Merging Globally Rigid Formations of Mobile Autonomous Agents

Tolga Eren
Columbia University
Department of Computer Science
eren@cs.columbia.edu

Brian D.O. Anderson
Australian National University
and National ICT Australia
Brian.Anderson@nicta.com.au

Walter Whiteley
York University
Department of Mathematics
whiteley@mathstat.yorku.ca

A. Stephen Morse
Yale University
Department of Electrical Engineering
as.morse@yale.edu

Peter N. Belhumeur
Columbia University
Department of Computer Science
belhumeur@cs.columbia.edu

Abstract

This paper focuses on developing techniques and strategies for the analysis and design of sensor and network topologies required to merge globally rigid formations for cooperative tasks. Central to the development of these techniques and strategies is the use of tools from rigidity theory, and graph theory.

1. Introduction and Preliminaries

A *formation* is defined as a group of mobile agents moving in real 2- or 3-dimensional space. A formation is *rigid* if the distance between each pair of agents does not change over time, at least under ideal conditions. A formation is called *globally rigid*, if the distance between each pair of agents is unambiguous. In a rigid formation, distances between agents are held fixed by measurements and information gathered through "sensing and communication links" between agents. Distances between all agent pairs can be held fixed by directly measuring distances between only some agents and keeping them at desired values. It is also true that it is not necessary to have sensing and communication links between each pair of agents to create a globally rigid formation. We refer the reader to [1, 3, 4] for an introduction to the approaches based on rigidity and global rigidity for maintaining formations of autonomous agents.

A key element in all future multi-agent systems will be the role of sensor and communications networks as an integral part of coordination. One of the challenges in building sensor and communications networks between agents is the "topology" of the network. By *topology*, we mean the interconnection structure of sensing and communication links among agents. In other words, topology refers to the network's layout. Two networks have the same topology if the interconnection structure is the same, although the networks may differ in physical interconnections, distances between agents, transmission rates, and signal types. Network topologies, energy efficiency and communication bandwidth are critically important for autonomous systems involving mobile underwater, ground and air vehicles, and for sensor networks. Hence strategies that make efficient use of power and energy are beneficial. Therefore, we use topologies for providing sensing and communications with the minimum number of links, and propose methods requiring the minimum number of changes in the set of links in merging rigid sub-formations.

By *merging*, we mean two types of operations. By the first type of merging, we mean the following: Two globally rigid sub-formations are merged to form one single globally rigid formation. Finding new links to be inserted between these two sub-formations, which will make the whole formation globally rigid, is the first type of the merging problem (see Figure 1). By the second type of merging, we mean the following: Two globally rigid sub-formations, which share some agents and links, form one single formation. Finding which agents need to be shared between these two sub-formations so that the whole formation is globally rigid, is the second type of the merging problem (see Fig-

AAMAS'04, July 19-23, 2004, New York, New York, USA.

ure 2). During a merging operation, it is a natural starting point to preserve the links in each pre-merged rigid sub-formation. Hence a reasonable goal in the first type of a merging operation is to create a new post-merged rigid formation by inserting minimum number of links between sub-formations. The goal in the second type of a merging operation is to share minimum number of agents between sub-formations.

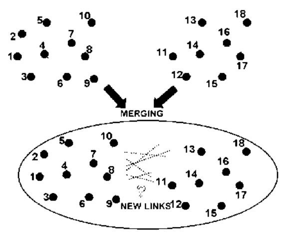

Figure 1. The first type of the merging problem. See text for explanation.

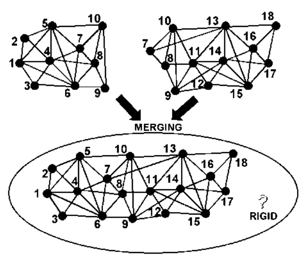

Figure 2. The second type of the merging problem. See text for explanation.

By a d-dimensional *point formation* at $p \triangleq$ column $\{p_1, p_2, \ldots, p_n\}$, written $\mathbb{F}_p$, is meant a set of n points $\{p_1, p_2, \ldots, p_n\}$ in $\mathbb{R}^d$ together with a set $\mathcal{L}$ of k *maintenance links*, labelled (i, j), where i and j are distinct integers in $\{1, 2, \ldots, n\}$; the *length* of link (i, j) is the Euclidean distance between points p_i and p_j. For our purposes, a point formation $\mathbb{F}_p = (\{p_1, p_2, \ldots, p_n\}, \mathcal{L})$ provides a natural high-level model for a set of n agents moving in real 2- or 3- dimensional space. In this context, the points p_i represent the positions of agents in $\mathbb{R}^d$ $\{d = 2 \text{ or } 3\}$ and the links in $\mathcal{L}$ label those specific agent pairs whose inter-agent distances are to be maintained over time.

2. Results

Now we present the main results of this paper. All the proofs of the theorems can be found in [2].

Connecting Globally Rigid Sub-Formations in 2-Dimensional Space:

Theorem 1. *Suppose that two globally rigid sub-formations $\mathbb{F}_1$ and $\mathbb{F}_2$, are connected by a set of links $\mathcal{L}$. Then $\mathbb{F}_1 \cup \mathbb{F}_2 \cup \mathcal{L}$ is globally rigid if the following two conditions hold: (i) The end points of $\mathcal{L}$ has at least three points in $\mathbb{F}_1$ and has at least three points in $\mathbb{F}_2$. (ii) There are at least four links in $\mathcal{L}$.*

Connecting Globally Rigid Sub-Formations in 3-Dimensional Space:

Theorem 2. *Assume that two globally rigid sub-formations $\mathbb{F}_1$ and $\mathbb{F}_2$, are connected by a set of links $\mathcal{L}$. Then $\mathbb{F}_1 \cup \mathbb{F}_2 \cup \mathcal{L}$ is globally rigid if the following two conditions hold: (i) The end points of $\mathcal{L}$ has at least four points in $\mathbb{F}_1$ and has at least four points in $\mathbb{F}_2$. (ii) There are at least seven links in $\mathcal{L}$.*

Globally Rigid Sub-Formations Sharing Points in 2-Dimensional Space:

Theorem 3. *If two globally rigid sub-formations, $\mathbb{F}_1$ and $\mathbb{F}_2$, share at least three points, then the formation $\mathbb{F}_1 \cup \mathbb{F}_2$ is globally rigid.*

Globally Rigid Sub-Formations Sharing Points in 3-Dimensional Space:

Theorem 4. *If two globally rigid sub-formations, $\mathbb{F}_1$ and $\mathbb{F}_2$, share at least four points, then the formation $\mathbb{F}_1 \cup \mathbb{F}_2$ is globally rigid.*

References

[1] T. Eren, B.D.O. Anderson, A.S. Morse, W. Whiteley, and P.N. Belhumeur. Information structures to control formation splitting and merging. In *Proceedings of the American Control Conference*, Boston, Massachusetts, USA, 2004. Available at http://www.cs.columbia.edu/~eren.

[2] T. Eren, B.D.O. Anderson, W. Whiteley, A.S. Morse, and P.N. Belhumeur. Merging globally rigid formations of mobile autonomous agents. *Preprint*, 2004. Available at http://www.cs.columbia.edu/~eren.

[3] T. Eren, D. Goldenberg, W. Whiteley, A.S. Morse, B.D.O. Anderson, and P.N. Belhumeur. Rigidity, computation, and randomization in network localization. In *Proceedings of the IEEE INFOCOM Conference*, Hong Kong, 2004. Available at http://www.cs.columbia.edu/~eren.

[4] T. Eren, W. Whiteley, A.S. Morse, B.D.O. Anderson, and P.N. Belhumeur. Information structures to secure control of globally rigid formations. In *Proceedings of the American Control Conference*, Boston, Massachusetts, USA, 2004. Available at http://www.cs.columbia.edu/~eren.