# W4281 - Introduction to Quantum Computing

## Homework 1

due date: Thursday 6/2/2005

Note: for this homework we use the 6 single qubit gates $X$, $Y$, $Z$ (Pauli matrices), $H$ (Hadamard gate), $S$ (phase gate), and $T$ ($\pi/8$ gate) as defined in class or in Nielsen and Chuang, p. 174.

**Exercise 1 (10 points):**

In this problem you compute the exponential $e^M$ of a matrix $M$. The definition of the exponential of a matrix can be found in Nielsen and Chuang, p. 75.

Let $X$, $Y$, $Z$ be the Pauli matrices and let $\alpha$ be a complex number. Compute

$$e^{\alpha X},\ e^{\alpha Y},\ e^{\alpha Z}.$$

**Exercise 2 (10 points):**

Show that the Hadamard gate $H$ can be written as

$$H = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\langle 0| + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\langle 1|$$

and that the $n$-fold tensor product of $H$ with itself is

$$H^{\otimes n} = \frac{1}{2^{n/2}} \sum_{j,k=0}^{2^n-1} (-1)^{j\cdot k} |j\rangle \langle k|$$

where $j \cdot k$ denotes

$$j_0 k_0 + j_1 k_1 + \ldots + j_{n-1} k_{n-1}$$

if the binary expansions of $j$ and $k$ are

$$j = (j_{n-1} j_{n-2} \cdots j_0)_2 = \sum_{l=0}^{n-1} 2^l j_l$$

$$k = (k_{n-1} k_{n-2} \ldots k_0)_2 = \sum_{l=0}^{n-1} 2^l k_l.$$

**Exercise 3 (10 points):**

Write a program to solve the following problem:

*Input:* Three integers $j,k,m$, with $m, j \in \{0,1,\ldots,2^k-1\}$.

*Output:* The probability to measure the outcome $j$ of the $k$-qubit state

$$|\psi\rangle = U_1 \otimes U_2 \otimes \ldots \otimes U_k |m\rangle ,$$

where the $U_l$, $l \in \mathbb{N}$, are the following matrices:

$$\begin{aligned} U_1 &= U_7 = \ldots = X \\ U_2 &= U_8 = \ldots = Y \\ U_3 &= U_9 = \ldots = Z \\ U_4 &= U_{10} = \ldots = H \\ U_5 &= U_{11} = \ldots = S \\ U_6 &= U_{12} = \ldots = T. \end{aligned}$$

**Your algorithm should be linear in** $k$.

*Hint:* You only have to compute the probability for the $j$ given as input. This probability is given by

$$\left|\langle j| U_1 \otimes U_2 \otimes \ldots \otimes U_k |m\rangle\right|^2 .$$

*Example:* For $j=2$, $k=2$, $m=1$ this probability is

$$|\langle 2| X \otimes Y |1\rangle|^2 = \left|\left\langle 10_2\right| X \otimes Y \left|01_2\right\rangle\right|^2 = \left|-i\left\langle 10_2|10_2\right\rangle\right|^2 = |-i|^2 = 1.$$