$	reduce sample complexity of *any* property to $O(1)$
	MA proofs of proximity vs. standard testers	quadratically stronger	exponentially stronger
	Probabilistic (MA) vs. deterministic (NP) verification	nearly equivalent	NP proofs of proximity are extremely weak
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FUNCTIONS VS DISTRIBUTIONS

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OPEN PROBLEMS

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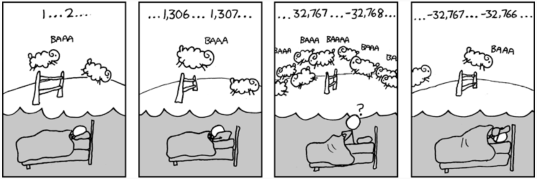

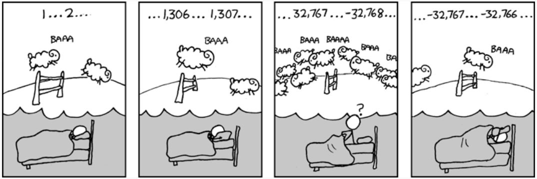

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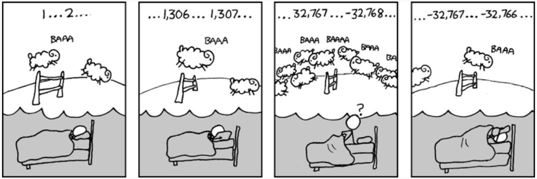

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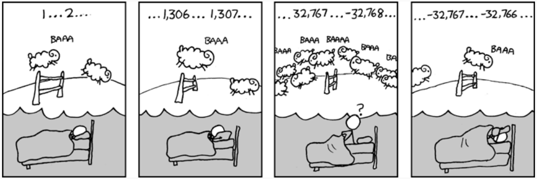

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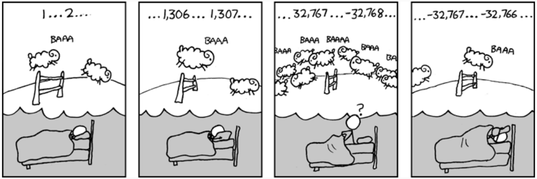

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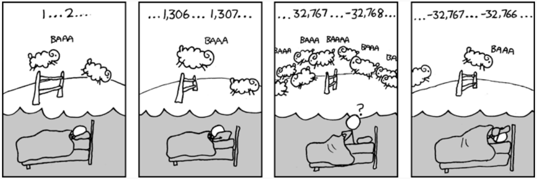

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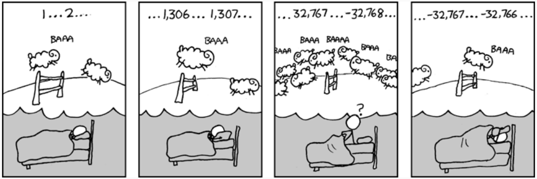

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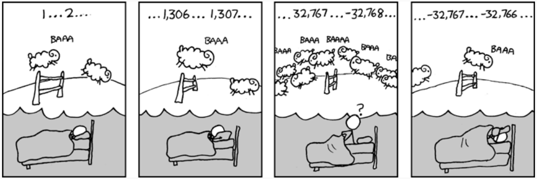

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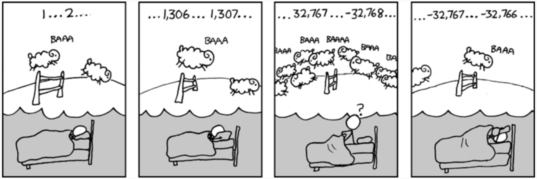

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Is AM strictly stronger than MA?

Thank you!

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