Data Structures in Java

Lecture 20: Algorithm Design Techniques

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Algorithms and Problem Solving

- Purpose of algorithms: find solutions to problems.
- Data Structures provide ways of organizing data such that problems can be solved more efficiently
  - Examples: Hashmaps provide constant time access by key, Heaps provide a cheap way to explore different possibilities in order…
- When confronted with a new problem, how do we:
  - Get an idea of how difficult it is?
  - **Develop an algorithm to solve it?**

Common Types of Algorithms

- Greedy Algorithms
- Divide and Conquer
- Dynamic Programming

- We have already seen some examples for each.
- We will look at the general techniques and some additional examples.

Greedy Algorithms

"Take what you can get now"

- Algorithm uses multiple "phases" or "steps". In each phase a local decision is made that appears to be good.
- Making a local decision is fast (often O(log N) time). Examples: Dijkstra's, Prim's, Kruskal's
- Greedy algorithms assume that making ***locally optimal decisions*** leads to a ***global optimum.***
  - This works for some problems.
  - For many others it doesn't. Greedy algorithms are still useful to find approximate solutions.

ASCII Encoding

Character	Decimal	Binary
⋮		
A	65	1000001
B	66	1000010
C	67	1000011
D	68	1000100
E	69	1000101
⋮		
a	97	1100000
b	98	1100001
c	99	1100010
d	100	1100011
e	101	1100100
⋮		

- The ASCII codec contains 128 characters (about 100 printable characters + special chars).
- Each character needs $\lceil \log 128 \rceil = 7$ bits of space.

Can we store data more efficiently?

A 5-Character Alphabet

Character	Decimal	Binary
a	0	“000"
e	1	“001"
i	2	“010"
s	3	“011"
t	4	“100"
space	5	“101"
newline	6	“110"

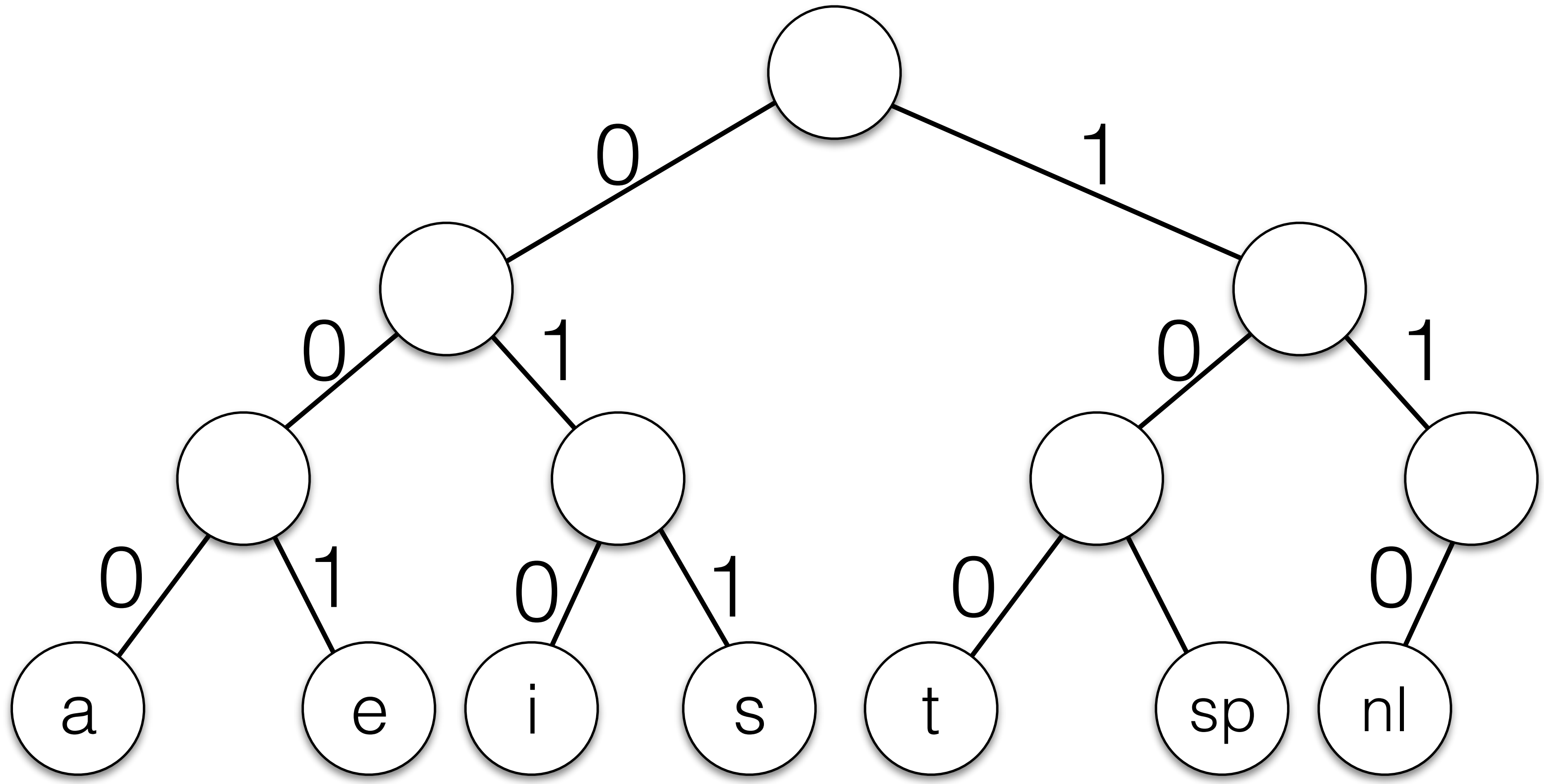
A 5-Character Alphabet

Assume we see each character with a certain frequency in a textfile. We can then compute the total number of bits required to store the file.

Character	Decimal	Binary Code	Frequency	Total bits
a	0	"000"	10	30
e	1	"001"	15	45
i	2	"010"	12	36
s	3	"011"	3	9
t	4	"100"	4	12
space	5	"101"	13	39
newline	6	"110"	1	3
				Total: 175

Prefix Trees

Character	Binary
a	“000"
e	“001"
i	“010"
s	“011"
t	“100"
space	“101"
newline	“110"



d_i depth of character i

f_i frequency of i in the file

$$\text{file size} = \sum_i d_i f_i$$

Prefix Trees

Character	Binary
a	“000"
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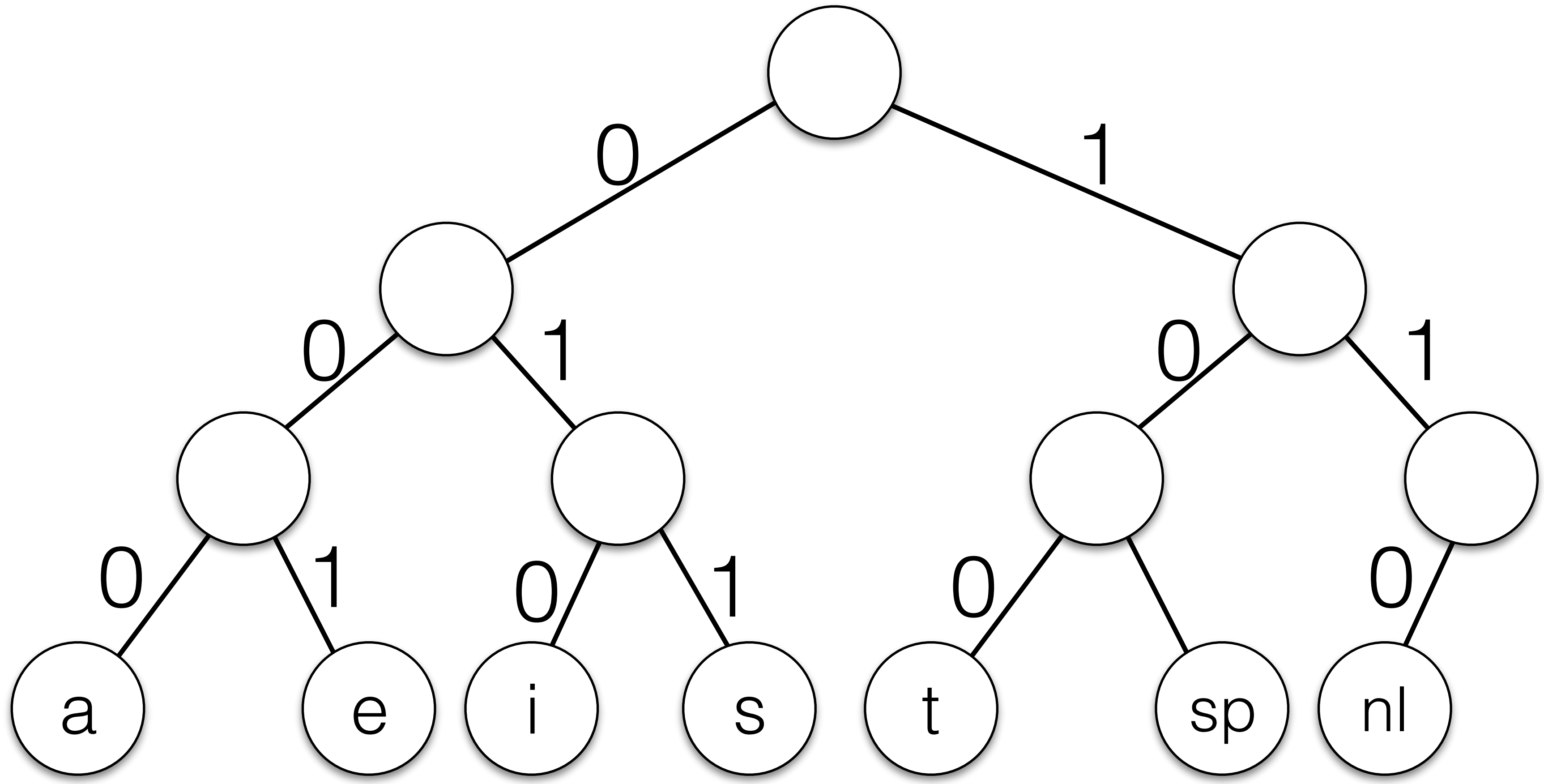

d_i depth of character i

$$\text{file size} = \sum_i d_i f_i$$

f_i frequency of i in the file

Can we restructure the tree to minimize the file size?

Prefix Trees

Character	Binary
a	“000"
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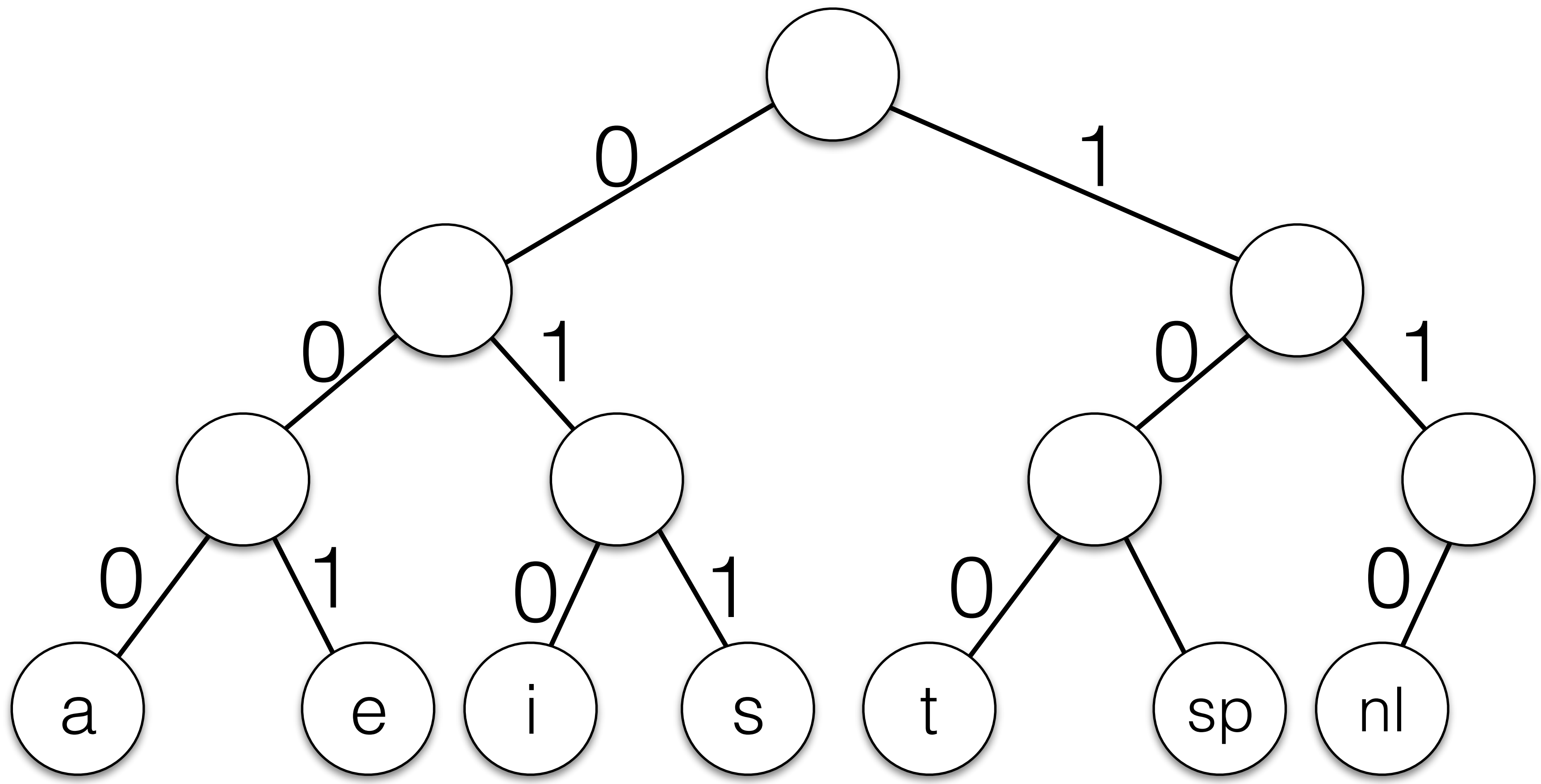
newline	“110"



d_i depth of character i

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f_i frequency of i in the file

Prefix “11” is not used for any other character than *nl.*

Prefix Trees

Character	Binary
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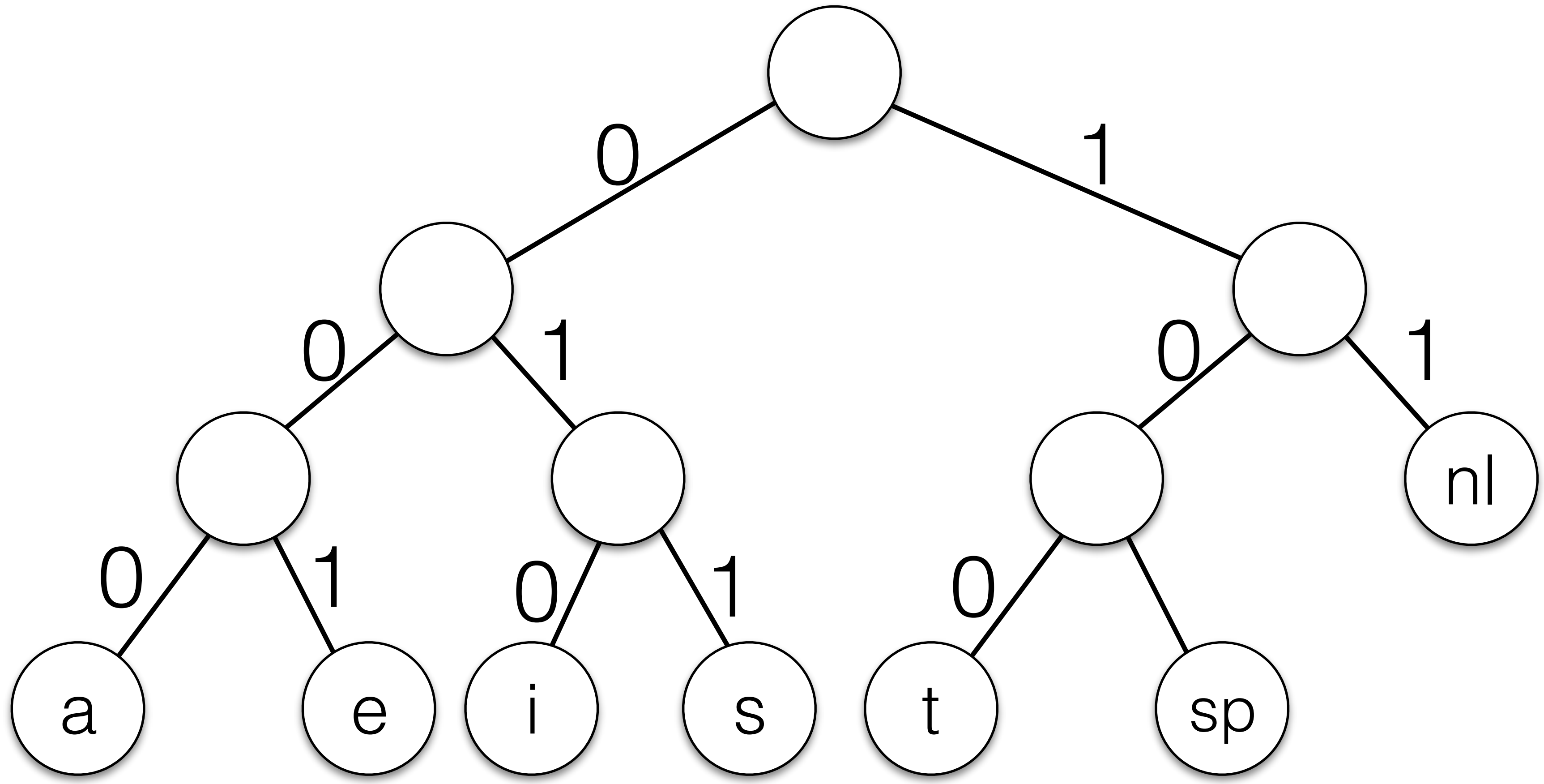
e	“001"
i	“010"
s	“011"
t	“100"
space	“101"
newline	“11"



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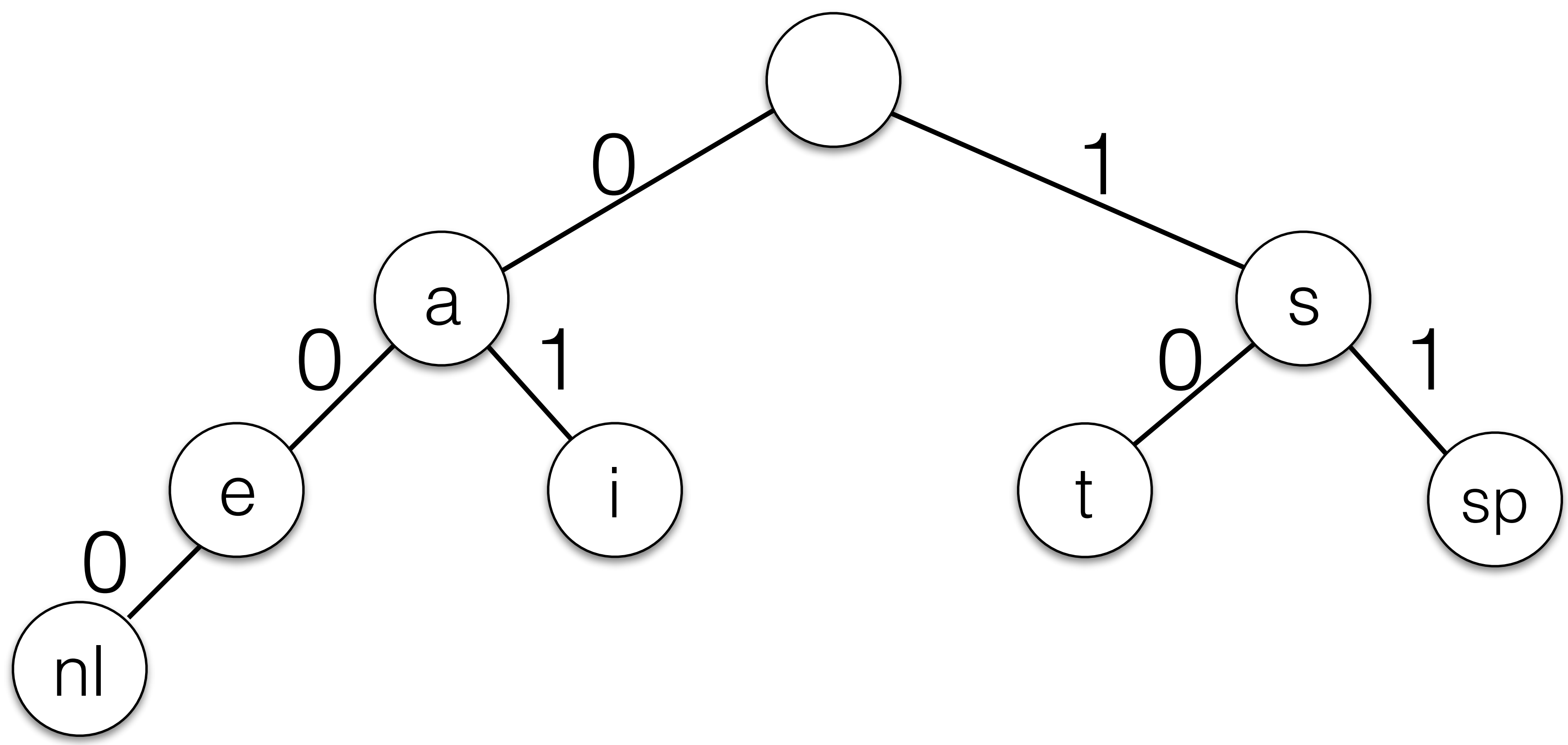
Prefix Trees

We cannot place characters on interior nodes, or else encoded sequences would be ambiguous.



000110

Prefix Trees

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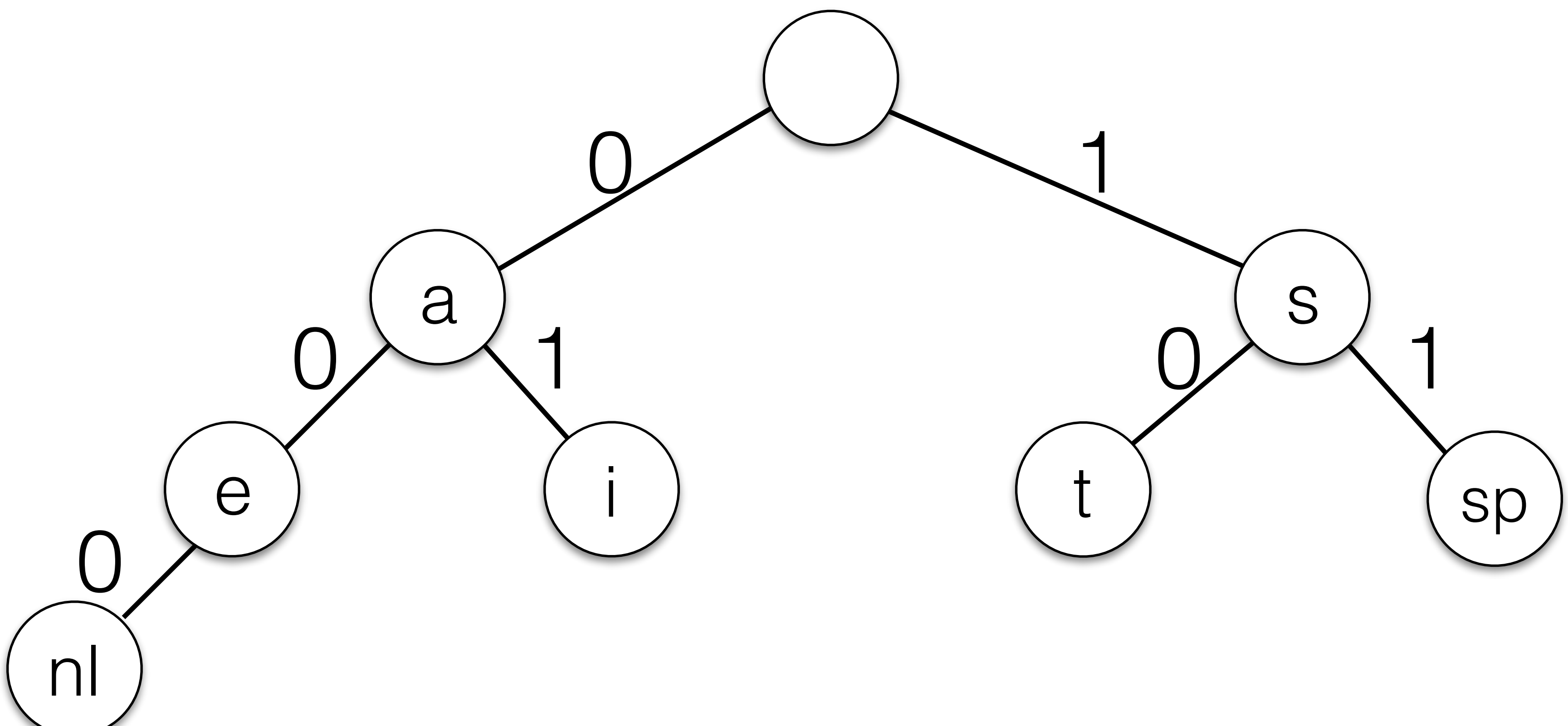

00_01_**10**

e i t

Prefix Trees

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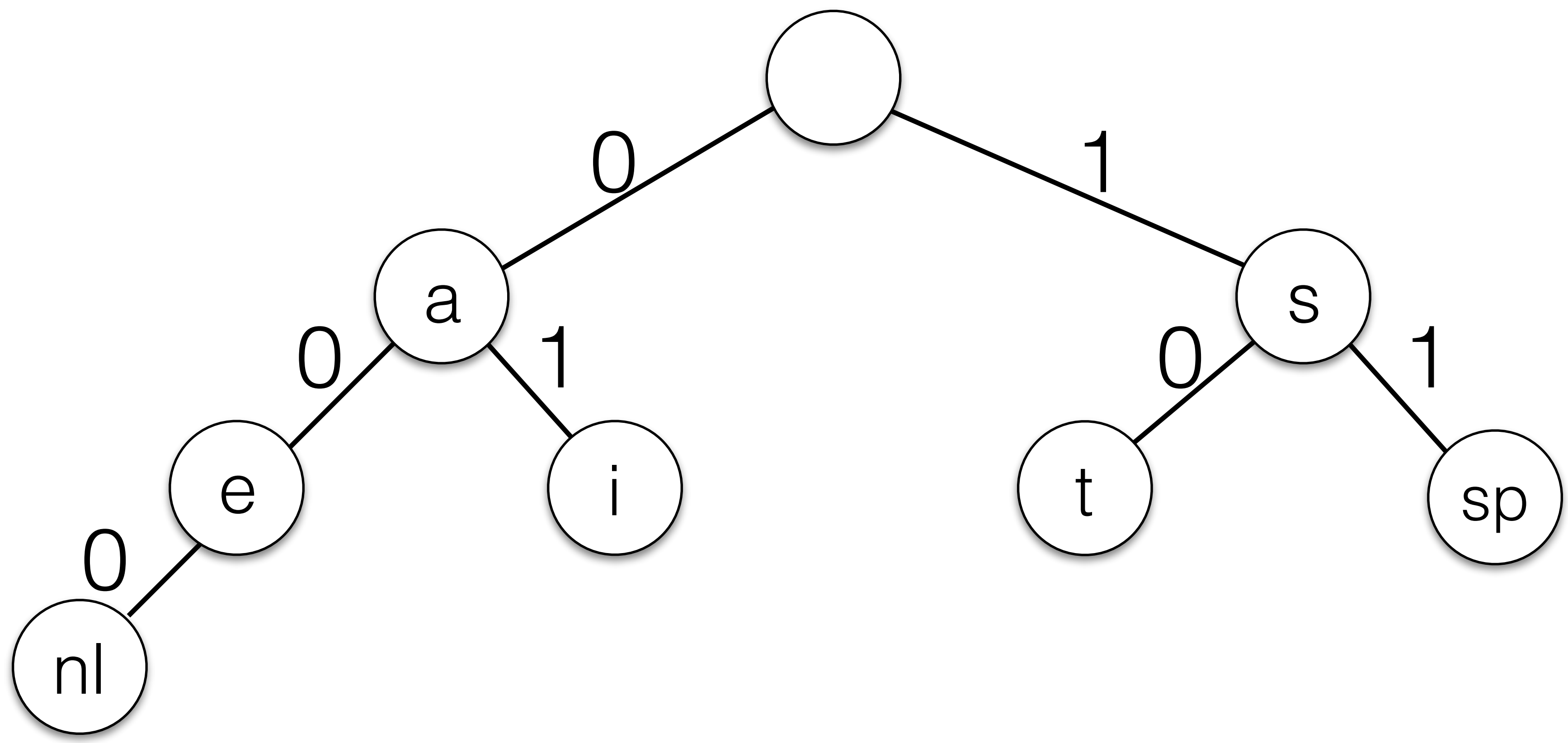
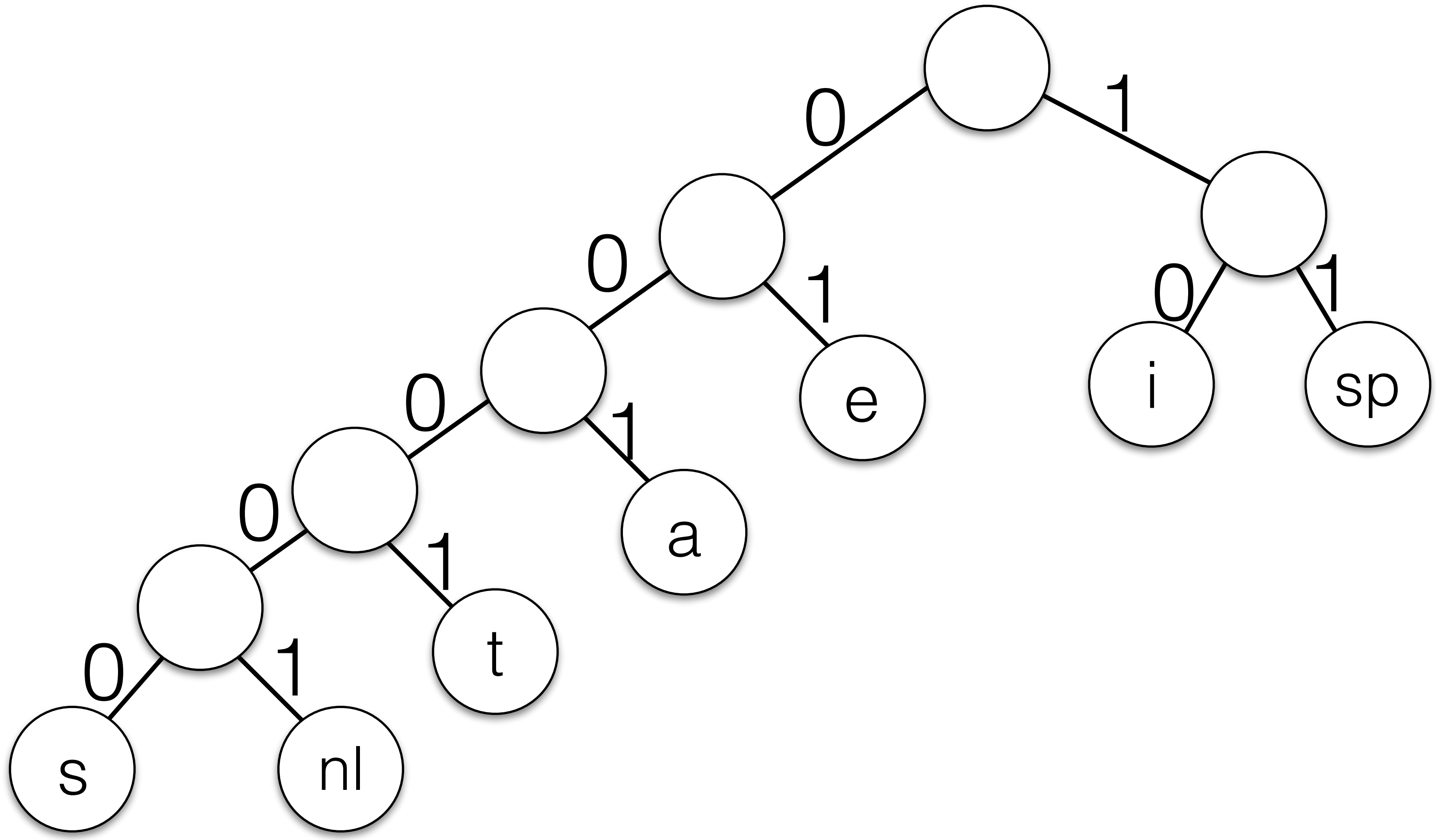


000 *1 1* **0**
nl sp t

Huffman Code

chr	bin	f_i
e	“01"	15
sp	“11"	13
i	“10"	12
a	“001”	10
t	“0001”	4
s	“00000”	3
nl	“00001"	1



$$\text{file size} = \sum_i d_i f_i$$

- All characters are at leaves.
- Frequent characters have short codes.
- Rare characters have long codes.

Huffman Code

chr	bin	f_i
e	“01"	15
sp	“11"	13
i	“10"	12
a	“001”	10
t	“0001”	4
s	“00000”	3
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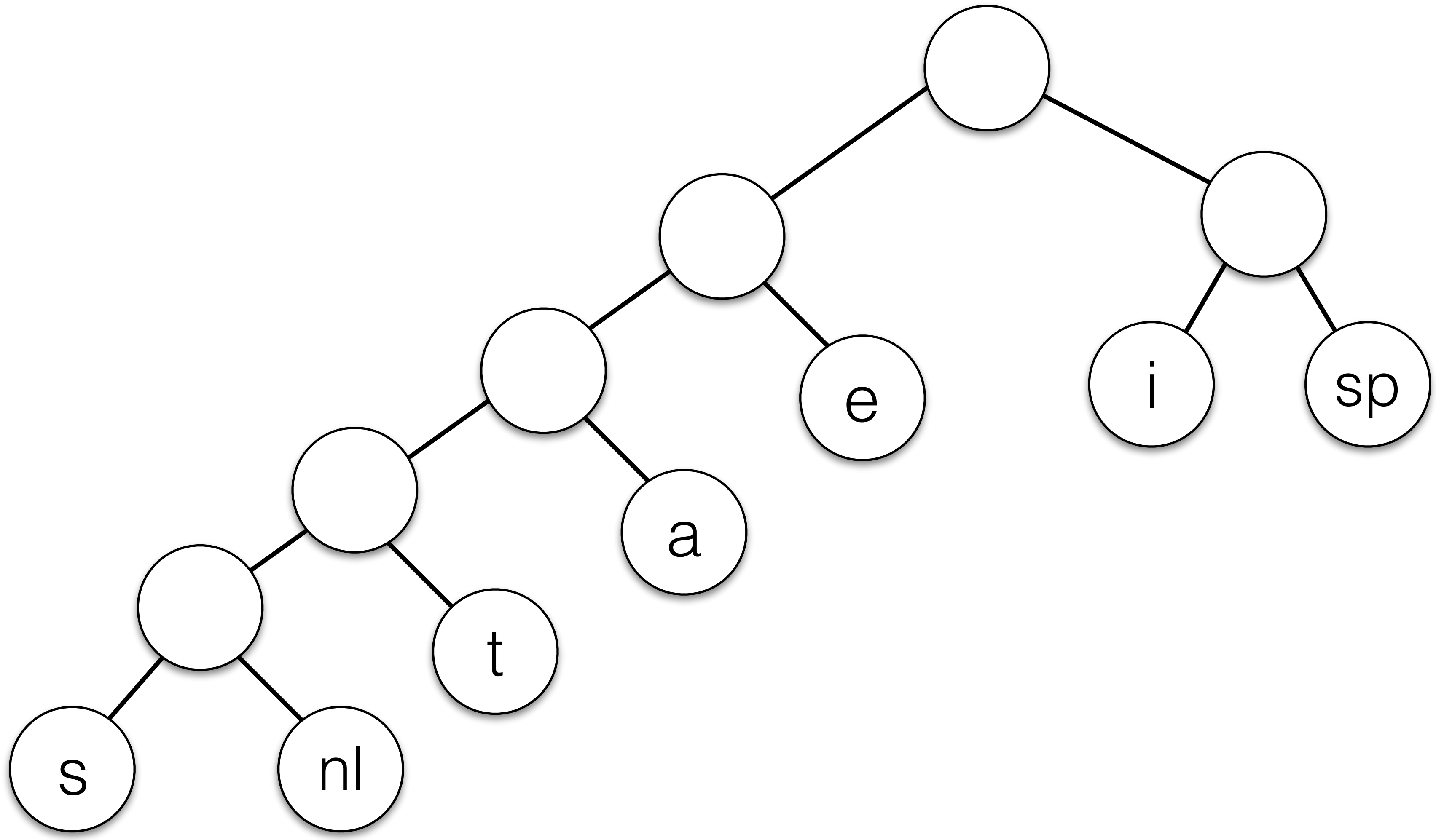
Total size: 146



0000001001000100001

- This example: Save 16% space compared to standard coding.
- Typically much better compression (for larger files and alphabets).

Huffman Code

chr	**bin**	**f_i**
e	“01"	15
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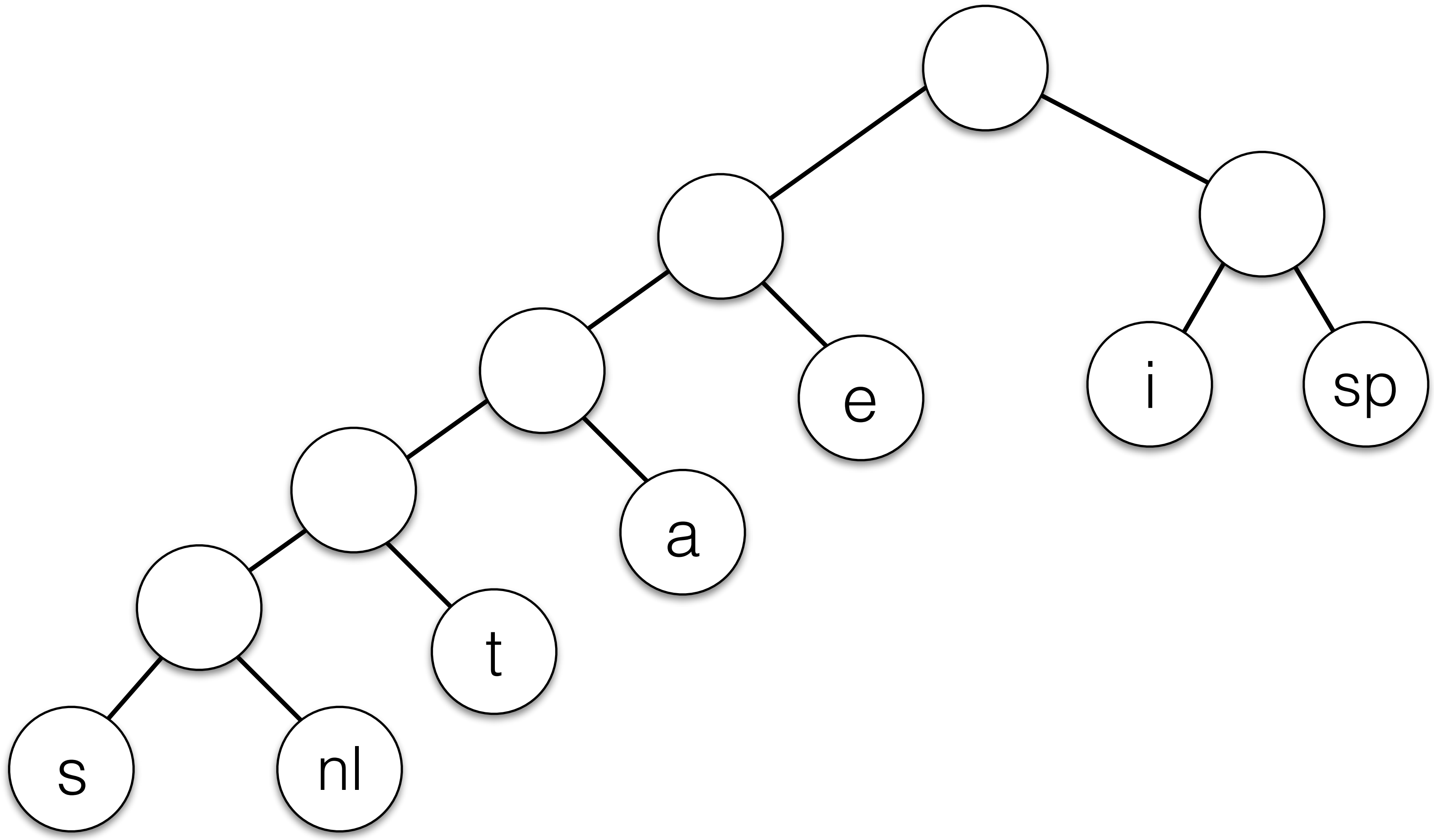

0000001**001**0001**00001**

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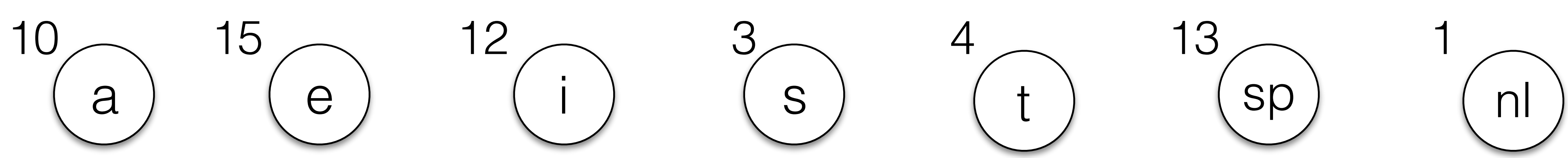
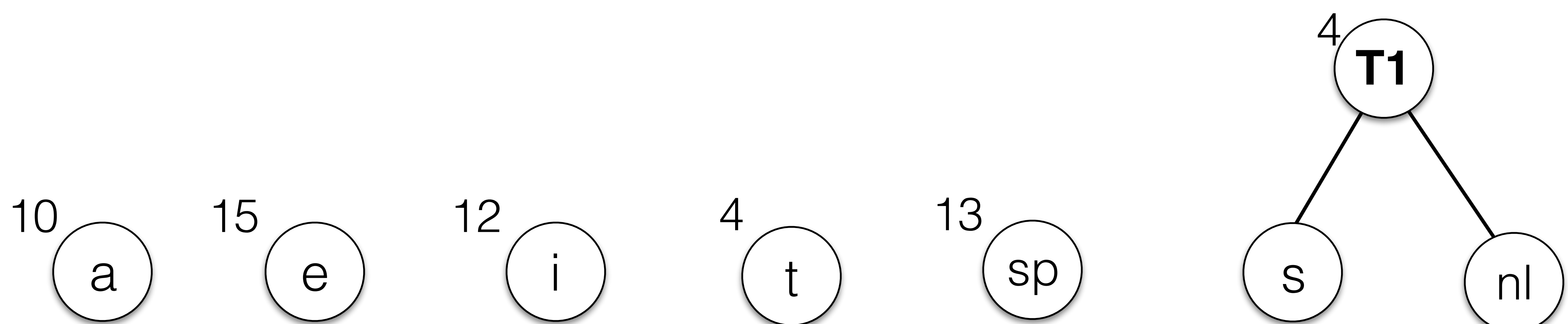
Huffman's Algorithm



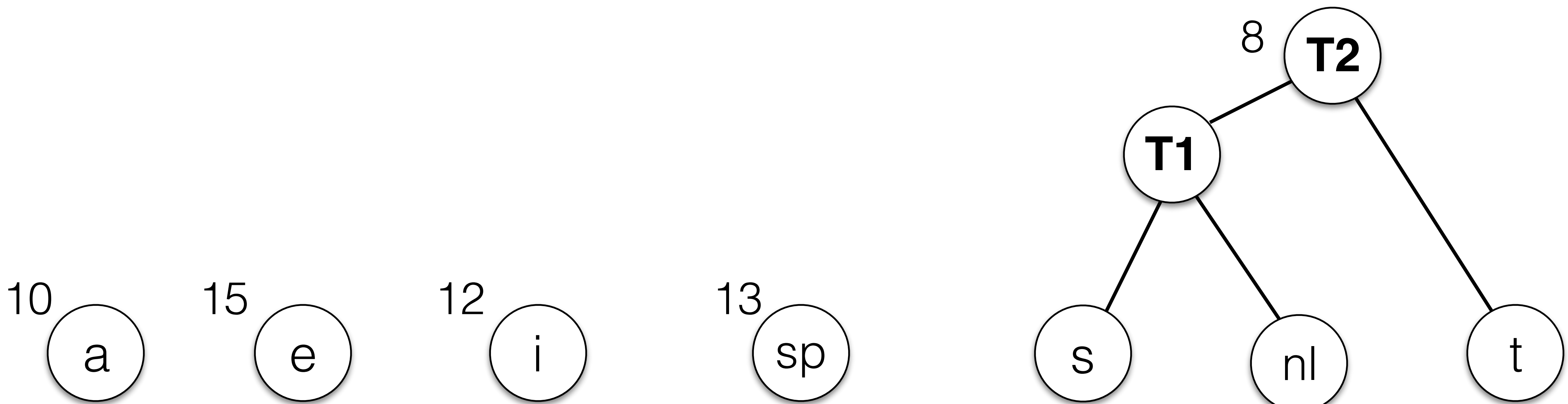
- Maintain a forest of prefix trees.
- Weight of a tree T = sum of frequencies of characters in T.

Huffman's Algorithm



- In every phase:
  - Choose the two trees with smallest weight and merge them.

Huffman's Algorithm



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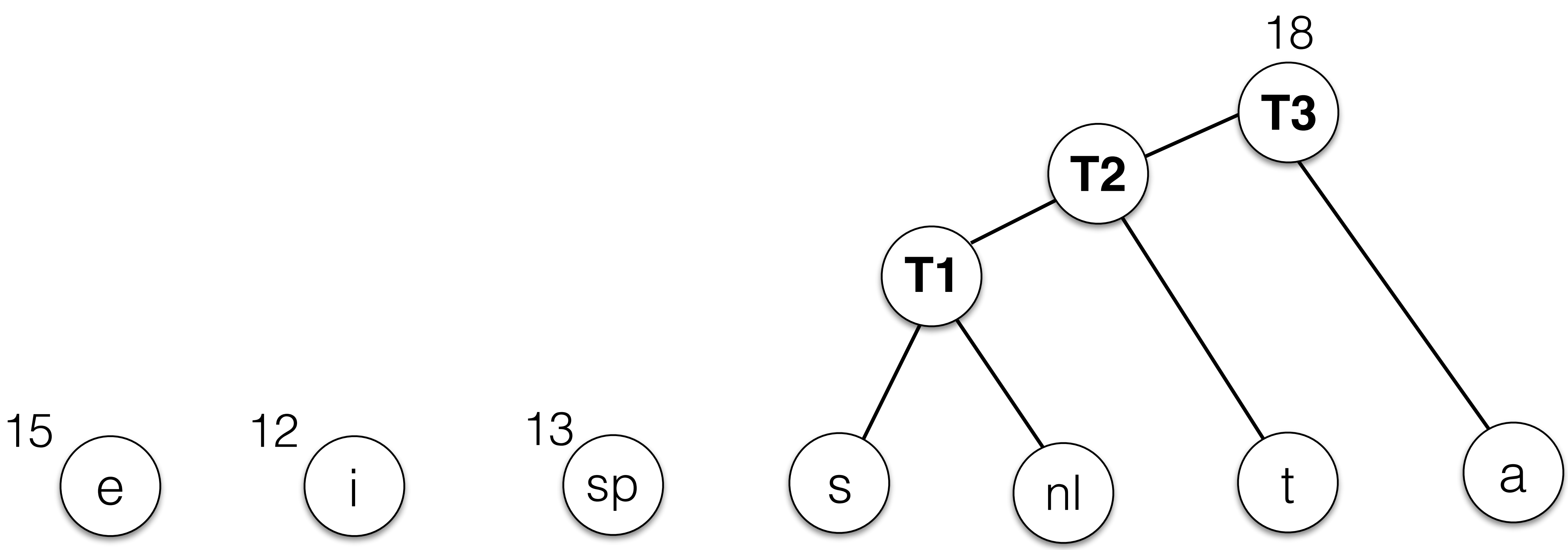
Huffman's Algorithm



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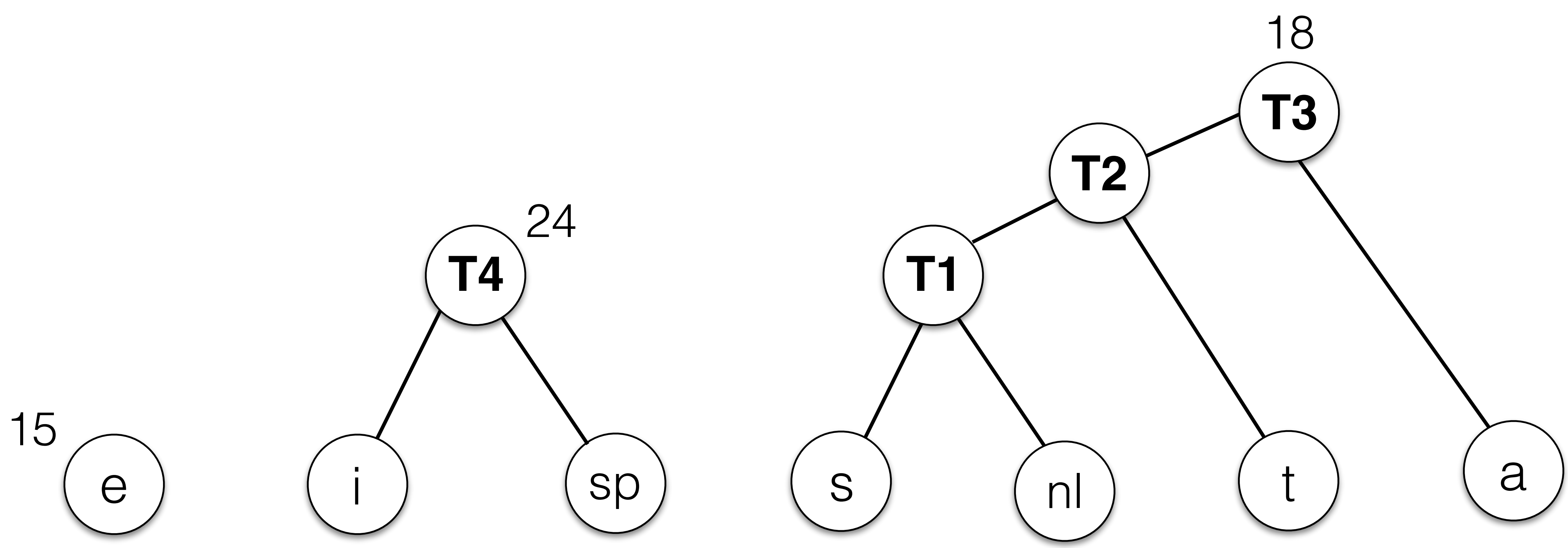
Huffman's Algorithm



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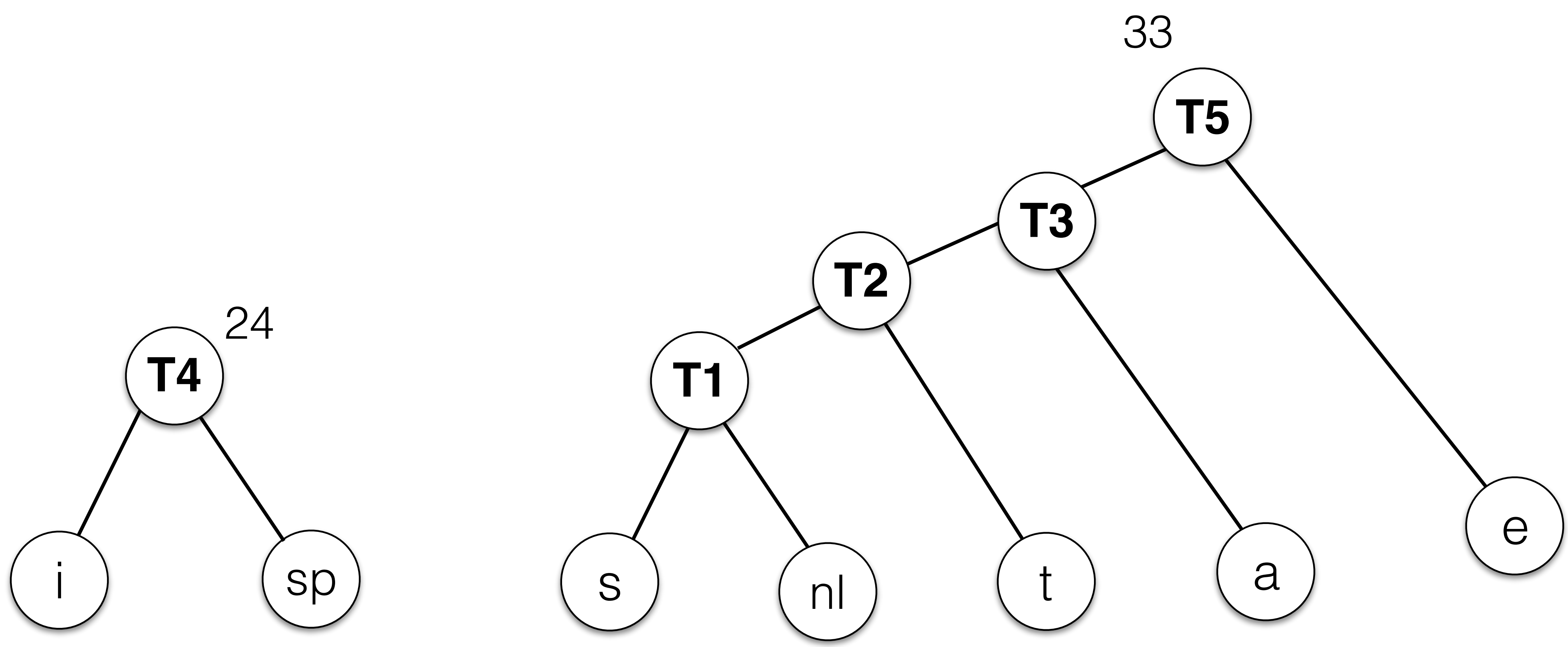
Huffman's Algorithm



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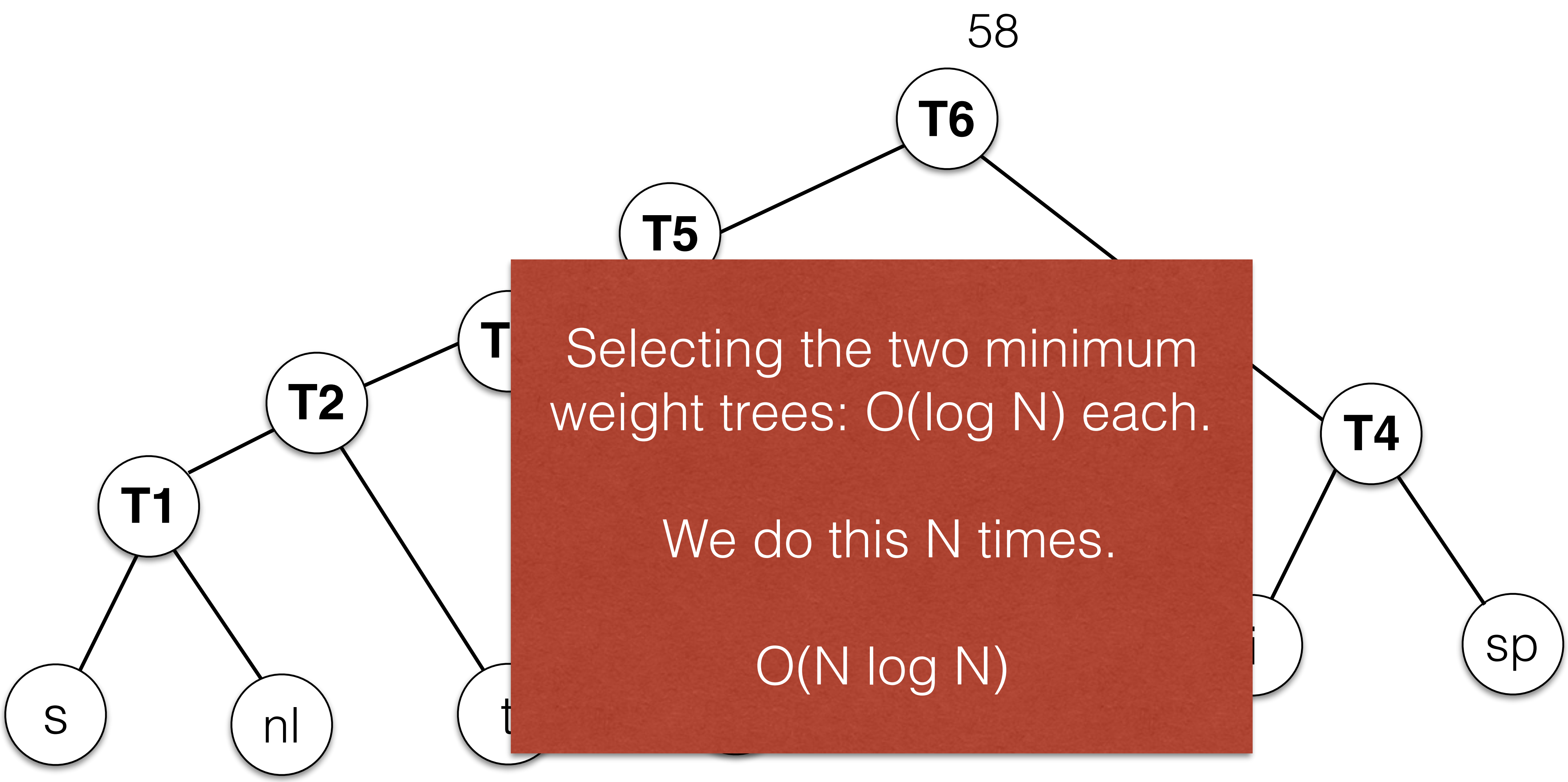
Huffman's Algorithm



Selecting the two minimum weight trees: O(log N) each.

We do this N times.

O(N log N)

- This is clearly a greedy algorithm as we consider then two lowest-weight trees at any level.
- Keep the trees in the forest on a heap.

Divide and Conquer Algorithms

- Algorithms consist of two parts:
  - Divide: Decompose the problem into smaller sub-problems. Solve each problem recursively (down to the base case).
  - Conquer: Solve the problem by combining solutions to the sub-problem.

Divide and Conquer Example Algorithms

- Merge Sort. Quick Sort.
- Binary Search.
- Towers of Hanoi.
- These algorithms work efficiently because:
  - The subproblems are independent.
  - Solving the subproblems first makes the overall problem easier.

Merge Sort

- Split the array in half, recursively sort each half.
- Merge the two sorted lists.

34	8	64	2	51	32	21	1

Merge Sort

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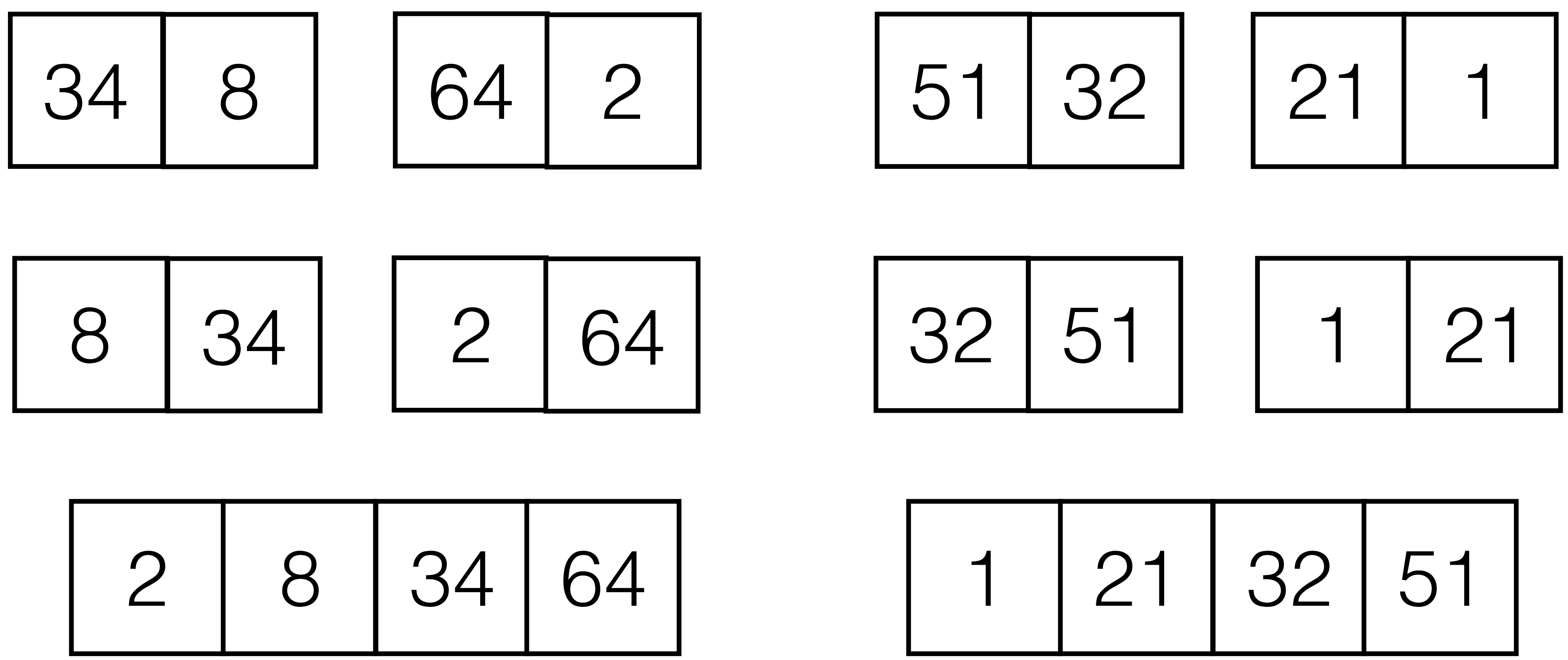

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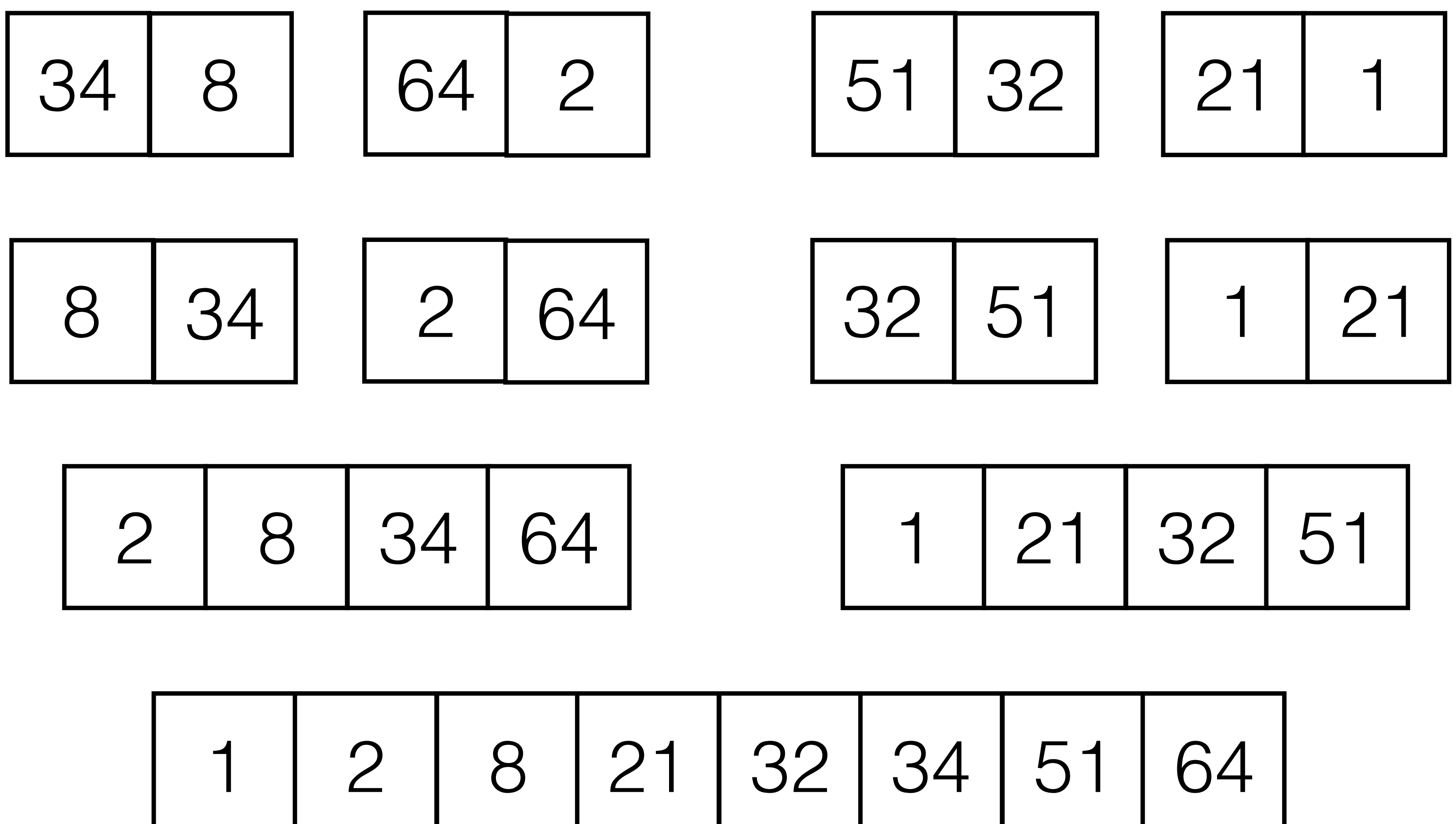


Merge Sort Running Time

- Base case: N=1 (sort a 1-element list). T(1) = 1
- Recurrence: T(N) = 2 T(N/2) + N

Merge the two halfs

Recursively sort each half

Running Time Analysis for Merge Sort and Quick Sort with Perfect Pivot.

$$T(N) = 2 \cdot T(\frac{N}{2}) + N$$

$$= 2 \cdot (2 \cdot T(\frac{N}{4}) + \frac{N}{2}) + N \qquad = 4 \cdot T(\frac{N}{4}) + N + N$$

$$= 2^k \cdot T(\frac{N}{2^k}) + k \cdot N$$

assume $k = \log N$

$$= N \cdot T(1) + logN \cdot N$$

$$= N + N \cdot \log N = \Theta(N \log N)$$

Running time of Divide and Conquer Algorithms: "Master Theorem"

Most divide and conquer algorithms have the following running time equation:

$$T(N) = aT(N/b) + \Theta(N^k)$$

The "Master Theorem" states that this recurrence relation has the following solution:

$$T(N) = \begin{cases} O(N^{\log_b a}) & \text{if } a > b^k \\ O(N^k \log N) & \text{if } a = b^k \\ O(N^k) & \text{if } a < b^k \end{cases}$$

Master Theorem: MergeSort

$$aT(N/b) + \Theta(N^k) = \begin{cases} O(N^{\log_b a}) & \text{if } a > b^k \\ O(N^k \log N) & \text{if } a = b^k \\ O(N^k) & \text{if } a < b^k \end{cases}$$

Example: Merge Sort $T(N) = 2 \cdot T(\frac{N}{2}) + N$

$$a = 2 \quad b = 2 \quad k = 1$$

This is Case 2: $T(N) = O(N \log N)$

Dynamic Programming Algorithms

- In some cases, recursive algorithms (such as the ones used for Divide and Conquer algorithms) won't work.
- That's because the solution to a subproblem is used more than once.
  - Merge Sort works because each partition is processed *exactly once.*
- Dynamic Programming algorithms solve this problem by systematically recording the solution to sub-problems in a table and re-using them later.

Broken Fibonacci

$$F_1 = 1, F_2 = 2$$

$$F_{k+1} = F_k + F_{k-1}$$

1,1,2,3,5,8,13,21,..

```
public int fibonacci(int k) throws IllegalArgumentException{
    if (k < 1) {
      throw new IllegalArgumentException("Expecting a positive integer.");
    }
    if (k == 1 | k == 2) {
      return 1;
    } else {
      return fibonacci(k-1) + fibonacci(k-2);
    }
}
```

Base case: 1 step T(1) = O(c), T(2) = O(c)

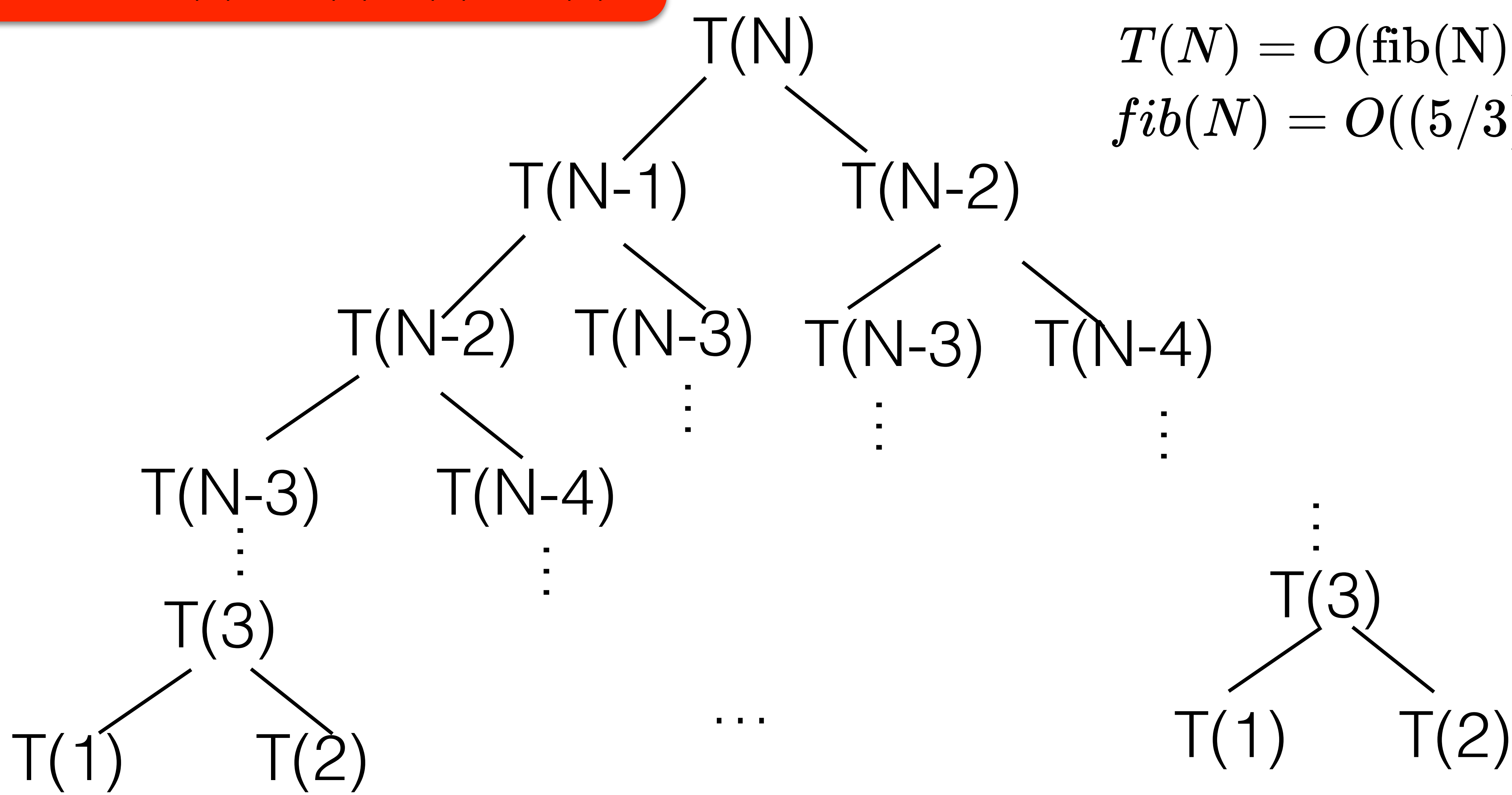
Recursive calls: T(k) = O(T(k-1) + T(k-2))

Analyzing the Recursive Fibonacci Solution

Recursive calls: T(k) = O(T(k-1) + T(k-2))
Base case: T(1) = O(c), T(2) = O(c)

each node is one recursive call

$$T(N) = O(\text{fib(N)})$$
$$fib(N) = O((5/3)^N)$$



Dynamic Programming
Fibonacci

```
public int fibonacci(int k) throws IllegalArgumentException{

  if (k < 1) {
    throw new IllegalArgumentException("Expecting a positive integer.");
  }
  int b = 1; //k-2
  int a = 1; //k-1
  for (int i=3; i<=k; i++) {
    int new_fib = a + b;
    b = a;
    a = new_fib;
  }
  return a;
}
```

$T(N) = O(N)$

Longest Increasing Subsequence

- Given a sequence of numbers, find the longest increasing (not necessarily contiguous) subsequence.

5 2 8 6 3 6 9 7

5 8 9

Longest Increasing Subsequence

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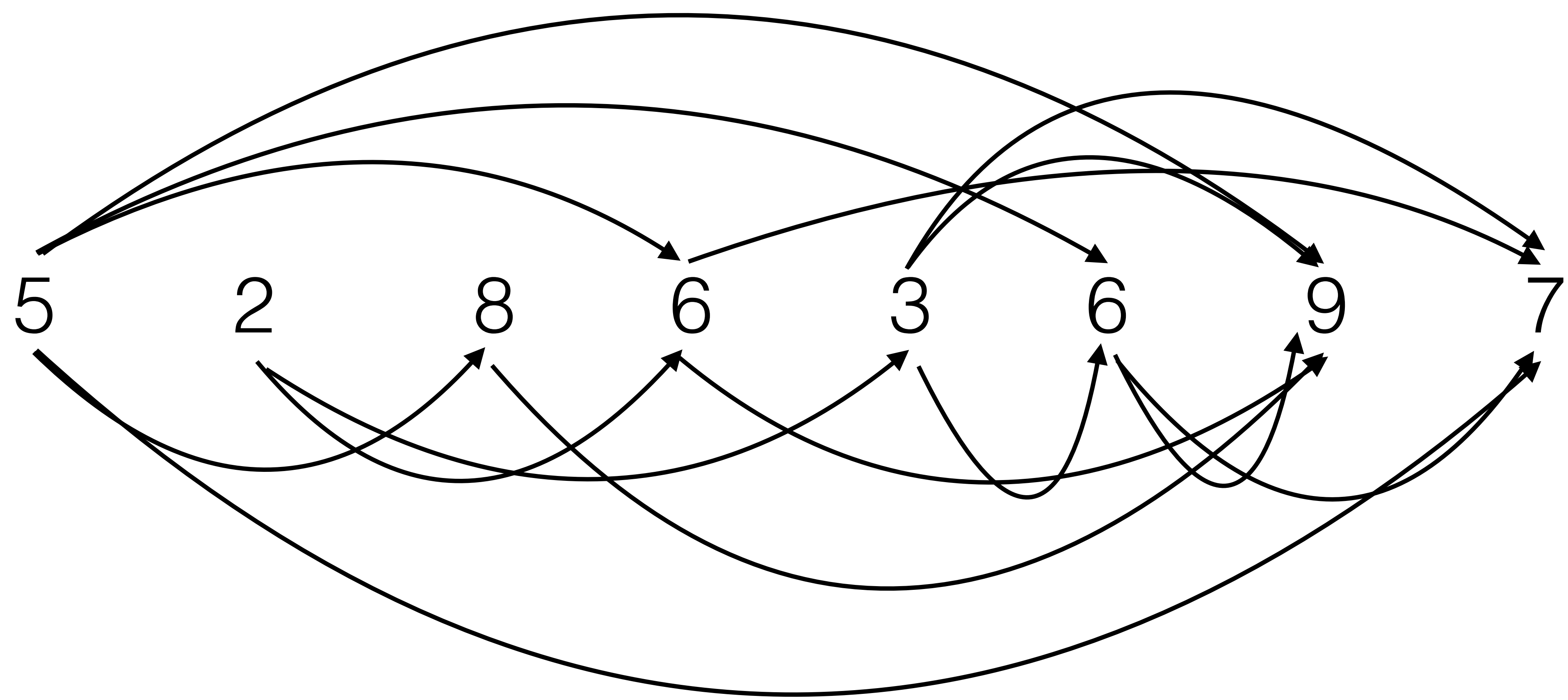
5 2 8 6 3 6 9 7

2 3 6 7

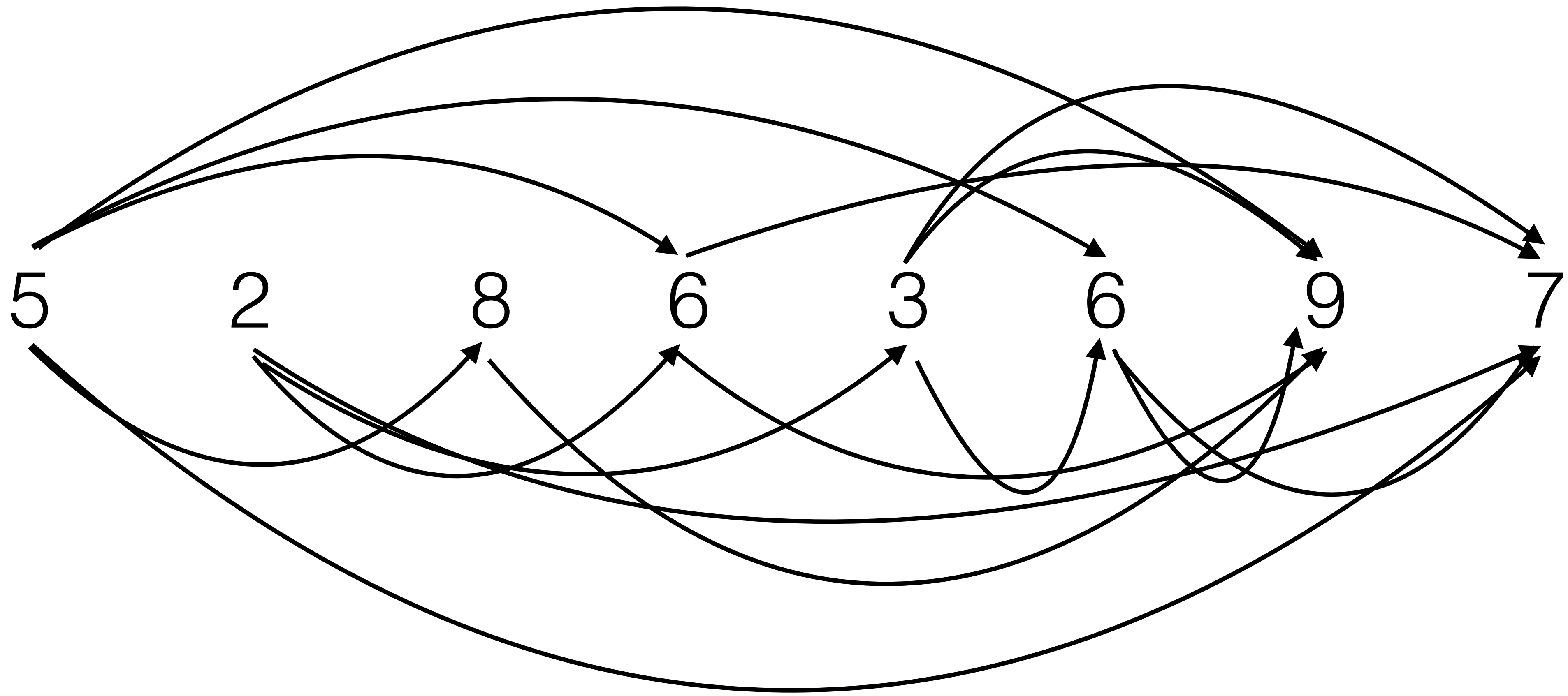
Longest Increasing Subsequence

- We can think of this problem as a graph problem.



This is a DAG. Our goal is to find the longest path.

Longest Increasing Subsequence: Recursive Solution

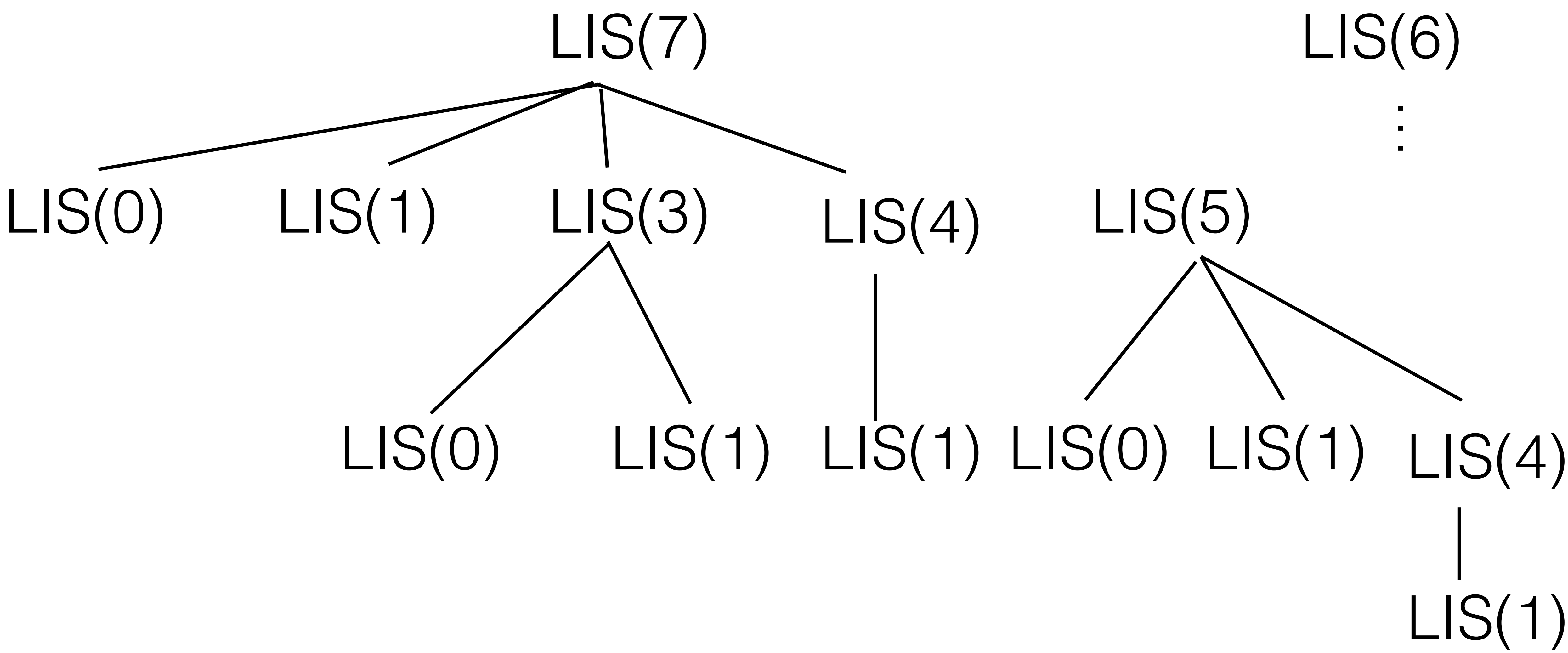


Step 1: Reducing the problem to easier subproblems (recursive divide-and-conquer solution)

```
LIS(i) {
    return max( {LIS(j) for j=j..i-1 if a[j] < a[i]} ) + 1
}
```

Longest Increasing Subsequence: Recursive Solution

```
LIS(i) {
    return max( {LIS(j) for j=0..i-1 if a[j] < a[i]} ) + 1
}
```



Longest Increasing Subsequence: Dynamic Programming

```
L = new Integer[n];
for i = 1…n {
    L[j] = max( {L(j) for j=0..i-1 if a[j] < a[i]} ) + 1
}
```

i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
a[i]	5	2	8	6	3	6	9	7
L[i]	0	0						

Longest Increasing Subsequence: Dynamic Programming

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L[i]	0	0	1	1	1	2		

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L[i]	0	0	1	1	1	2	3	

Longest Increasing Subsequence: Dynamic Programming

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Longest Increasing Subsequence: Dynamic Programming

$O(N^2)$

```
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for i = 1…n {
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}
```

i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
a[i]	5	2	8	6	3	6	9	7
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Dynamic Programming Example: Minimum Edit Distance

- The Minimum *Edit Distance (Levenshtein Distance)* between two strings *s* and *t* is the minimal number of **insertions**, **deletions**, and **substitutions** needed to convert s into t.

SATURDAY

SUNDAY

Dynamic Programming Example: Minimum Edit Distance

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SATURDAY

delete A

STURDAY

SUNDAY

Dynamic Programming Example: Minimum Edit Distance

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SATURDAY

delete A

STURDAY

delete T

SURDAY

SUNDAY

Dynamic Programming Example: Minimum Edit Distance

- The Minimum *Edit Distance* between two strings *s* and *t* is the minimal number of **insertions**, **deletions**, and **substitutions** needed to convert s into t.

SATURDAY

delete A

STURDAY

Minimum Edit Distance = 3

delete T

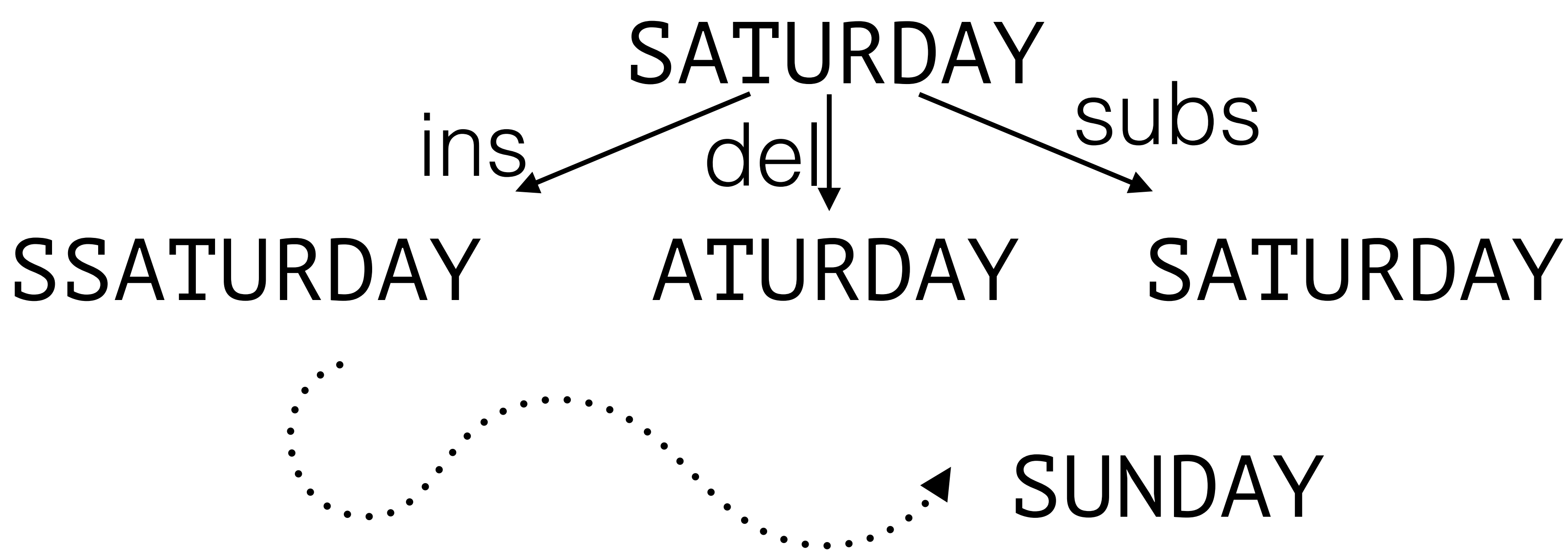
SURDAY

Substitute R with N

SUNDAY

Edit Distance as Search

- Initial state s, Goal state t.
- Try each operation for each letter.
- Try to find the shortest path.



- Search space is HUGE and contains **many duplicate states.**

Dynamic Programming Algorithm for Edit Distance

- Assume we have two strings
  $s = s_1, s_2, \ldots, s_n$ and $t = t_1, t_2, \ldots, t_m$

- Let D(i,j) be the minimum edit distance between s[0..i] and t[0..j].

- For example s = `SATURDAY`, t = `SUNDAY`
  D(2,3) = 2

$$\texttt{SA} \xrightarrow[\text{subs A / U}]{} \texttt{SU} \xrightarrow[\text{insert N}]{} \texttt{SUN}$$

Dynamic Programming Algorithm for Edit Distance

- Let D(i,j) be the minimum edit distance between s[0..i] and t[0..j].
- Basic approach:
  - Fill a table by computing D(i,j) for all ($0 < i < n$) and ($0 < j < m$).
  - Do this "bottom-up", starting with small i and j. Table entries for larger i and j are based on previous entries.

Dynamic Programming Algorithm for Edit Distance

For i = 1..n $D(i,0) = i$
For j = 1..m $D(0,j) = j$

For i = 1..n {
  For j = 1..m {

$$D(i,j) = min\begin{cases} D(i-1,j\qquad) + 1 \\ D(i\qquad,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} \end{cases}$$

  }
}

Edit Distance Example

initialization

$$D(i,0) = i$$

$$D(0,j) = j$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2						
S	1						
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j \quad\quad) + 1 & \text{deletion} \\ D(i \quad\quad , j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix} \text{subst} \\ \text{or equal} \end{matrix} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2						
S	1						
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j \quad\quad) + 1 & \text{deletion} \\ D(i \quad\quad, j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix}\text{subst} \\ \text{or equal}\end{matrix} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2						
S	**1**	0					
-	**0**	**1**	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j) + 1 & \text{deletion} \\ D(i, j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j & \text{subst} \\ 0 \text{ if } s_i = s_j & \text{or equal} \end{cases} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2						
S	1	**0**	1				
-	0	**1**	**2**	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j) + 1 & \text{deletion} \\ D(i, j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix}\text{subst} \\ \text{or equal}\end{matrix} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2						
S	1	0	**1**	2			
-	0	1	**2**	**3**	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j \quad\quad) + 1 & \text{deletion} \\ D(i \quad\quad , j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j & \text{subst} \\ 0 \text{ if } s_i = s_j & \text{or equal} \end{cases} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2						
S	1	0	1	**2**	3		
-	0	1	2	**3**	**4**	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j \quad\quad) + 1 & \text{deletion} \\ D(i \quad\quad , j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix}\text{subst} \\ \text{or equal}\end{matrix} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2						
S	1	0	1	2	**3**	4	
-	0	1	2	3	**4**	**5**	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j \quad\quad) + 1 & \text{deletion} \\ D(i \quad\quad , j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix} \text{subst} \\ \text{or equal} \end{matrix} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2						
S	1	0	1	2	3	**4**	5
-	0	1	2	3	4	**5**	**6**
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j) + 1 & \text{deletion} \\ D(i, j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{array}{l}\text{subst} \\ \text{or equal}\end{array} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	**2**	1					
S	**1**	**0**	1	2	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j \quad\quad) + 1 & \text{deletion} \\ D(i \quad\quad , j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \text{subst or equal} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2	**1**	1				
S	1	**0**	**1**	2	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j \quad\quad) + 1 & \text{deletion} \\ D(i \quad\quad, j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \text{subst or equal} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2	1	**1**	2			
S	1	0	**1**	**2**	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min\begin{cases} D(i-1, j \quad\quad) + 1 & \text{deletion} \\ D(i \quad\quad , j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix}\text{subst} \\ \text{or equal}\end{matrix} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2	1	1	**2**	3		
S	1	0	1	**2**	**3**	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j\quad\quad) + 1 & \text{deletion} \\ D(i\quad\quad, j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix}\text{subst} \\ \text{or equal}\end{matrix} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2	1	1	2	**3**	3	
S	1	0	1	2	**3**	**4**	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j \quad\quad) + 1 & \text{deletion} \\ D(i \quad\quad , j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j & \text{subst} \\ 0 \text{ if } s_i = s_j & \text{or equal} \end{cases} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3						
A	2	1	1	2	3	**3**	4
S	1	0	1	2	3	**4**	**5**
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j \qquad) + 1 & \text{deletion} \\ D(i \qquad, j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix}\text{subst} \\ \text{or equal}\end{matrix} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4						
T	3	2	2	2	3	4	4
A	2	1	1	2	3	3	4
S	1	0	1	2	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j\quad\ ) + 1 & \text{deletion} \\ D(i\quad\ , j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix}\text{subst} \\ \text{or equal}\end{matrix} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5						
U	4	3	2	3	3	4	5
T	3	2	2	2	3	4	4
A	2	1	1	2	3	3	4
S	1	0	1	2	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min\begin{cases} D(i-1,j\quad) + 1 & \text{deletion} \\ D(i\quad, j-1) + 1 & \text{insertion} \\ D(i-1,j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \text{subst or equal} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6						
R	5	4	3	3	4	4	5
U	4	3	2	3	3	4	5
T	3	2	2	2	3	4	4
A	2	1	1	2	3	3	4
S	1	0	1	2	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j \quad\quad) + 1 & \text{deletion} \\ D(i \quad\quad , j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix}\text{subst} \\ \text{or equal}\end{matrix} \end{cases}$$

D(i,j)							
Y	8						
A	7						
D	6	5	4	4	3	4	5
R	5	4	3	3	4	4	5
U	4	3	2	3	3	4	5
T	3	2	2	2	3	4	4
A	2	1	1	2	3	3	4
S	1	0	1	2	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min\begin{cases} D(i-1, j\quad\quad) + 1 & \text{deletion} \\ D(i\quad\quad, j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix}\text{subst} \\ \text{or equal}\end{matrix} \end{cases}$$

D(i,j)							
Y	8						
A	7	6	5	5	4	3	4
D	6	5	4	4	3	4	5
R	5	4	3	3	4	4	5
U	4	3	2	3	3	4	5
T	3	2	2	2	3	4	4
A	2	1	1	2	3	3	4
S	1	0	1	2	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j \quad\quad) + 1 & \text{deletion} \\ D(i \quad\quad , j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix}\text{subst} \\ \text{or equal}\end{matrix} \end{cases}$$

D(i,j)							
Y	8	7	6	6	5	4	3
A	7	6	5	5	4	3	4
D	6	5	4	4	3	4	5
R	5	4	3	3	4	4	5
U	4	3	2	3	3	4	5
T	3	2	2	2	3	4	4
A	2	1	1	2	3	3	4
S	1	0	1	2	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$
D(i,j) = min\begin{cases} D(i-1, j \quad\quad) + 1 & \text{deletion} \\ D(i \quad\quad , j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j & \text{subst} \\ 0 \text{ if } s_i = s_j & \text{or equal} \end{cases} \end{cases}
$$

D(i,j)							
Y	8	7	6	6	5	4	3
A	7	6	5	5	4	3	4
D	6	5	4	4	3	4	5
R	5	4	3 subs	3	4	4	5
U	4	3	2	3	3	4	5
T	3	2	2	2	3	4	4
A	2	1	1	2	3	3	4
S	1	0	1	2	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min\begin{cases} D(i-1, j\quad\quad) + 1 & \text{deletion} \\ D(i\quad\quad, j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix}\text{subst} \\ \text{or equal}\end{matrix} \end{cases}$$

D(i,j)							
Y	8	7	6	6	5	4	3
A	7	6	5	5	4	3	4
D	6	5	4	4	3	4	5
R	5	4	3 subs	3	4	4	5
U	4	3	2	3	3	4	5
T	3	2	2	2	3	4	4
A	2	1	1	2	3	3	4
S	1	0	1	2	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1,j) + 1 & \text{deletion} \\ D(i,j-1) + 1 & \text{insertion} \\ D(i-1,j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \begin{matrix}\text{subst} \\ \text{or equal}\end{matrix} \end{cases}$$

D(i,j)							
Y	8	7	6	6	5	4	3
A	7	6	5	5	4	3	4
D	6	5	4	4	3	4	5
R	5	4	3 subs	3	4	4	5
U	4	3	2	3	3	4	5
T	3	2	2 ins	2	3	4	4
A	2	1	1 ins	2	3	3	4
S	1	0	1	2	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**

$$D(i,j) = min \begin{cases} D(i-1, j) + 1 & \text{deletion} \\ D(i, j-1) + 1 & \text{insertion} \\ D(i-1, j-1) + \begin{cases} 1 \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases} & \text{subst or equal} \end{cases}$$

SUNDAY

$\downarrow$ +A

SAUNDAY

$\downarrow$ + T

SATUNDAY

$\downarrow$ N/R

SATURDAY

D(i,j)							
Y	8	7	6	6	5	4	3
A	7	6	5	5	4	3	4
D	6	5	4	4	3	4	5
R	5	4	3 subs	3	4	4	5
U	4	3	2	3	3	4	5
T	3	2	2	2	3	4	4
A	2	1 ins	1	2	3	3	4
S	1	0 ins	1	2	3	4	5
-	0	1	2	3	4	5	6
	-	**S**	**U**	**N**	**D**	**A**	**Y**