Data Structures and Algorithms

Session 9. February 18, 2009

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Announcements

- Homework 2 is up. Due Feb. 23
  - Problem 2 (Weiss 3.7), trimToSize() creates a new array of same size as list, copies each element.
  - Problem 5 (Weiss 3.9), we want a linear time algorithm

Review

- Binary Search Trees
  - Basic operations: insert, findMin/Max, contains
  - Delete
  - Average depth analysis

Today’s Plan

- Brief look at tradeoffs
- Balanced (AVL) Binary Search Trees
  - AVL Tree property
  - Tree Rotations
  - Worst case depth analysis

Tradeoffs

	insert	remove	lookup	index
ArrayList	**O(N)**	**O(N)**	**O(N)**	**O(1)**
LinkedList	**O(1)**	**O(1)**	**O(N)**	**O(N)**
Stack/Queue	**O(1)**	**O(1)**	N/A	N/A
BST	**O(d)=O(N)**	**O(d)=O(N)**	**O(d)=O(N)**	N/A
AVL	**O(log N)**	**O(log N)**	**O(log N)**	N/A

- There may not be free lunch, but sometimes there's a cheaper lunch

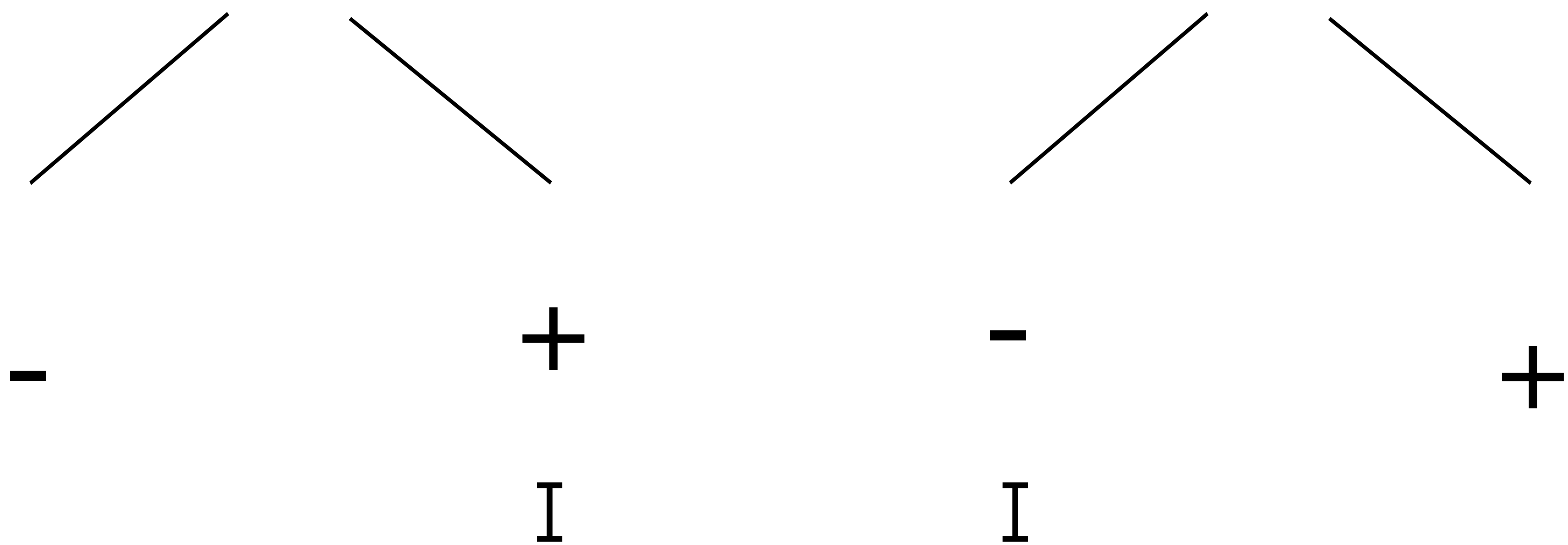
Question

- How do we visit the contents of a binary search tree in ascending order?

AVL Trees

- Motivation: want height of tree to be close to log N
- AVL Tree Property:
  For each node, all keys in its left subtree are less than the node's and all keys in its right subtree are greater. **Furthermore, the height of the left and right subtrees differ by at most 1**

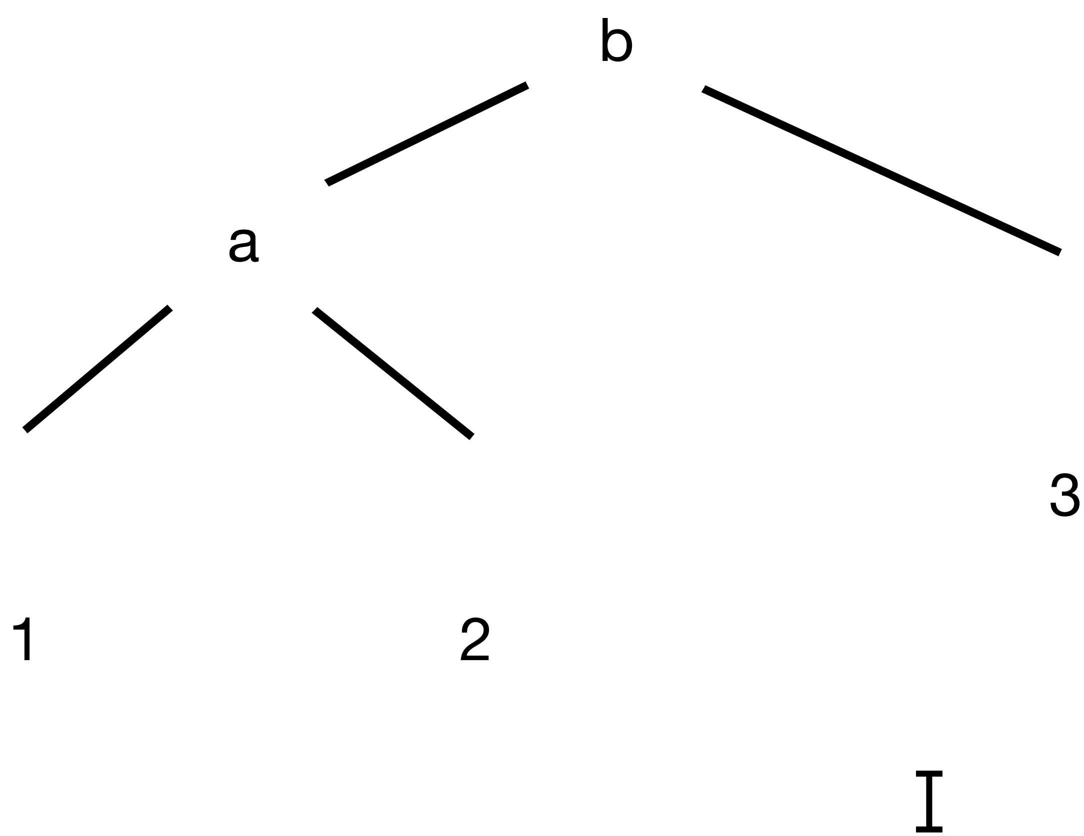
AVL Tree Visual



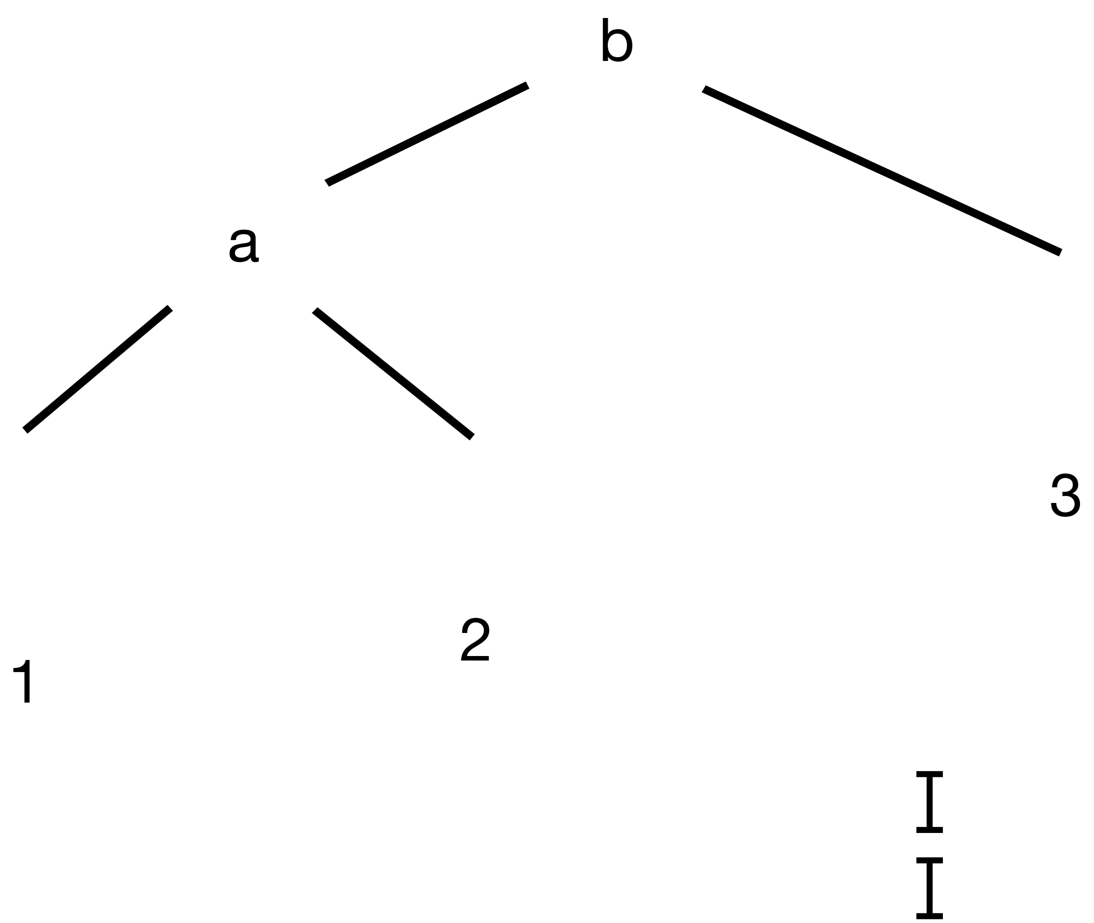
Tree Rotations

- To balance the tree after an insertion violates the AVL property,
  - rearrange the tree; make a new node the root.
  - This rearrangement is called a **rotation.**
  - There are 2 types of rotations.

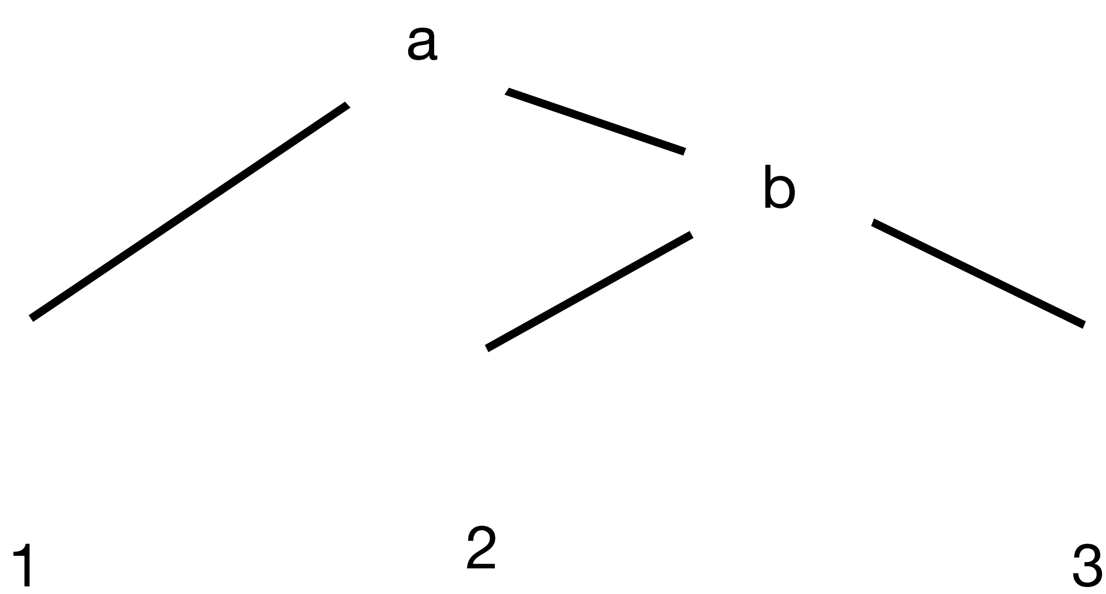
AVL Tree Visual: Before insert



AVL Tree Visual: After insert



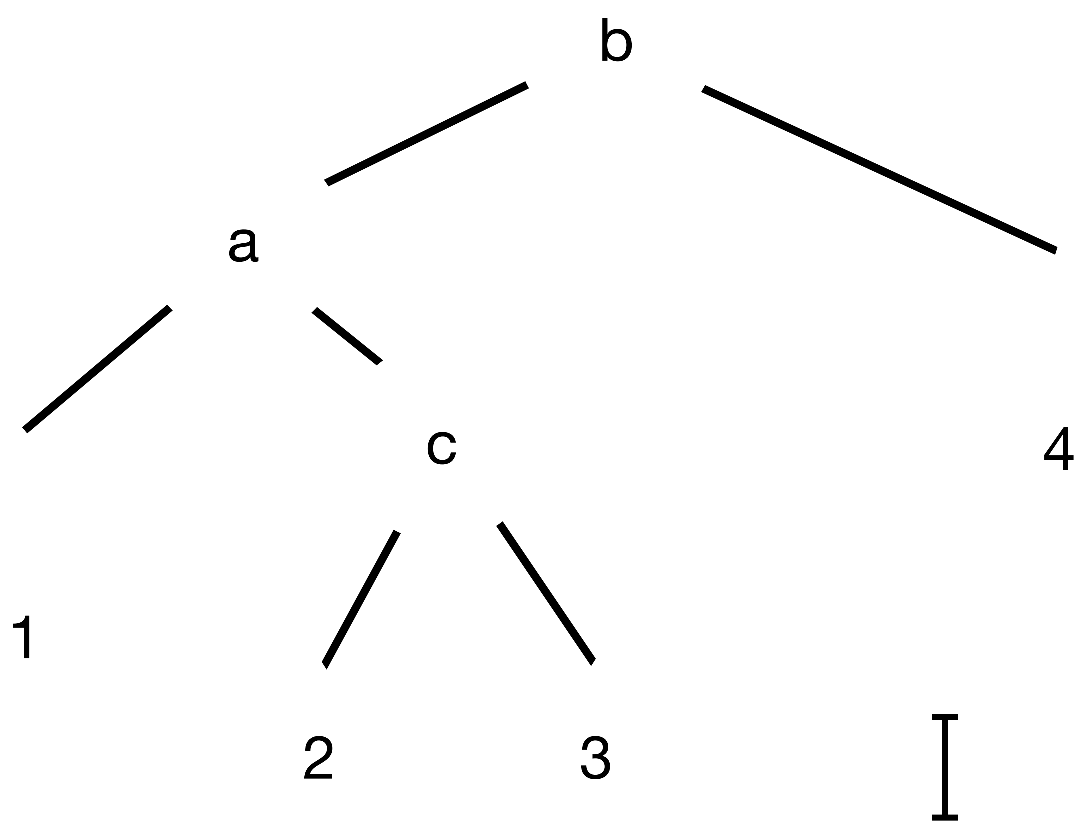
AVL Tree Visual: Single Rotation



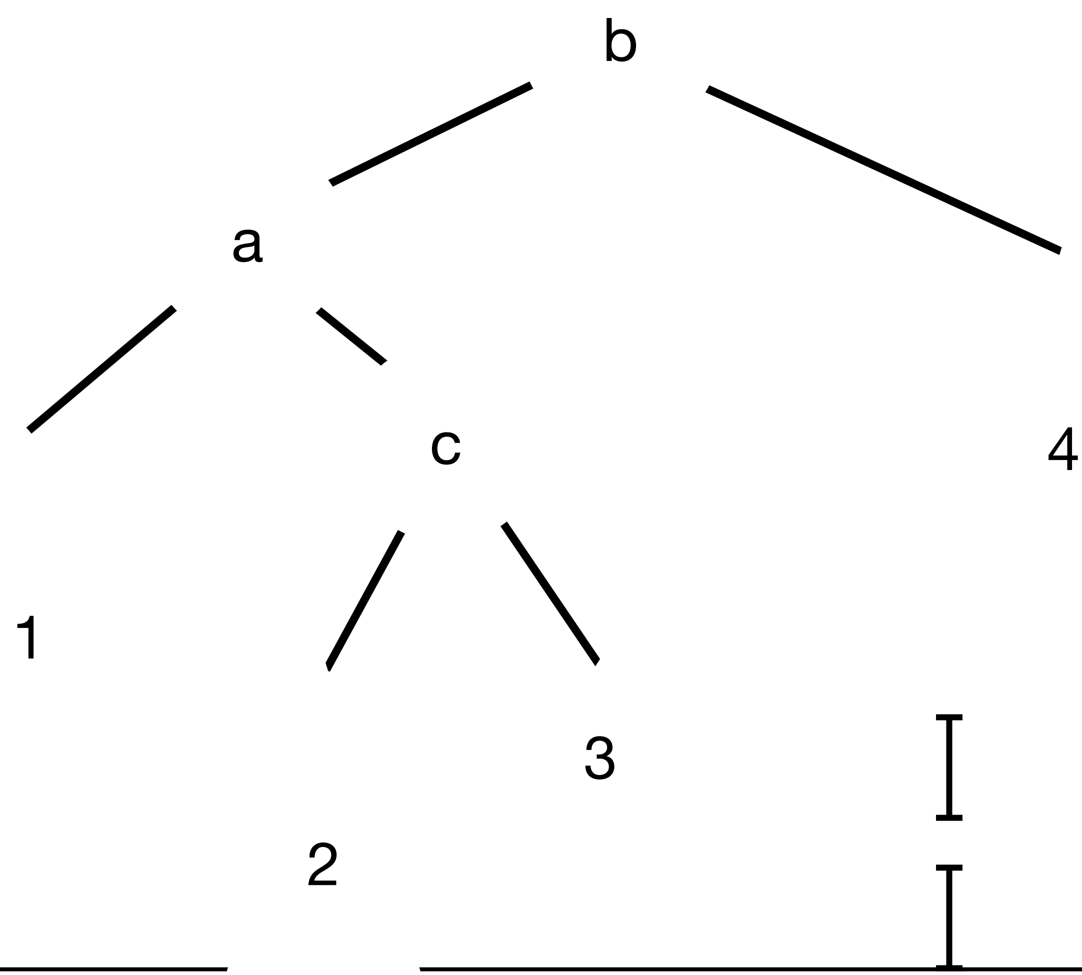
AVL Tree
Single Rotation

- Works when new node is added to outer subtree (left-left or right-right)
- What about inner subtrees? (left-right or right-left)

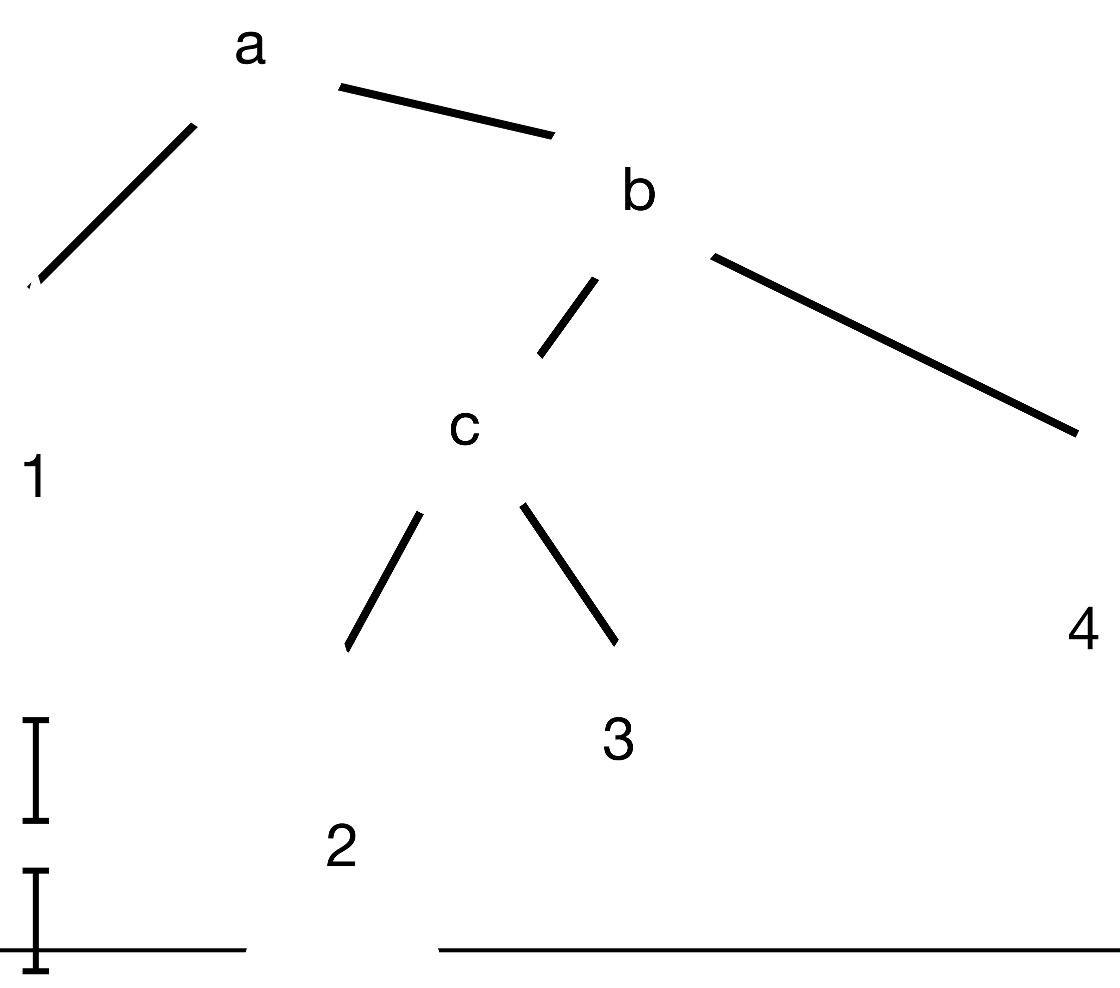
AVL Tree Visual: Before Insert 2



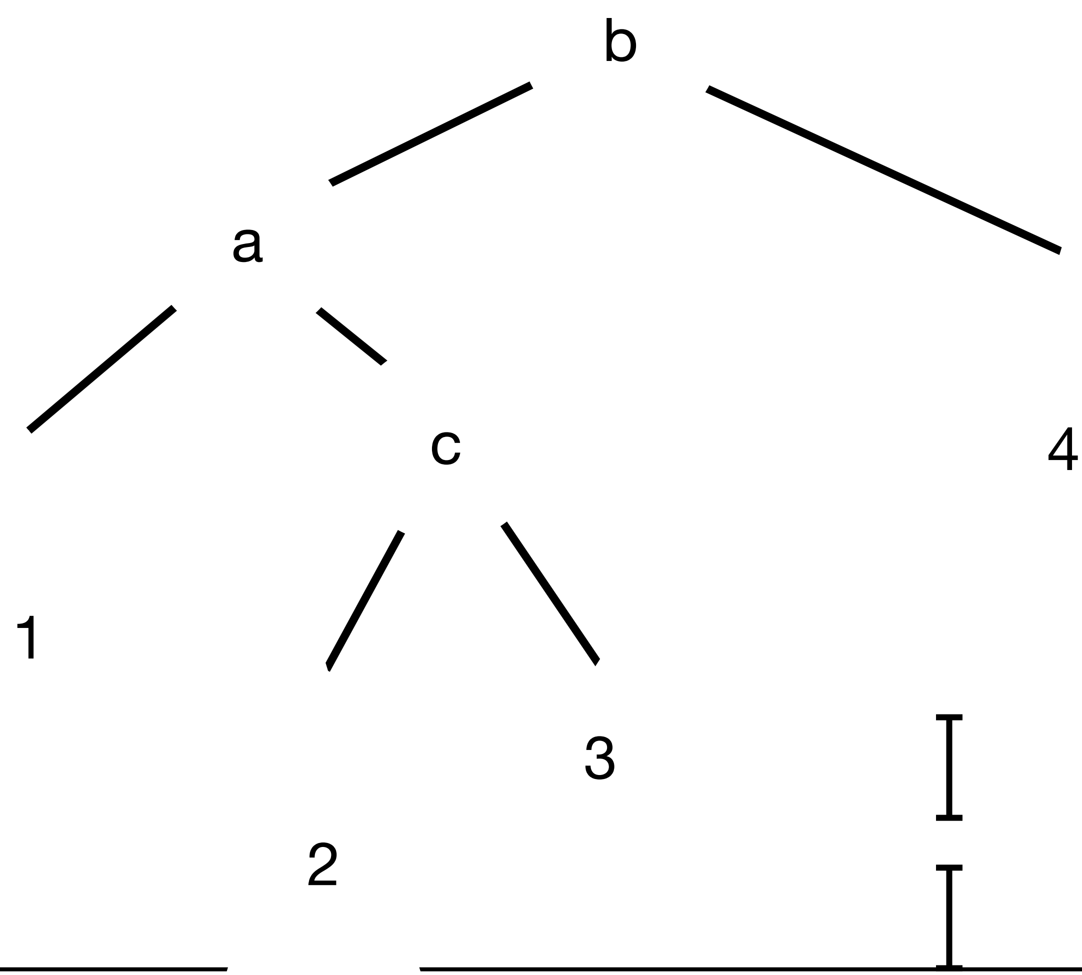
AVL Tree Visual: After Insert 2



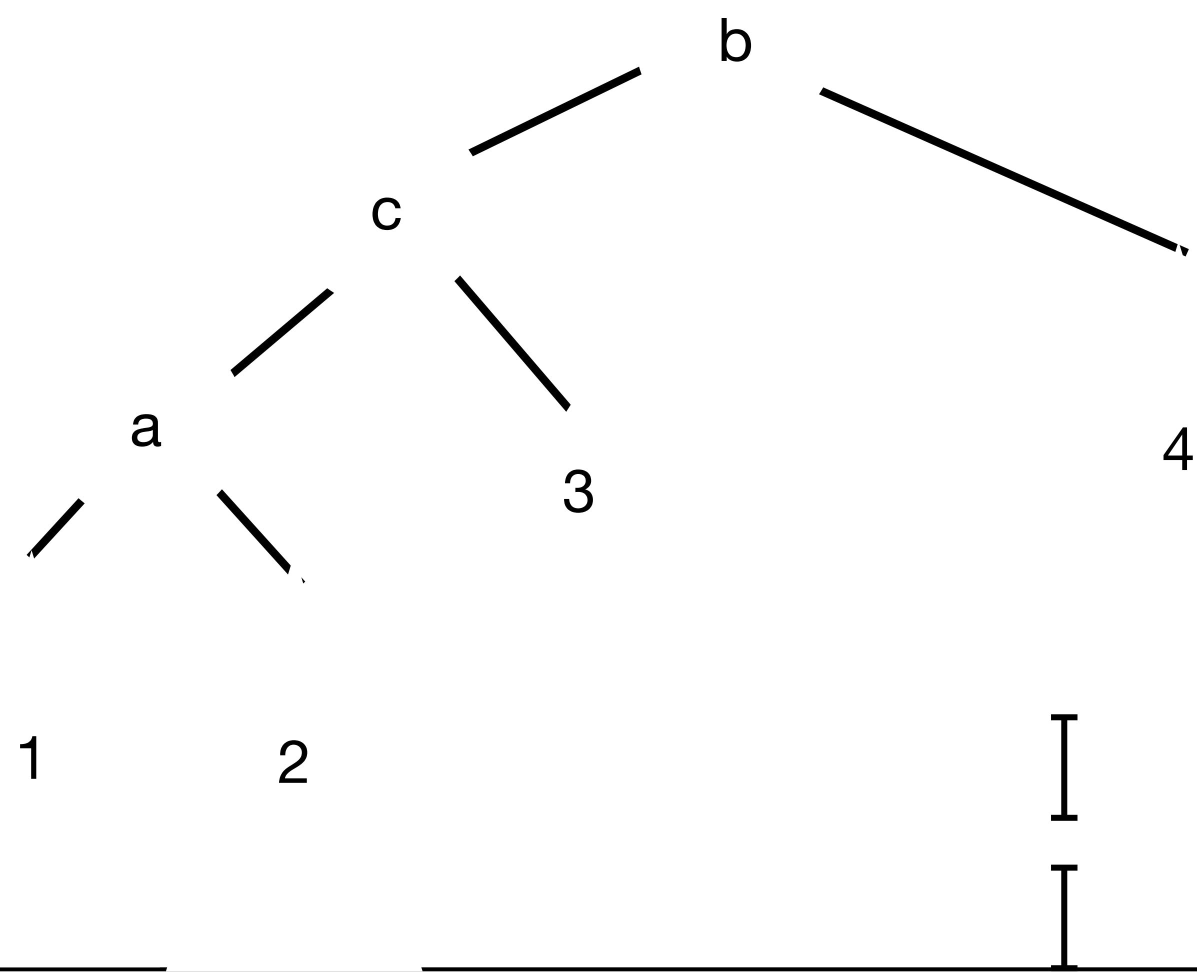
AVL Tree Visual: Single Rotation Fails



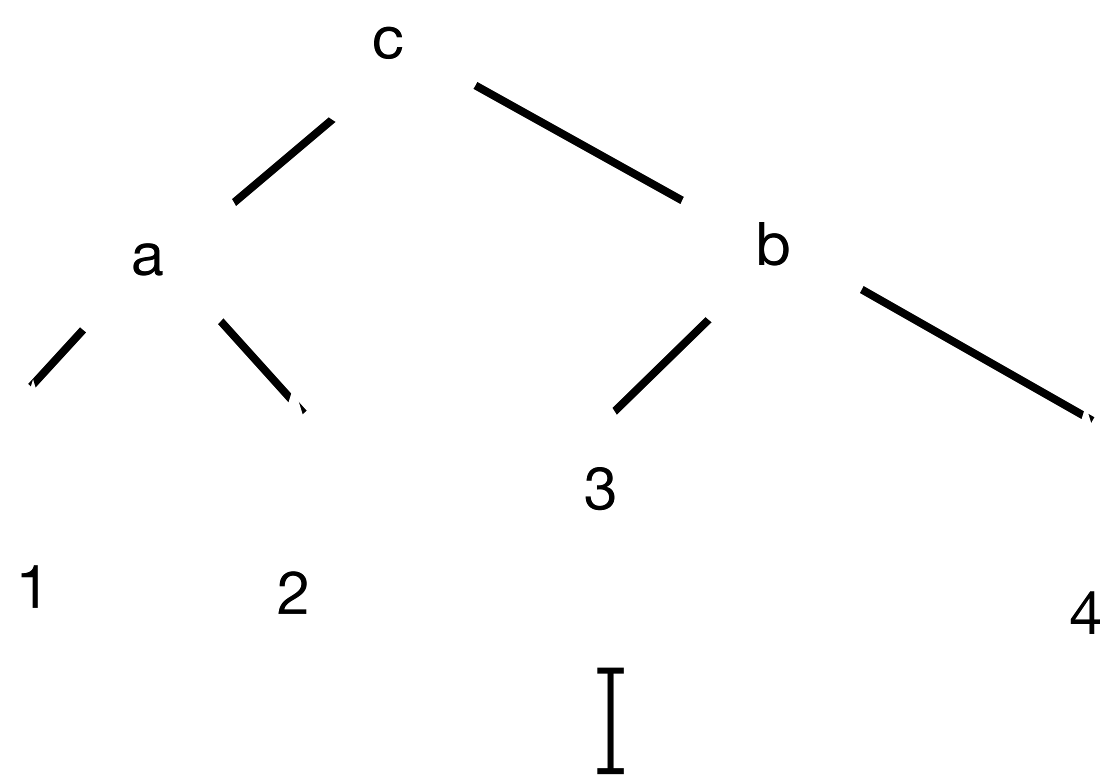
AVL Tree Visual: Double Rotation



AVL Tree Visual: Double Rotation



AVL Tree Visual: Double Rotation



Rotation running time

- Constant number of link rearrangements
- Double rotation needs twice as many, but still constant
  - So AVL rotations do not change O(d) running time of all BST operations*

- * remove() can require up to O(d) rotations.

Depth analysis

- Worst case: minimum number of nodes in an AVL tree of height *h*: *N(h)*
- *N(1) = 1, N(2) = 2*
- For greater heights, the total number of nodes includes:
  - the root node
  - the # of nodes in a subtree of size *h-1*
  - the # of nodes in a subtree of size *h-2*

$$N(h) = 1 + N(h-1) + N(h-2)$$

Depth analysis

$$N(h) = 1 + N(h-1) + N(h-2)$$

- Recursively subbing in the formula above, we get

$$\begin{aligned} N(h) &= 1 + (1 + N(h-2) + N(h-3)) + \\ &\quad (1 + N(h-3) + N(h-4)) \end{aligned}$$

- Combine all the newly generated 1's, then recurse

$$\begin{aligned} N(h) &= 1 + 2 \\ &\quad +(1 + N(h-3) + N(h-4)) \\ &\quad +(1 + N(h-4) + N(h-5)) \\ &\quad +(1 + N(h-4) + N(h-5)) \\ &\quad +(1 + N(h-5) + N(h-6)) \end{aligned}$$

Depth analysis

- Each time we recurse, we generate a new constant, which is the count of the number of evaluations we did.
- We can recurse h/2 times before at least one N(h-k)=N(0)
- Therefore, we can lower bound

$$N(h) > \sum_{i=0}^{h/2} 2^i > 2^{h/2}$$

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- Solve for h,

$$\begin{aligned} \log(N) &> h/2 \\ 2\log(N) &> h \\ h &= O(\log N) \end{aligned}$$

- Recap:
  - We analyzed minimum number of nodes necessary to cause height *h*
  - We lower bounded that minimum; a formula that is even worse than the worst case
  - We showed that worst case means the height is still O(log N)

Looking Forward

- AVL Trees aggressively guarantee log running time
  - Every operation is now log running time
  - May be overkill

Reading

- Weiss Section 4.5: Splay Trees