W4281 - Introduction to Quantum Computing

Homework 3

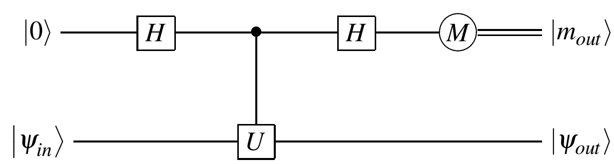
due date: Thursday 06/30/2005

Exercise 1 (10 points): Consider projective measurements in the computational basis.

a.) A single qubit matrix U has the eigenvalues ± 1 and the orthonormal eigenvectors $|\psi_1\rangle$, $|\psi_2\rangle$, i.e. we can write it as,

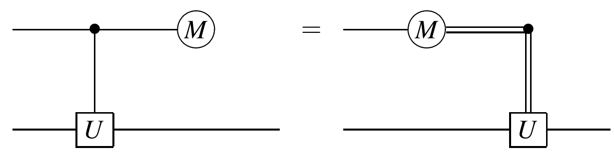
$$U = |\psi_1\rangle\langle\psi_1| - |\psi_2\rangle\langle\psi_2|.$$

Consider the following circuit (the M in the circle is a measurement):



Find $|m_{out}\rangle$ and $|\psi_{out}\rangle$, and explain how they are related to $|\psi_1\rangle$ and $|\psi_2\rangle$.

b.) Prove that measurement commutes with controls. That is, for an arbitrary single qubit unitary U we have



Exercise 2 (10 points):

Let f be a function $f : \{0,1,\ldots,2^n-1\} \to \{0,1,\ldots,2^m-1\}$ and

$$U_f |j\rangle |k\rangle = |j\rangle |k \oplus f(j)\rangle,$$

with $|j\rangle|k\rangle \in \mathbb{C}^{2^n} \otimes \mathbb{C}^{2^m}$ and $\oplus$ denoting addition modulo 2^m.

For an arbitrary integer $\ell \geq 0$ find the matrix representation of

$$U_f^\ell = \underbrace{U_f U_f \ldots U_f}_{\ell \text{ times}}.$$

Exercise 3 (10 points):

a.) Write a program to simulate the Deutsch-Josza quantum algorithm. Here "simulate" means that your algorithm actually does the same steps as in the quantum algorithm and not only returns a precomputed answer.

b.) Suppose that a Boolean function $f : \{0,1,\ldots,2^n-1\} \to \{0,1\}$, with $n \geq 2$, is constant, i.e., $f(j) = f(0)$, or $\frac{3}{4}$-balanced, i.e., $|\{j \,|\, f(j) = 1\}| = \frac{3}{4}2^n$.

Modify the Deutsch-Josza algorithm to check that f is constant with probability $p > \frac{1}{2}$.

c.) Write a program to simulate the quantum algorithm of b.).