Data Structures in Java

Session 8

Instructor: Bert Huang

http://www1.cs.columbia.edu/~bert/courses/3134

Announcements

- Homework 2 released on website
  - Due Oct. 6th at 5:40 PM (next class)
- Post homework to Shared Files, Homework #2
- Distinguished Lecture

CS Distinguished Lecture

- Network Evolution, Network Economics, and Network Innovation
  - Andrew Odlyzko
- Monday, October 5^{th}.
- 11:00 AM Davis Auditorium Schapiro/CEPSR

Review

- Lists, Stacks, Queues in Linux
- Introduction to Trees
  - Definitions
  - Tree Traversal Algorithms
- Binary Trees

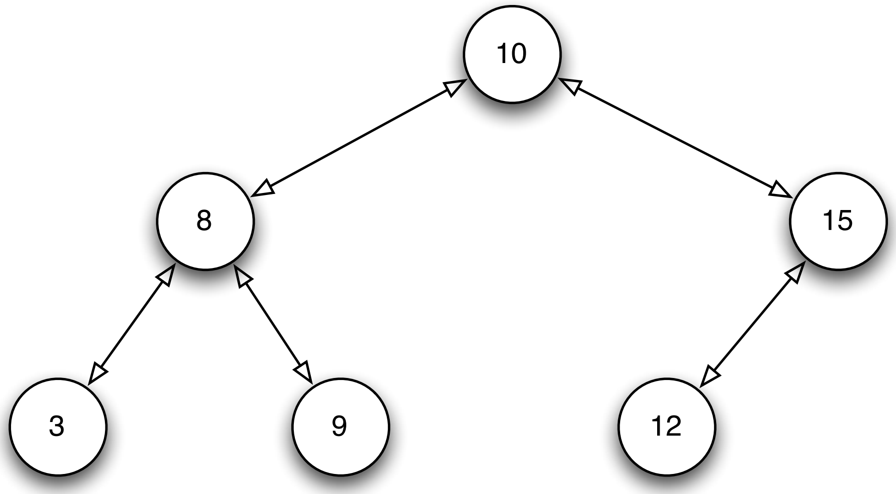
Today's Plan

- Search Tree ADT
- Binary search tree
  - implementation
  - running time analysis

Search (Tree) ADT

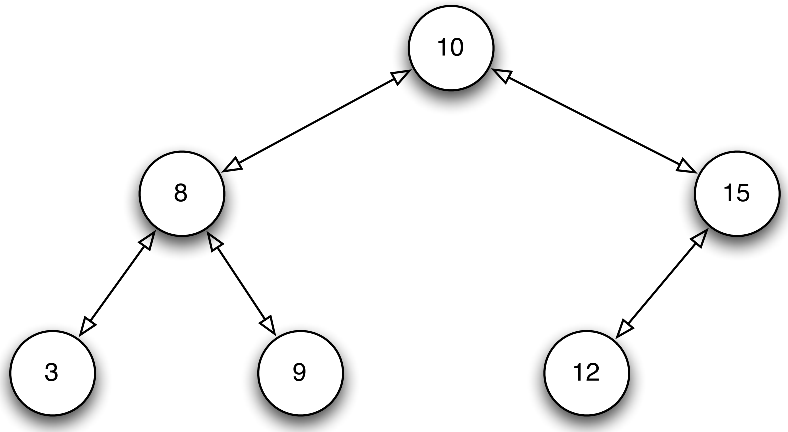
- ADT that allows insertion, removal, and searching by **key**
  - A **key** is a value that can be compared
  - In Java, we use the **Comparable** interface
  - Comparison must obey transitive property
- Search ADT doesn't use any index

Binary Search Tree



- Binary Search Tree Property:
  Keys in left subtree are less than root.
  Keys in right subtree are greater than root.
- BST property holds for all subtrees of a BST

Inserting into a BST



- Compare new value to current node, if greater, insert into right subtree, if lesser, insert into left subtree
- **insert(x, Node t)**
  if **(t == null)** return new Node(x)
  if (**x > t.key**), then **t.right = insert(x, t.right)**
  if (**x < t.key**), then **t.left = insert(x, t.left)**
  return **t**

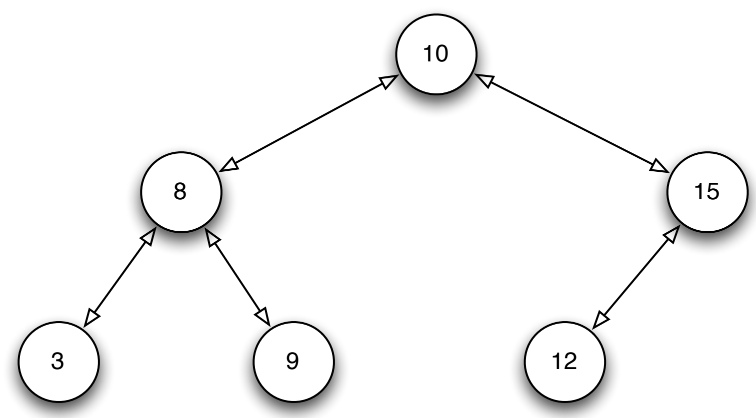
Searching a BST

- **findMin(t)** // return left-most node
  if (**t.left == null**) return **t.key**
  else return **findMin(t.left)**

- **search(x,t)** // similar to insert
  if (**t == null**) return **false**
  if (**x == t.key**) return **true**
  if (**x > t.key**), then return **search(x, t.right)**
  if (**x < t.key**), then return **search(x, t.left)**

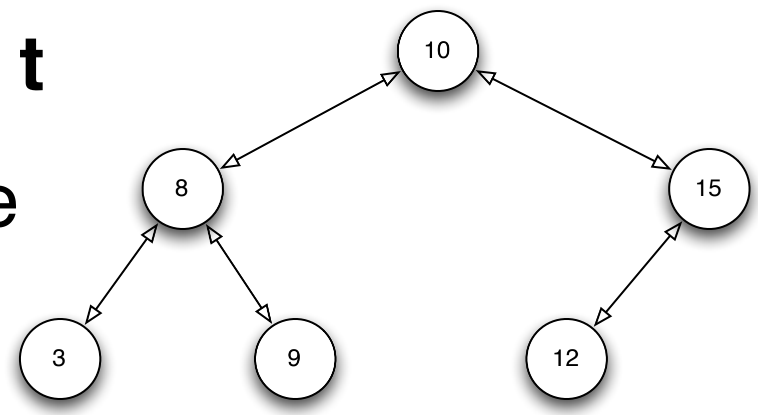
Deleting from a BST

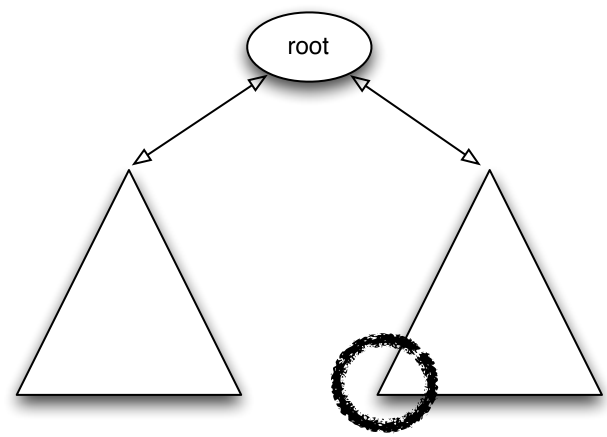
- Removing a leaf is easy, removing a node with one child is also easy
- Nodes with no grandchildren are easy
- What about nodes with grandchildren?



A Removal Strategy

- First, find node to be removed, **t**
- Replace with the smallest node from the right subtree
  - **a = findMin(t.right);**
    t.key = a.key;
- Then delete original smallest node in right subtree
  remove(a.key, t.right)



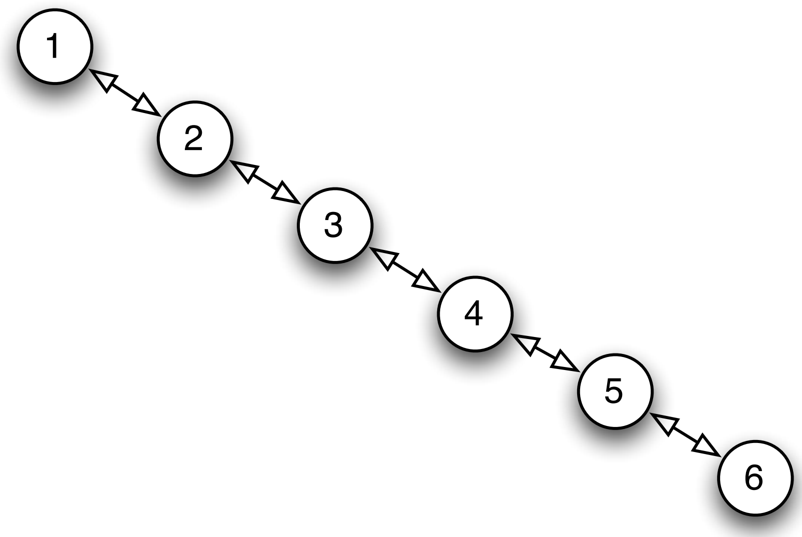


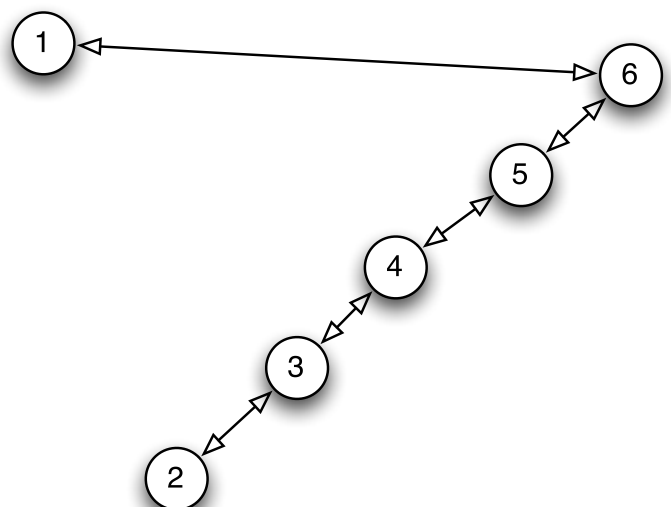
Sorting with BST

- Suppose we have a built BST
- How to print out nodes in order?
  - inorder traversal
- Running time?
  - O(N)

Worst Case

- Operations: insert, remove, search
- Worst case insert sequence: 1,2,3,4,5,6
  - search(6)
- Insert 1,6,5,4,3,2 Remove(1)





Average Case Analysis

- All operations run in O(d) time, but what is d?
  - Worst case d = N
  - Best case d = log(N+1)-1
  - Average case?

Average Case Analysis

- Consider the **internal path length**: the sum of the depths of all nodes in a tree
- Let **D(N)** be the internal path length for some tree **T** with **N** nodes*.
  - Suppose **i** nodes are in the left subtree of **T**.
  - Then $D(N) = D(i) + D(N - i - 1) + N - 1$

Average Case Analysis

- $D(N) = D(i) + D(N - i - 1) + N - 1$
  - Assume all insertion sequences are equally likely
  - Subtree sizes only depend on the 1st key inserted
    - all subtree sizes equally likely
- Average of **D(i)** (and **D(N-i-1)**) is $\frac{1}{N} \sum_{j=0}^{N-1} D(j)$

Average Case Analysis

- Average case **D(N)** then becomes

$$D(N) = \frac{2}{N}\left[\sum_{j=0}^{N-1} D(j)\right] + N - 1$$

- This is a **recurrence**, which can be solved to show that $D(N) = O(N \log N)$
  - (page 272-273 in Weiss)
- Then average depth over all **N** nodes is $O(\log N)$

Looking Forward

- How do we implement Search Trees that explicitly avoid worst case O(N) operations?
- Intuition: try to keep the tree balanced
- What is the cost of this balancing?

Reading

- This class: Section 4.3
- Next class: Section 4.4 – AVL Trees