W4281 - Introduction to Quantum Computing

Homework 2

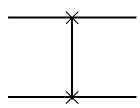
due date: Thursday 6/16/2005

Exercise 1 (10 points):

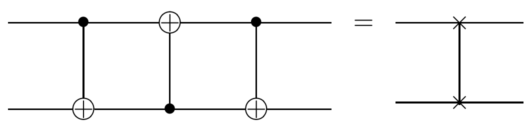
The swap gate is defined as

$$Q\,|a\rangle\,|b\rangle = |b\rangle\,|a\rangle \text{ for } a,b \in \{0,1\},$$

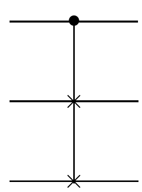
and its circuit representation is



1. Find the matrix representation of Q (and prove its correctness).
2. Prove that



3. The controlled-swap (Fredkin gate) is defined as



Find the matrix representation of this gate (and prove its correctness).

Exercise 2 (10 points):

Let $|\psi\rangle = H^{\otimes n}\,|0\rangle$, and let

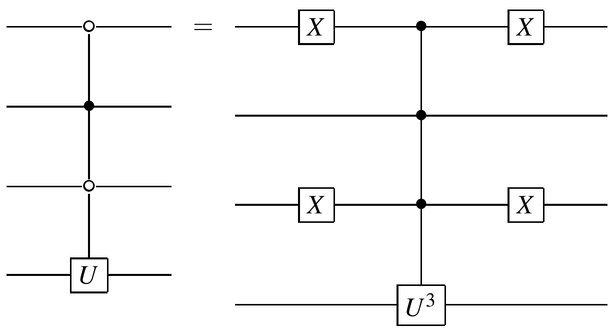
$$A = I - 2\,|\psi\rangle\,\langle\psi|\,.$$

Prove that

$$e^{i\pi A} = -I.$$

Exercise 3 (10 points):

Write a program which verifies whether the following circuits are equivalent:



The first circuit applies U if the first three qubits are $|0\rangle\,|1\rangle\,|0\rangle$ (the first and third dots should be empty).

Input: A 2×2 unitary matrix U:

$$U = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

Output: YES or NO depending on whether the circuits are (approximately) equivalent. If you are using a symbolic program like Mathematica it should be easy to check whether they actually are equivalent. If you are using a numerical package like Matlab, check that the circuits C_1 and C_2 are *approximately* equivalent, which means that for all $|\psi\rangle \in \mathbb{C}^{2^4}$, $\||\psi\rangle\| = 1$

$$\left\|C_1\,|\psi\rangle - C_2\,|\psi\rangle\right\| < \varepsilon.$$

A good choice for ε would be proportional to the rounding unit.

Test your program for $U \in \{X,Y,Z,H,S,T\}$ and document these test runs.