Data Structures and Algorithms

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Instructor: Bert Huang

http://www.cs.columbia.edu/~bert/courses/3137

Announcements

- Homework 2 is up. Due Feb. 23
  - Problem 2 (Weiss 3.7), trimToSize() creates a new array of same size as list, copies each element.

Review

- Introduction to Trees
  - Definitions
  - Tree Traversal Algorithms
- Binary Trees

Today's Plan

- Finish up examples of binary tree applications
- Binary Search Trees
  - Basic operations: insert, findMin/Max, contains
  - Delete
  - Average depth analysis

Full Binary Tree Depth

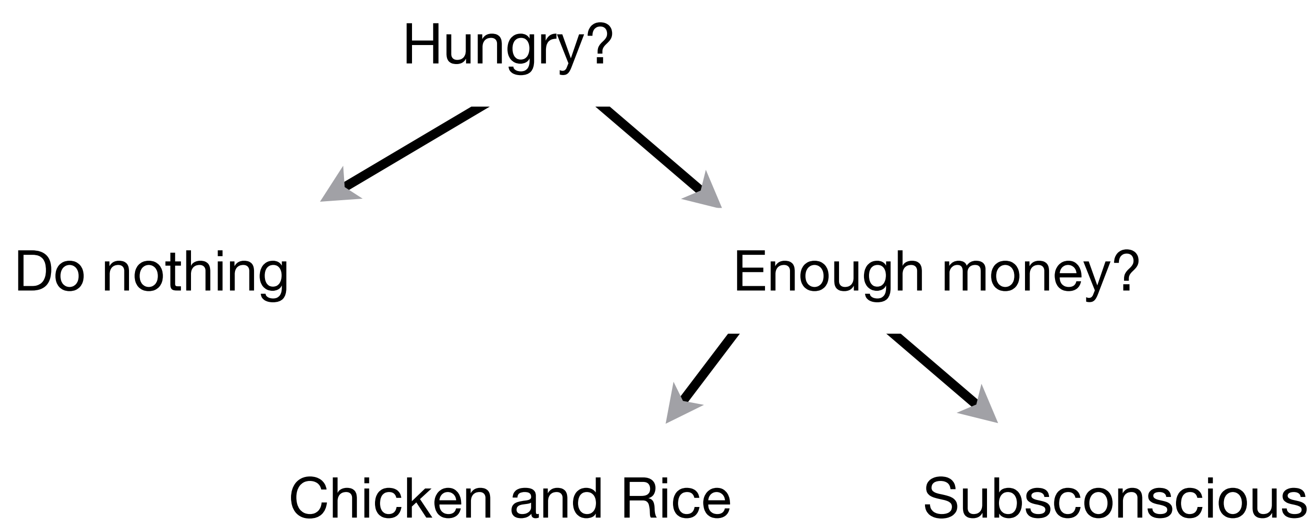
- The number of nodes at depth **d** is 2^d
- Total in a tree of depth **d** is $$\sum_{i=0}^{d} 2^i = 2^{d+1} - 1$$
    - (series identity)
- A perfect binary tree has $N = 2^{d+1} - 1$ nodes
- Solving for **d** finds $d = \log(N+1) - 1$

Expression Trees

- Expression Trees are yet another way to store mathematical expressions
  - ((x + y) * z)/300
- Note that the main mathematical operators have 2 operands each
- Inorder traversal reads back infix notation
- Postorder traversal reads postfix notation

Decision Trees

- It is often useful to design decision trees
- Left/right child represents yes/no answers to questions



Search (Tree) ADT

- ADT that allows insertion, removal, and searching by **key**
  - A **key** is a value that can be compared
  - In Java, we use the **Comparable** interface
  - Comparison must obey transitive property
- Notice that the Search ADT doesn't use any index

Binary Search Tree

- Binary Search Tree Property:
  Keys in left subtree are less than root.
  Keys in right subtree are greater than root.
- BST property holds for all subtrees of a BST

Inserting into a BST

- **insert(x)** is public method
- privately, use **insert(x, root)**
- **insert(x, Node t)**
  if **(t == null)** return new Node(x)
  if (**x > t.key**), then **t.right = insert(x, t.right)**
  if (**x < t.key**), then **t.left = insert(x, t.left)**
  return **t**

Searching a BST

- **findMin(t)**
  if (**t.left == null**) return **t.key**
  else return **findMin(t.left)**

- **contains(x,t)**
  if (**t == null**) return **false**
  if (**x == t.key**) return **true**
  if (**x > t.key**), then return **contains(x, t.right)**
  if (**x < t.key**), then return **contains(x, t.left)**

Deleting from a BST

- Removing a leaf is easy, removing a node with one child is also easy
- Nodes with no grandchildren are easy
- Nodes with both children and grandchildren need more thought
  - Why can’t we replace the removed node with either of its children?

A Removal Strategy

- First, find node to be removed, **t**
- Replace with the smallest node from the right subtree
  - **a = findMin(t.right);**
t.key = a.key;
- Then delete original smallest node in right subtree **remove(a.key, t.right)**

Average Case Analysis

- All operations run in O(d) time, but what is d?
  - Worst case d = N
  - Best case d = log(N+1)-1
  - Average case?

Average Case Analysis

- Consider the **internal path length**: the sum of the depths of all nodes in a tree
- Let **D(N)** be the internal path length for some tree **T** with **N** nodes*.
  - Suppose **i** nodes are in the left subtree of **T**.
  - Then $D(N) = D(i) + D(N - i - 1) + N - 1$

Average Case Analysis

- $D(N) = D(i) + D(N - i - 1) + N - 1$
  - Assume all insertion sequences are equally likely
  - Subtree sizes only depend on the 1st key inserted
    - all subtree sizes equally likely
- Average of **D(i)** (and **D(N-i-1)**) is $\frac{1}{N}\sum_{j=0}^{N-1} D(j)$

Average Case Analysis

- Average case **D(N)** then becomes

$$D(N) = \frac{2}{N}\left[\sum_{j=0}^{N-1} D(j)\right] + N - 1$$

- This is a **recurrence**, which can be solved to show that $D(N) = O(N \log N)$
  - (page 272-273 in Weiss)
- Then the average depth over all **N** nodes is $O(\log N)$

Looking Forward

- How do we implement Search Trees that explicitly avoid worst case O(N) operations?
  - What is the cost of avoiding worst case?

Reading

- Weiss Section 4.4: AVL Trees