# A Chasm Between Identity and Equivalence Testing with Conditional Queries

Joint work with Jayadev Acharya and Gautam Kamath

# Outline of the talk

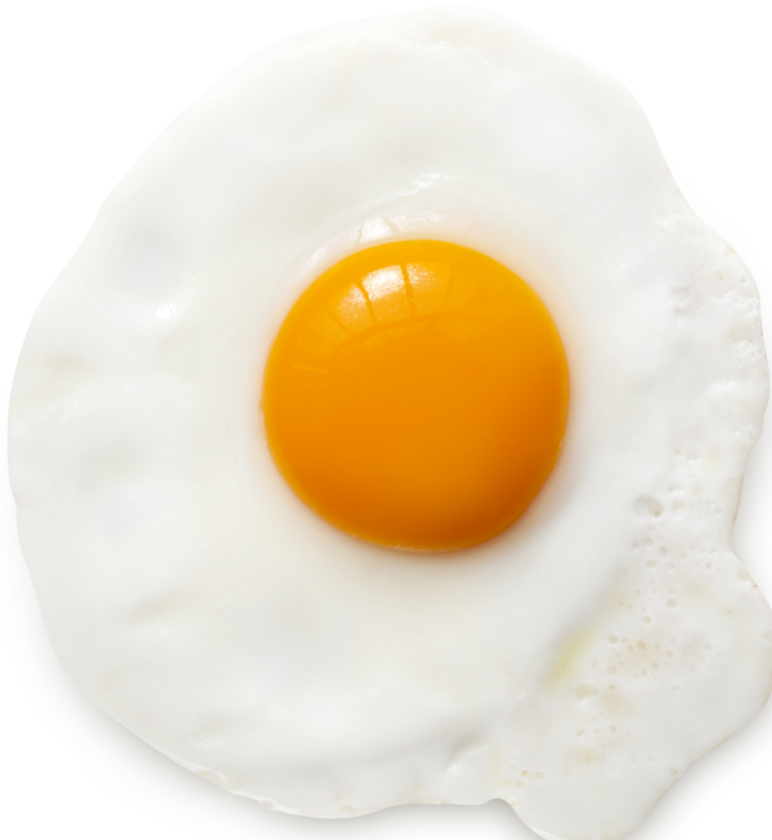
- Distribution Testing and Conditional Queries
- Our results
- Overview of the techniques and obstacles
- Open problems

# Distribution Testing

Subfield of **property testing**:

- Big (Unknown) Object O
- Fixed property (subset of Objects) P
- Is O in P, or *far* from every object in P?

# Distribution Testing



# Distribution Testing

Big Object:
  probability distribution D over n elements

Type of queries:
  independent samples from D

Distance measure (for "far"):
  Total variation distance ($L_1$)

# Distribution Testing: A Glimpse

Many results since seminal work of [BFRSW01]:

| | | |
|---|---|---|
| Testing uniformity: | $\Theta(\sqrt{n}/\epsilon^2)$ | [GR00,Pan08] |
| Testing identity: | $\Theta(\sqrt{n}/\epsilon^2)$ | [BFF+01,VV14] |
| Testing equivalence: | $\Theta(n^{2/3}/\epsilon^{4/3})$ | [BFR+10,CDVV14] |

*(+monotonicity, independence, membership to a class...)*

**Polynomial dependence on the domain size**

Given a fixed known D*, and samples from unknown arbitrary D, is D equal to D* or ε-far from it?

Given samples from unknown arbitrary D, is D uniform or ε-far from it?

Given samples from two unknown arbitrary D, D', is D equal to D' or ε-far from it?

Many results since se

Testing uniformity: $\Theta(\sqrt{n}/\epsilon^2)$ [GR00,Pan08]

Testing identity: $\Theta(\sqrt{n}/\epsilon^2)$ [BFF+01,VV14]

Testing equivalence: $\Theta(n^{2/3}/\epsilon^{4/3})$ [BFR+10,CDVV14]

*(+monotonicity, independence, membership to a class...)*

**Polynomial dependence on the domain size**

# Distribution Testing: A Twist

"Is $n^{\Theta(1)}$ our final answer ?"

More power to the testers: conditional sampling

[Chakraborty-Fischer-Goldhirsh-Matsliah'13, C-Ron-Servedio'12]

Tester chooses $S \subseteq [n]$

Gets sample drawn from $D_S$

# Distribution Testing: A Twist

- Generalizes sampling model
- Allows adaptivity
- (Several restricted variants)
- Natural in some settings

**“Power of comparisons”**

# Distribution Testing in $O(1)$

| | | |
|---|---|---|
| Testing uniformity: | $O(1)$ | [CRS15] |
| Testing identity: | $O(1)$ | [CRS15,FJO+15] |
| Testing equivalence: | $\log\log n$ | [CRS15,FJO+15] |

**"Should it be constant-query too?"**

# Distribution Testing ~~in O(1)~~

| | | |
|---|---|---|
| Testing uniformity: | $O(1)$ | [CRS15] |
| Testing identity: | $O(1)$ | [CRS15,FJO+15] |
| Testing equivalence: | $\boldsymbol{\Omega(\sqrt{\log\log n})}$ | [this work] |

**"Things are different now."**

# Our main result

**Theorem.** Any (adaptive) testing algorithm for equivalence in the conditional model must make $\boldsymbol{\Omega(\sqrt{\log\log n})}$ queries.

*"Know thy enemy — it helps for testing."*

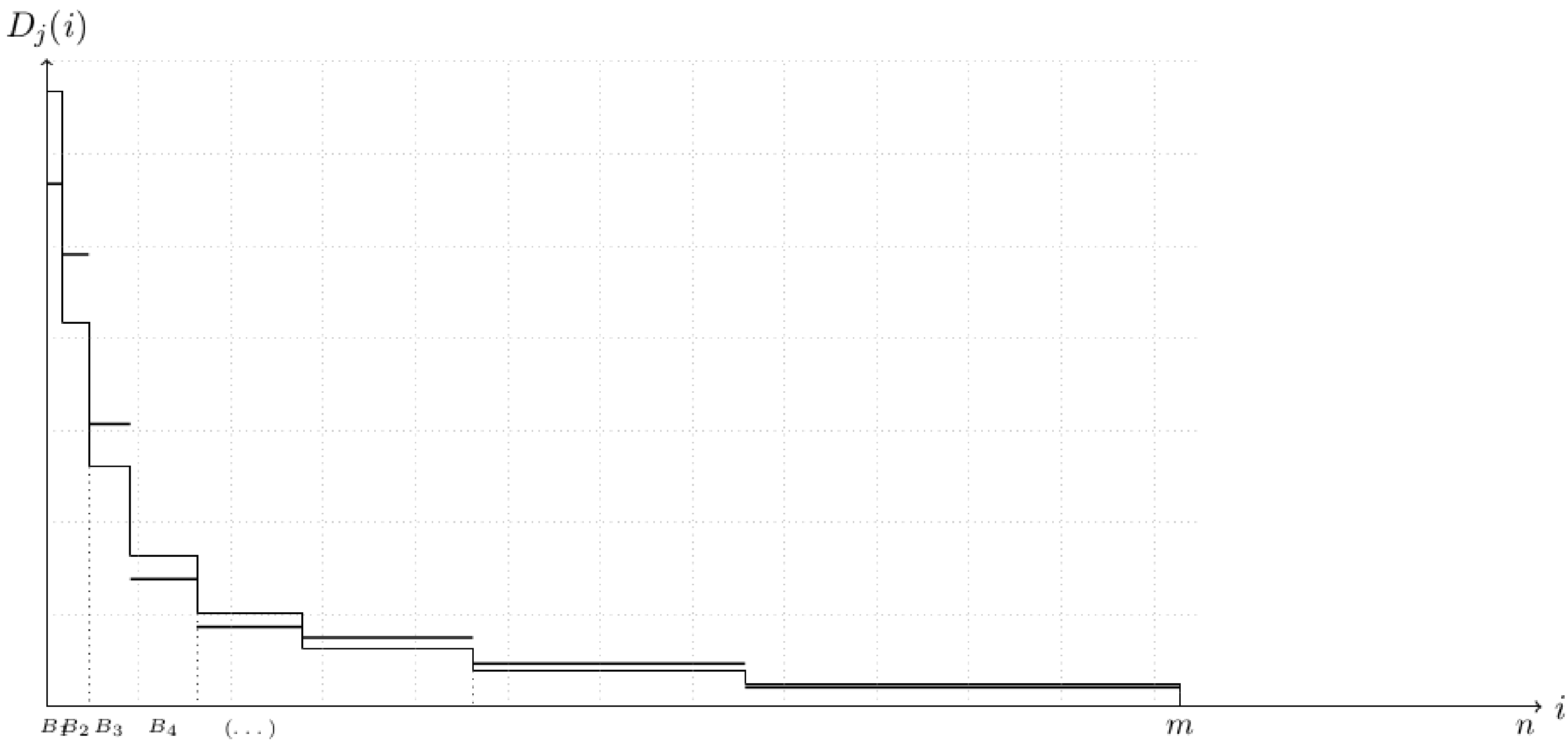
# Obstacles

- How to deal with adaptivity?
  *Lack of general lower bound techniques in this model.*
- How to deal with arbitrary queries?
  *The tester can pretty much do what it wants.*
- Cannot rely on the "usual tricks"
  *Heavy elements to hide the rest, only a few types of weights...*

# Ideas

- Bring in the concept of **adaptive core tester** of [CFGM13], + Yao's principle
- Adapt a lower bound construction of [CRS15] for a **restricted** conditional model (easy to beat in the full one).
- **Hide it** by scaling its support by a random factor.

# Construction



# Intuition

If you don't (roughly) guess the support size, you cannot learn *anything*...
But we show support size estimation is **hard**.

# Open Questions

- Can we get ε in the picture?
- Does *tolerant* testing behave like this? (chasm!)
- Develop general techniques for proving lower bounds?
- Characterize problems that are still hard in this model?

# Thank You.