W4281 - Introduction to Quantum Computing

Homework 5

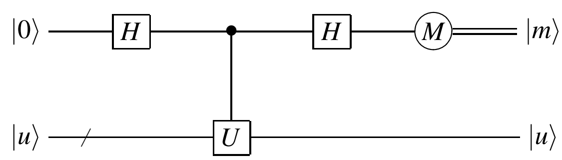
due date: Thursday, 7/28/05

Exercise 1 (10 points):

Kitaev's algorithm: Let U be a unitary transformation with eigenvector $|u\rangle$ such that

$$U|u\rangle = e^{2\pi i\varphi}|u\rangle .$$

Consider the following quantum circuit:



Show that $|m\rangle = |0\rangle$ with probability $p = \cos^2(\pi\varphi)$.

Exercise 2 (10 points):

Shift-invariance property of the Fourier transform:

a.) Show that the transformation U_k acting on q qubits and defined as

$$U_k|g\rangle = |g \oplus k\rangle = |g + k \mod 2^q\rangle$$

is unitary for $g, k = 0, 1, \ldots, 2^q - 1$.

b.) Show that for an arbitrary q qubit state

$$|\psi\rangle = \sum_{j=0}^{2^q-1} \alpha_j |j\rangle$$

the probabilities to measure the state $|g\rangle$, $g = 0, 1, \ldots, 2^q - 1$, when measuring $\mathscr{F}_{2^q}|\psi\rangle$ and $\mathscr{F}_{2^q}U_k|\psi\rangle$ are the same.

Exercise 3 (10 points):

Write a program to simulate Grover's search algorithm to find the unique index x_0 of a function $f : \{0,1\}^n \to \{0,1\}$ for which $f(x_0) = 1$.

The input should be given as

- The number of input bits for the function f as $n \in \mathbb{N}$.
- The index $x_0 = 0, 1, \ldots, 2^n - 1$ for which $f(x_0) = 1$ (the one we want to find).

Example: for $n = 3$ and $x_0 = 2$ we consider the function $f : \{0,1\}^n \to \{0,1\}$ defined as

$$f(x) = \begin{cases} 1 & \text{for } x = x_0 \\ 0 & \text{otherwise} \end{cases}$$

The function f now defines a $n+1$ qubit transformation

$$O\left|x\right\rangle\left|q\right\rangle = \left|x\right\rangle\left|q \oplus f(x)\right\rangle.$$

The complete algorithm can now be written as

$$((H^{\otimes n} S_0 H^{\otimes n}) \otimes I) O)^{\lceil \pi 2^{n/2}/4 \rceil} (H^{\otimes n} \otimes (HX)) \left|0\right\rangle^{\otimes n} \left|0\right\rangle$$

where S_0 is the following transform

$$S_0 = 2\left|0\right\rangle\left\langle 0\right| - I^{\otimes n}.$$

If we measure the final state of this algorithm, the first register should contain x_0 with probability greater than $3/4$.