Data Structures and Algorithms

Session 13. March 4, 2009

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Announcements

- Homework 3 is out. Due 3/9
- Sample midterm problems on Courseworks
- Midterm review March 9th
- Midterm Exam March 11th

Review

- Solving the Young Tableaux Recurrences
- buildHeap description and analysis
- HW2 solutions

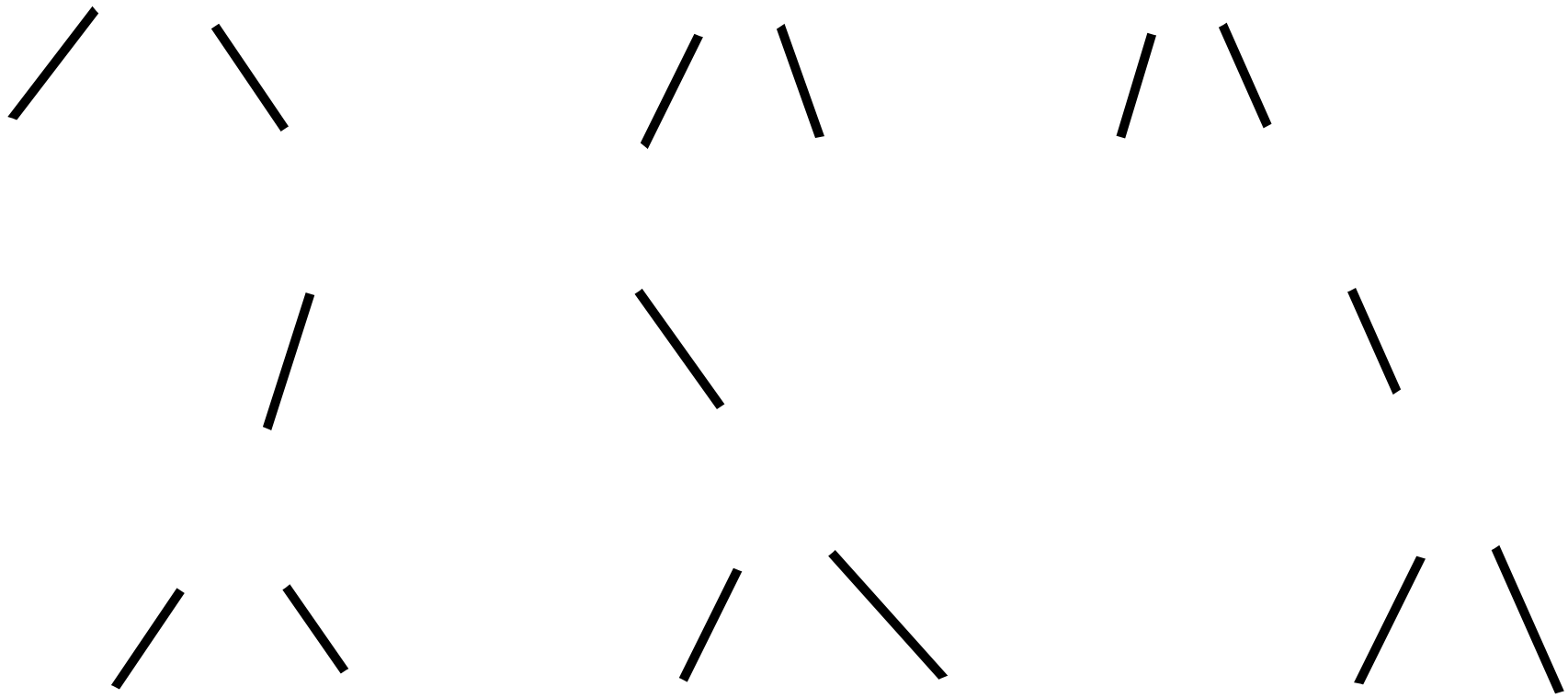
Today's Plan

- Clarification about isomorphism
- buildHeap example
- HeapSort and HeapSelect

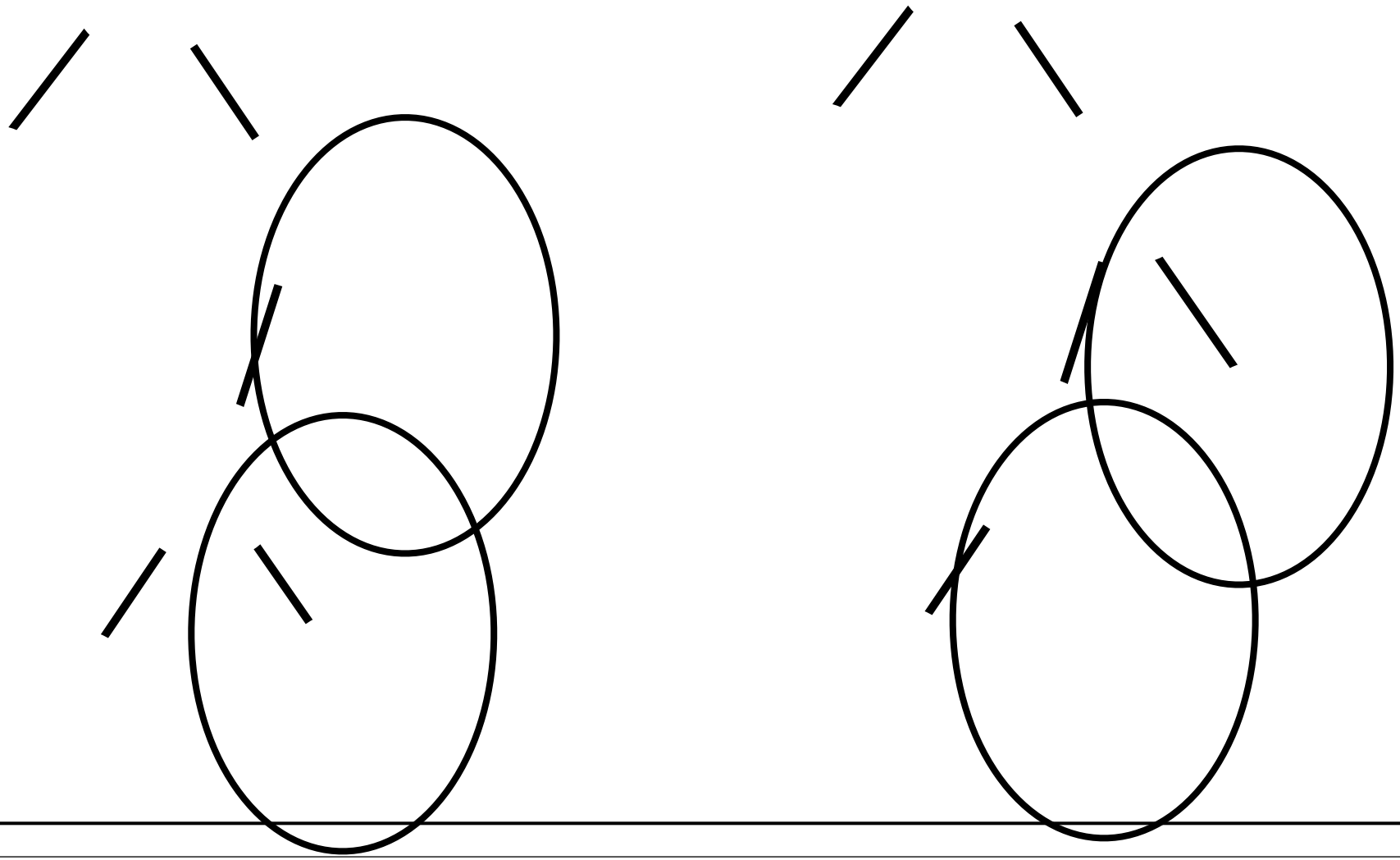
Isomorphism in Trees

- *iso* means equal, *morph* means shape
- Isomorphic means “having the same shape”
- Ignore left and right, element data
- Consider only *structural* properties:
  - How many children? How many siblings?
    How many cousins? How many grandchildren?

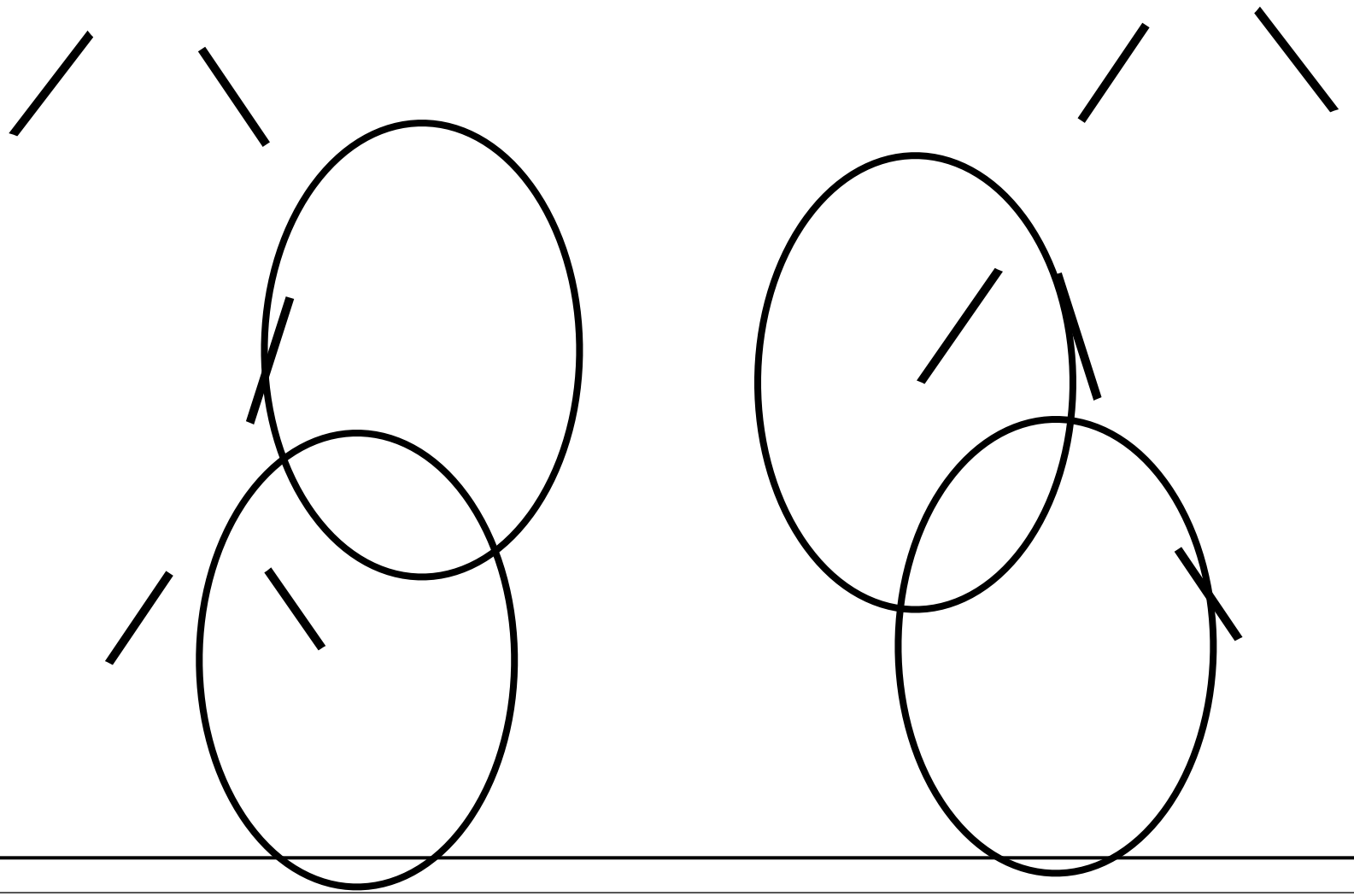
Isomorphic Binary Trees



Not Isomorphic Trees



Not Isomorphic Trees



buildHeap

- Start at deepest non-leaf node
  - in array, this is node N/2
- **percolateDown** on all nodes in reverse level-order
  - for i = N/2 to 1
    percolateDown(i)

6	3	11	7	14	8	5	15	1	2	4	13	9	10	12

Selection

- Recall the selection problem: findKth(k,A[ ])
  - find the kth smallest element in A
- Old method:
  - Sort the array, then return the kth element
  - Sort the first k elements, insert the remaining (N-k) elements in the proper place

HeapSelect

- run buildHeap on the array A
  - O(N)
- call deleteMin() k times
  - k O(log N) = O(k log N)
- HeapSelect runs in O(N+k log N)

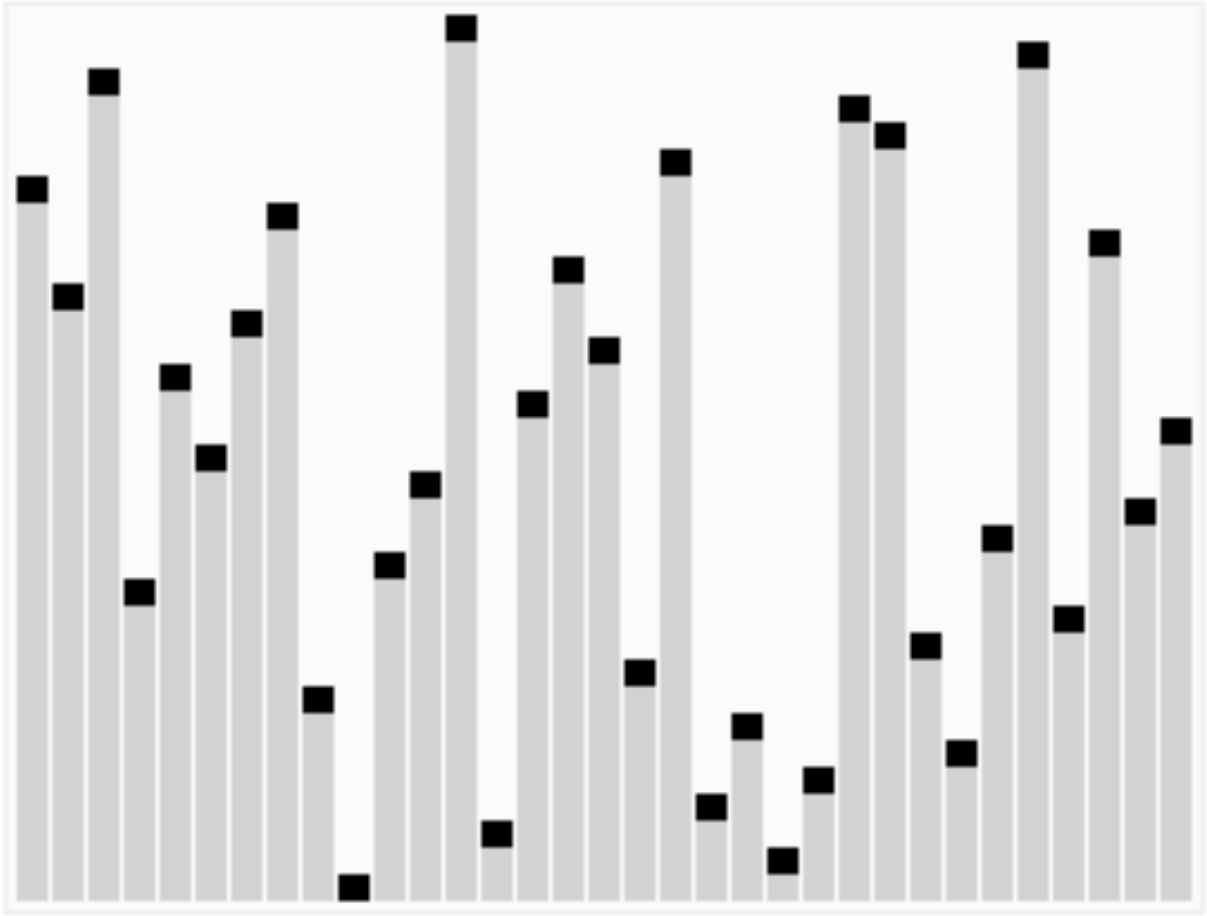
HeapSort

- Naturally, we can use this idea to sort the array
- Method 1:
  - buildHeap on the array
  - copy output of deleteMin into new array N times
  - buildHeap costs O(N), deleteMin costs O(log N)
    - Heapsort 1 costs O(N+N log N)=O(N log N)

HeapSort in Place

- We don’t need to allocate a new array
- Instead, use a **max-heap**
  - Reverse the heap order property: deleteMax
- After each deleteMax, heap size is 1 less
  - Stick the extracted max in the freed space

HeapSort Animation



Downloaded from http://en.wikipedia.org/wiki/Heapsort

Assignments

- Continue HW3
- Weiss 7.5 if you want to read about HeapSort
- Practice midterm samples