Data Structures in Java

Lecture 10: AVL Trees.

10/12/2015

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Balanced BSTs

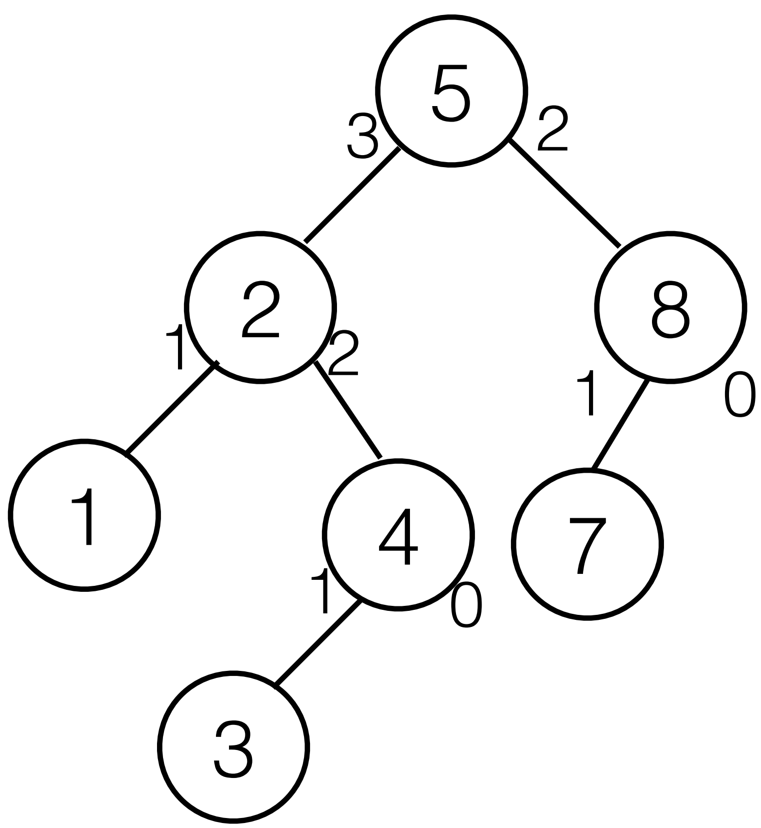
- Balance condition: Guarantee that the BST is always close to a complete binary tree (every node has exactly two or zero children).

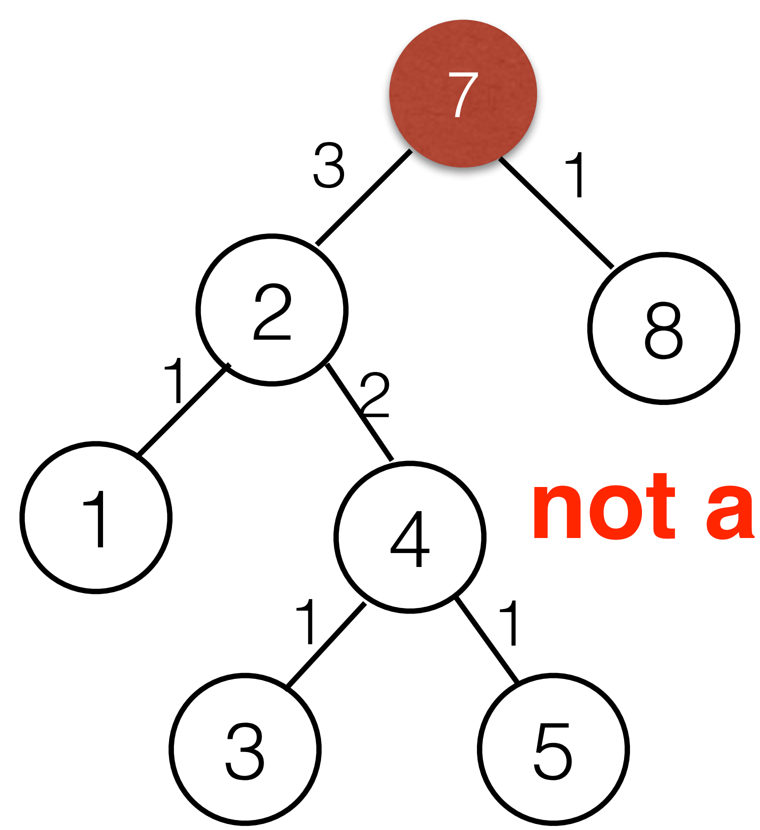
- Then the height of the tree will be O(*log N)* and all BST operations will run in *O(log N)*.

AVL Tree Condition

- An AVL Tree is a Binary Search Tree in which the following **balance condition** holds after each operation:

  - For every node, the height of the left and right subtree differs by at most 1.





not an AVL tree

AVL Trees

- Height of an AVL tree is at most
  $\sim 1.44 \log(N+2) - 1.328 = O(\log N)$

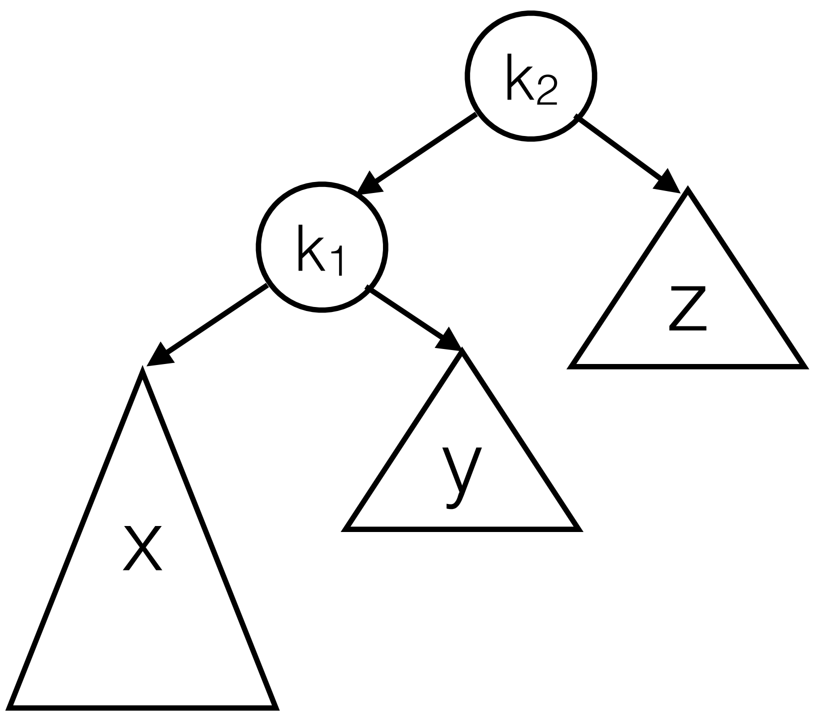
- How to maintain the balance condition?

  - Rebalance the tree after each change (insertion or deletion).

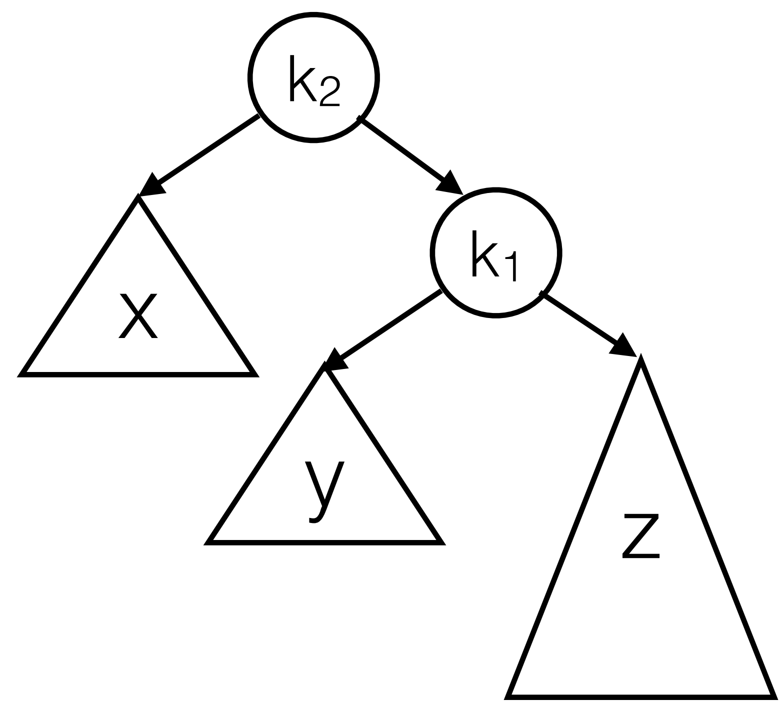
  - Rebalancing must be cheap.

“Outside” Imbalance

node k_2 violates the balance condition



left subtree of left child too high

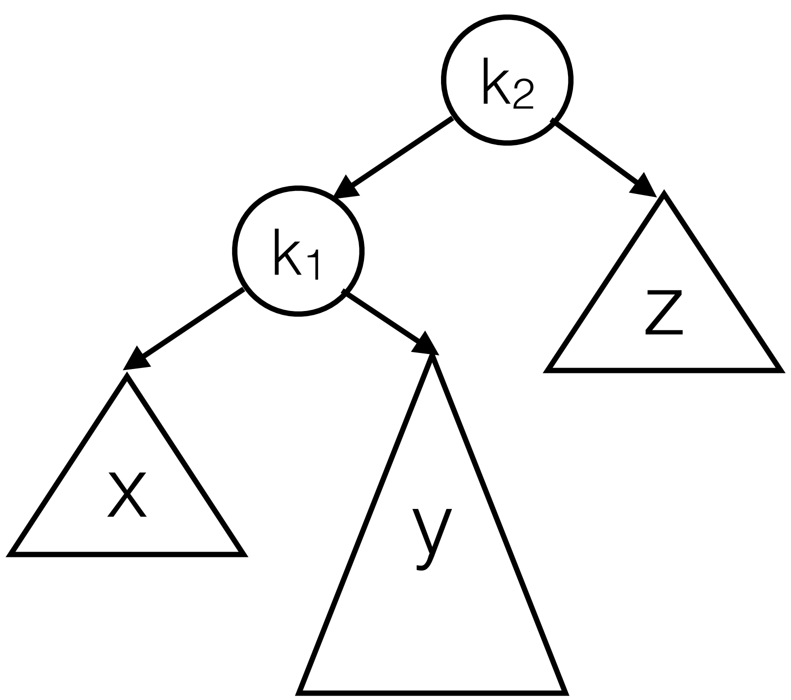


right subtree of right child too high

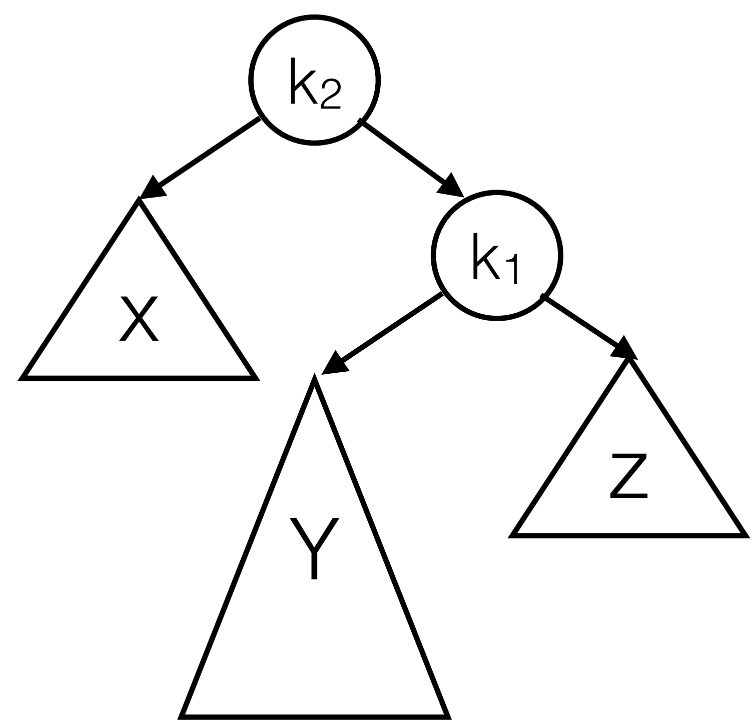
- Solution: Single rotation

“Inside” Imbalance

node k_2 violates the balance condition



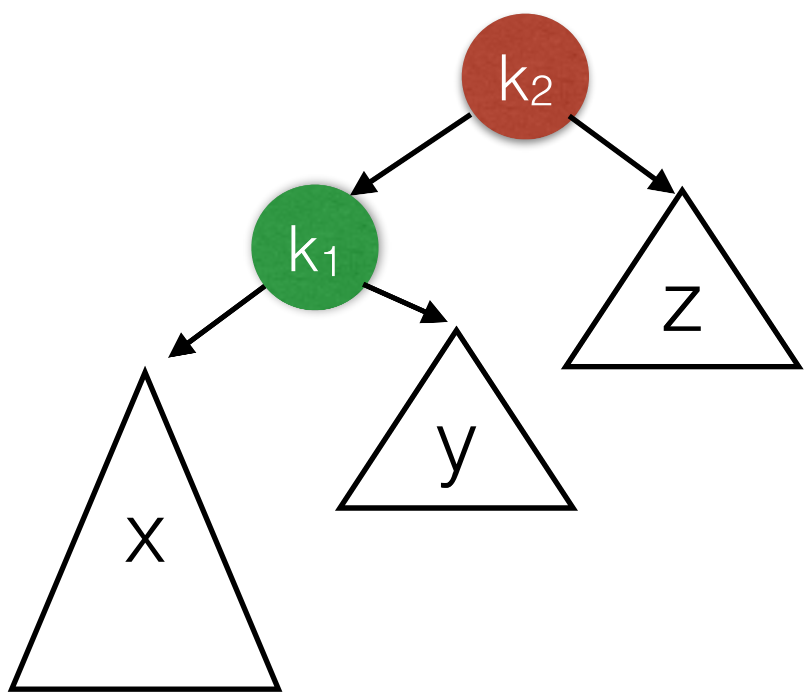
right subtree of left child too high



left subtree of right child too high

- Solution: Double rotation

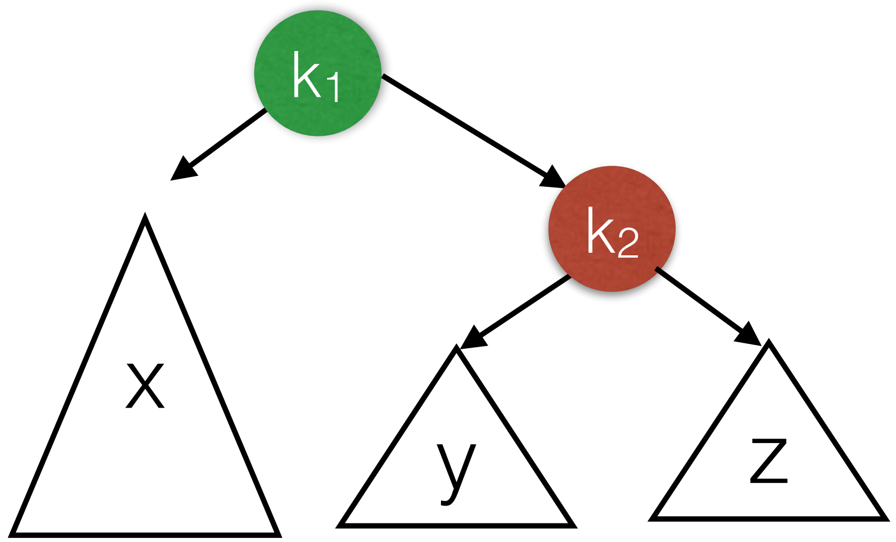
Single Rotation



Maintain BST property:

- x is still left subtree of k_{1}.
- z is still right subtree of k_2.
- For all values v in y: $k_1 < v < k_2$
  so y becomes new left subtree of k_2.

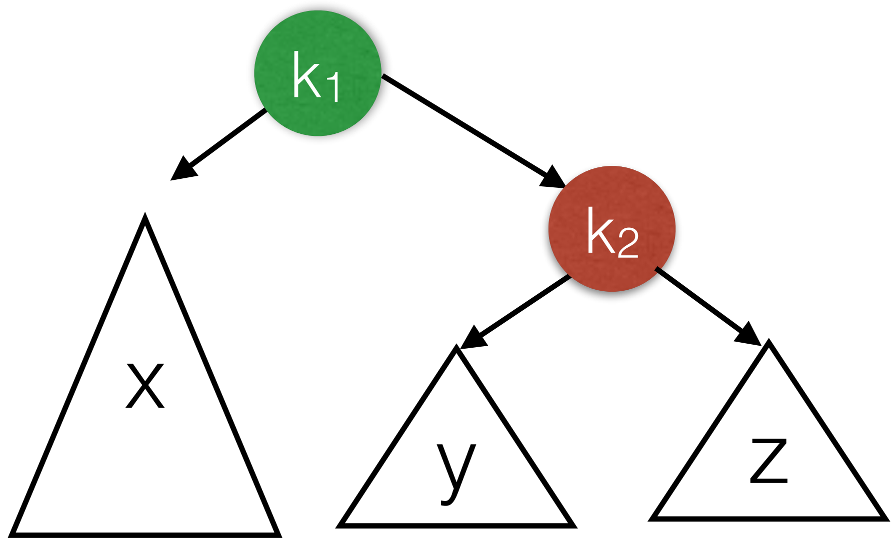
Single Rotation



Maintain BST property:

- x is still left subtree of k*1.*
- z is still right subtree of *k2.*
- For all values *v* in *y*: $k_1 < v < k_2$
  so y becomes new left subtree of *k2*.

Single Rotation



Modify 3 references:

- k_2.left = k_1.right
- k_1.right = k_2
- parent(k_2).left = k_1 or
  parent(k_2).right = k_1 or

Maintain BST property:

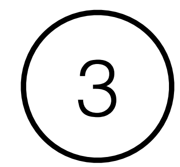
- x is still left subtree of k_1.
- z is still right subtree of k_2.
- For all values v in y: $k_1 < v < k_2$
  so y becomes new left subtree of k_2.

Maintaining Balance in an AVL Tree

- Assume the tree is balanced.
- After each insertion, find the lowest node *k* that violates the balance condition (if any).
- Perform rotation to re-balance the tree.
- Rotation maintains original height of subtree under *k* before the insertion. No further rotations are needed.

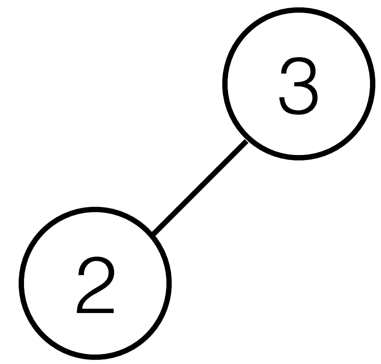
Single Rotation Example

insert(3)



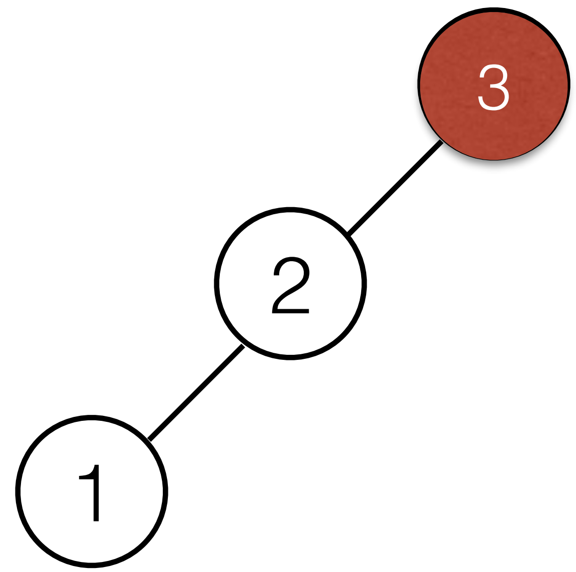
Single Rotation Example

insert(3)
insert(2)



Single Rotation Example

insert(3)
insert(2)
insert(1)   rotate_left(3)



Single Rotation Example

insert(3)

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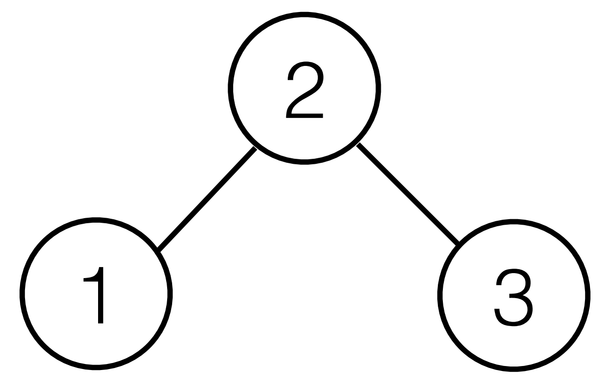

Single Rotation Example

insert(3)

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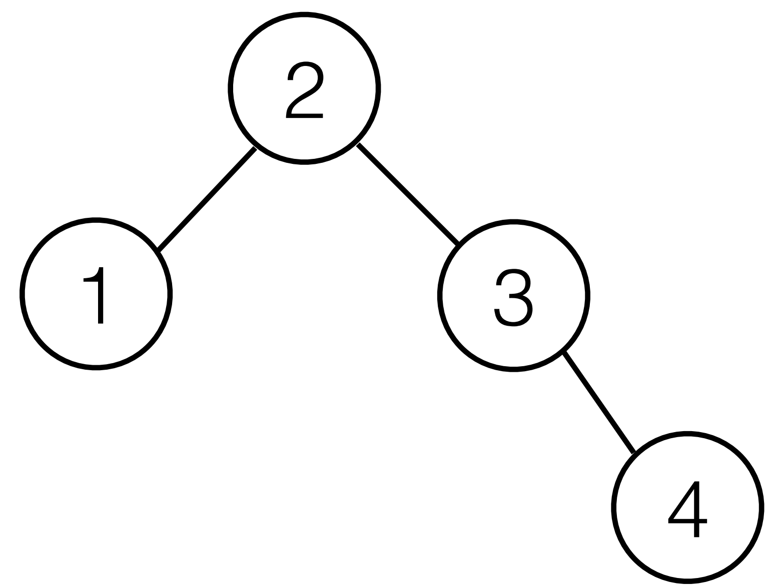
insert(1)   rotate_left(3)

insert(4)



Single Rotation Example

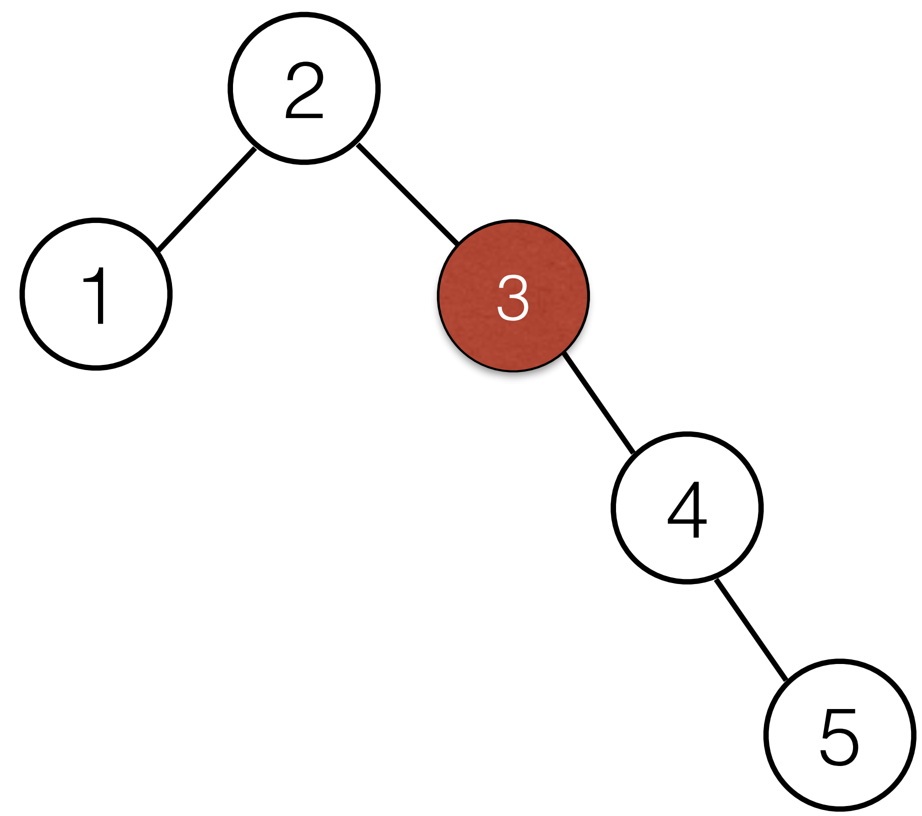
insert(3)

insert(2)

insert(1)   rotate_left(3)

insert(4)

insert(5)   rotate_right(3)



Single Rotation Example

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insert(1)   rotate_left(3)
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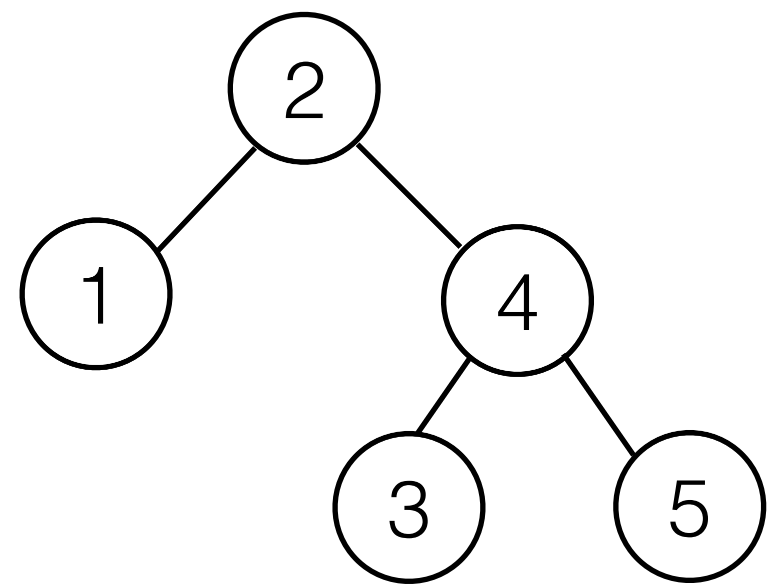

Single Rotation Example

insert(3)
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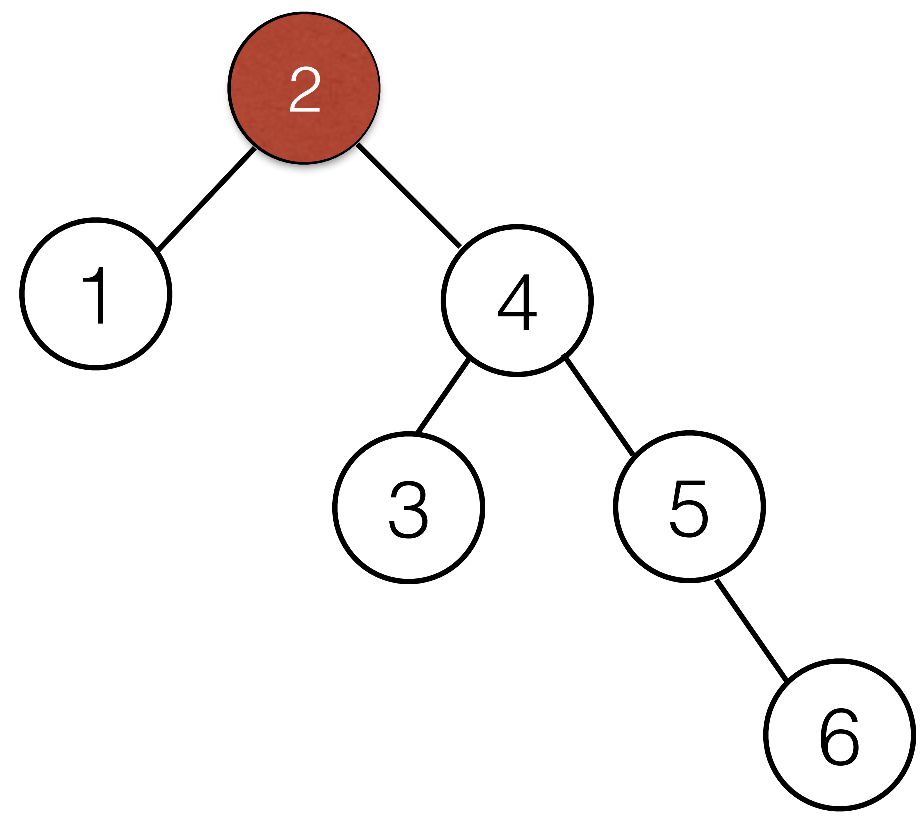
insert(5) rotate_right(3)
insert(6) rotate_right(2)



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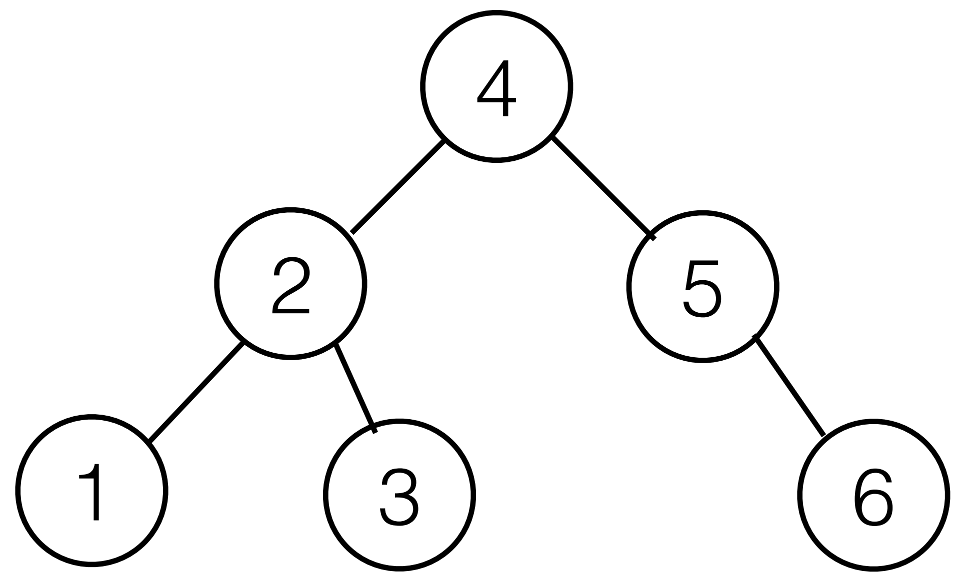

Single Rotation Example

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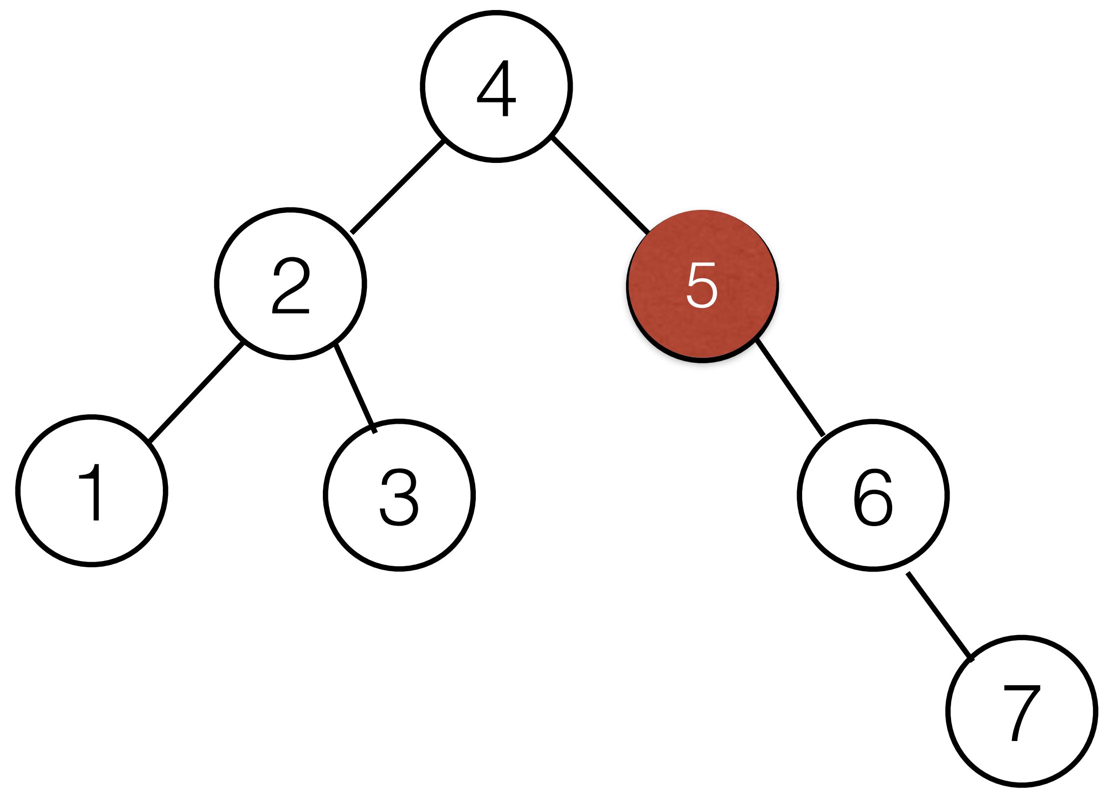
insert(7) rotate_right(5)



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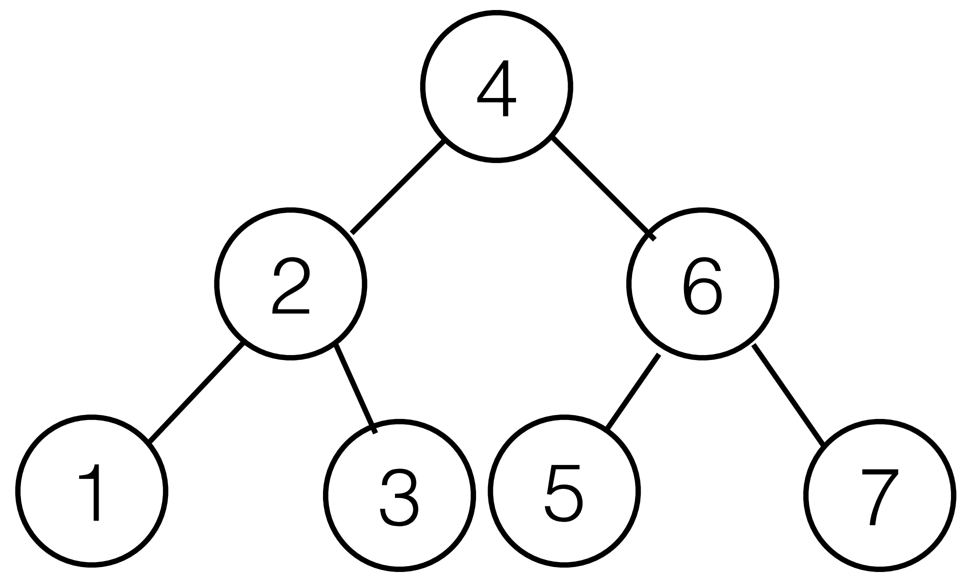
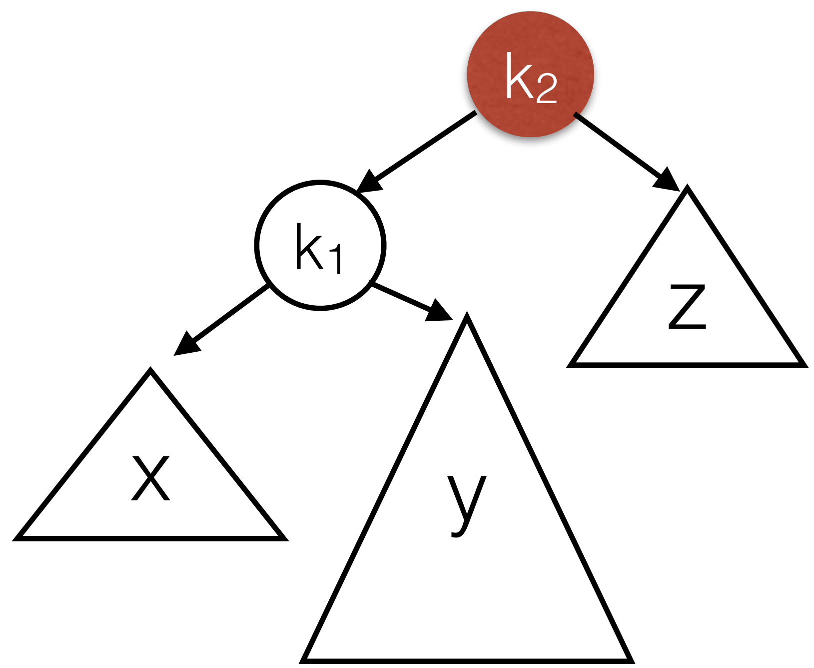


Single Rotation does not work for "Inside" Imbalance



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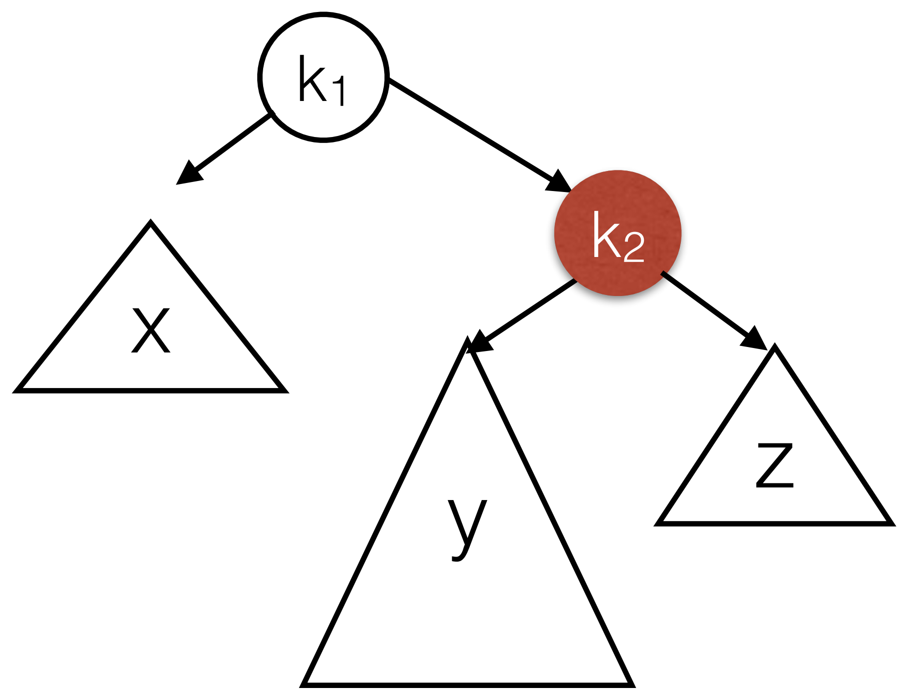


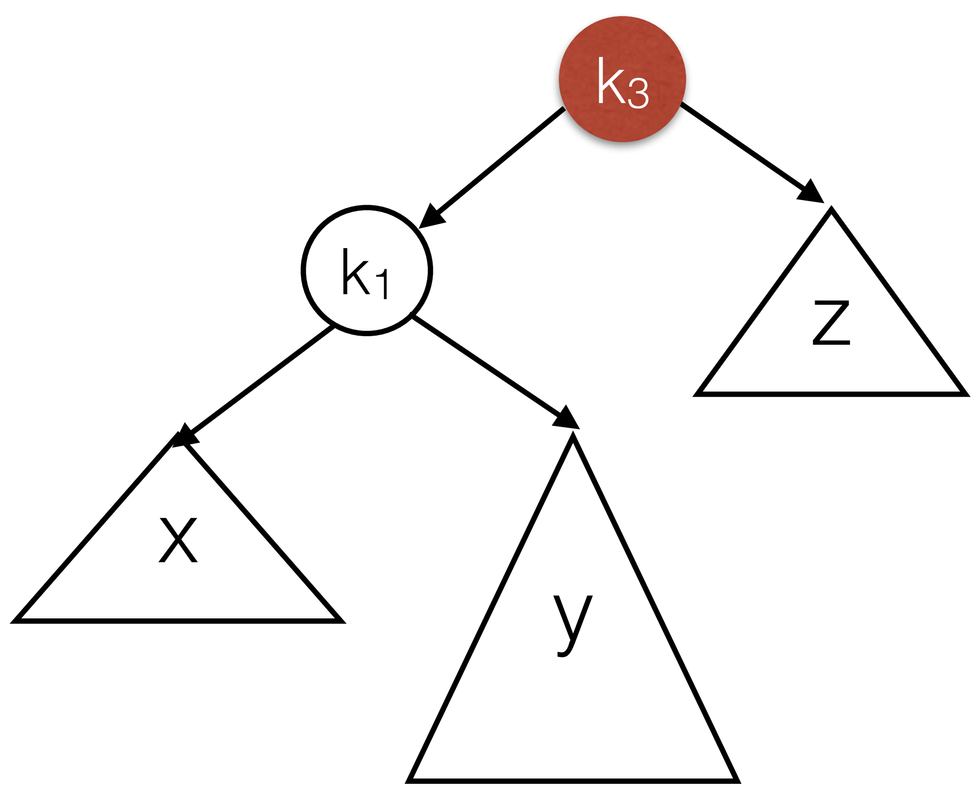
Result is not an AVL tree.

Now k_1 is violates the balance condition.

Problem: Tree y cannot move and it is too high.

Double Rotation (1)

- y is non-empty (imbalance due to insertion into y or deletion from z)
- so we can view y as a root and two sub-trees.



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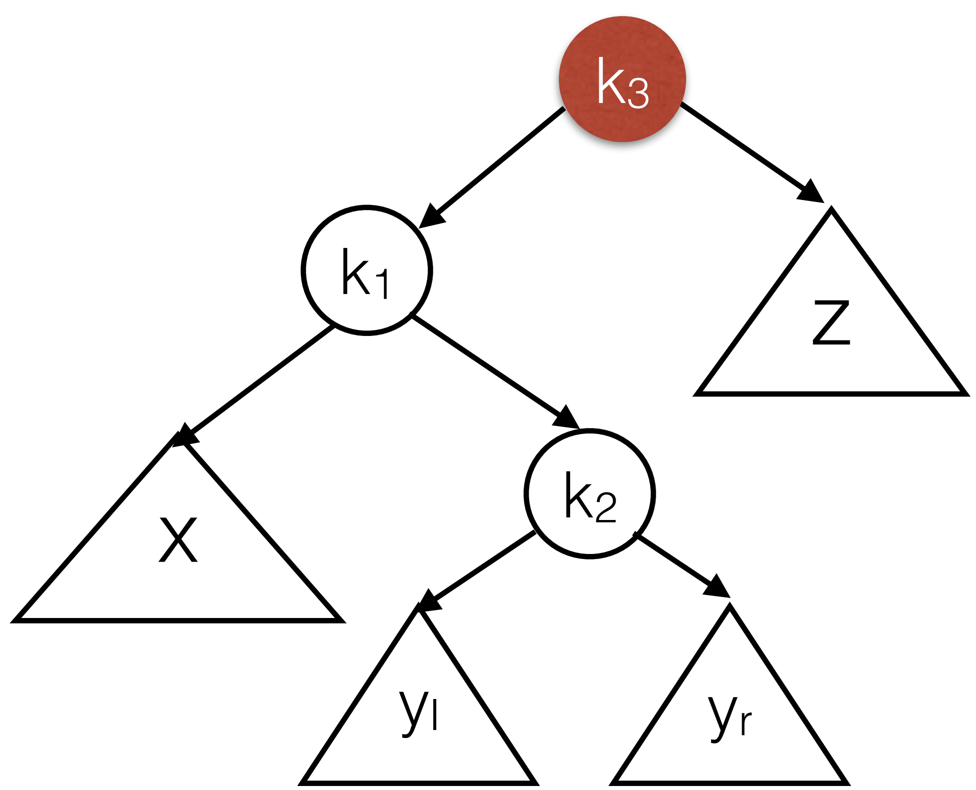
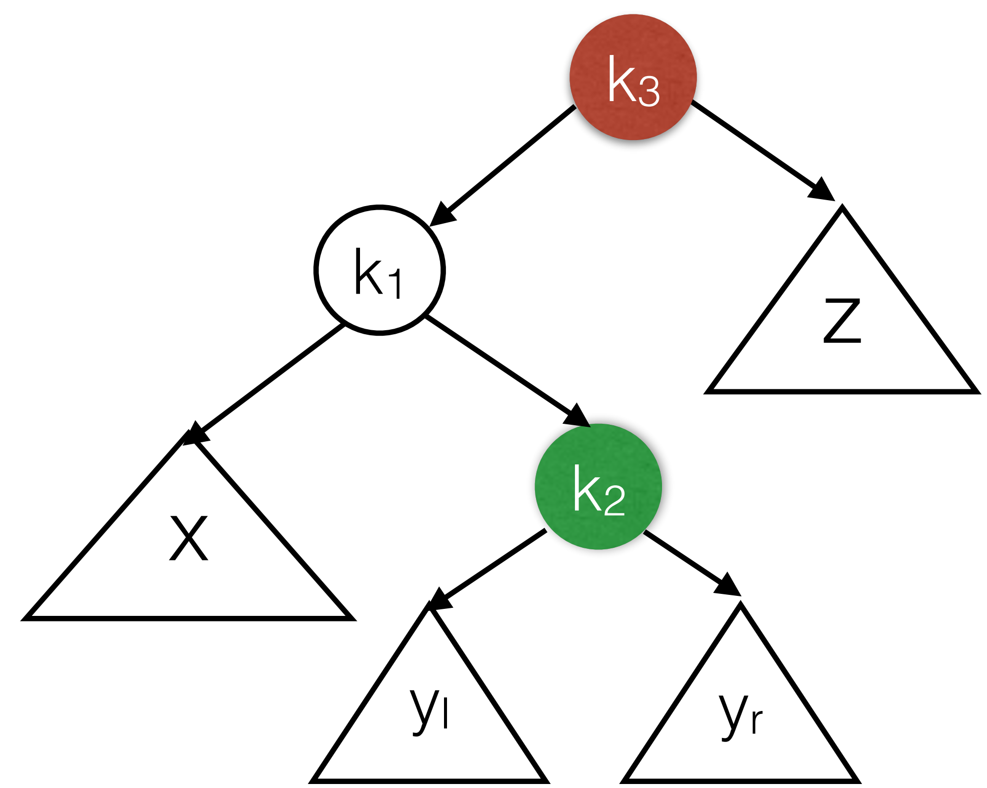


- either y_l or y_r is two levels deeper than z (or both are empty).

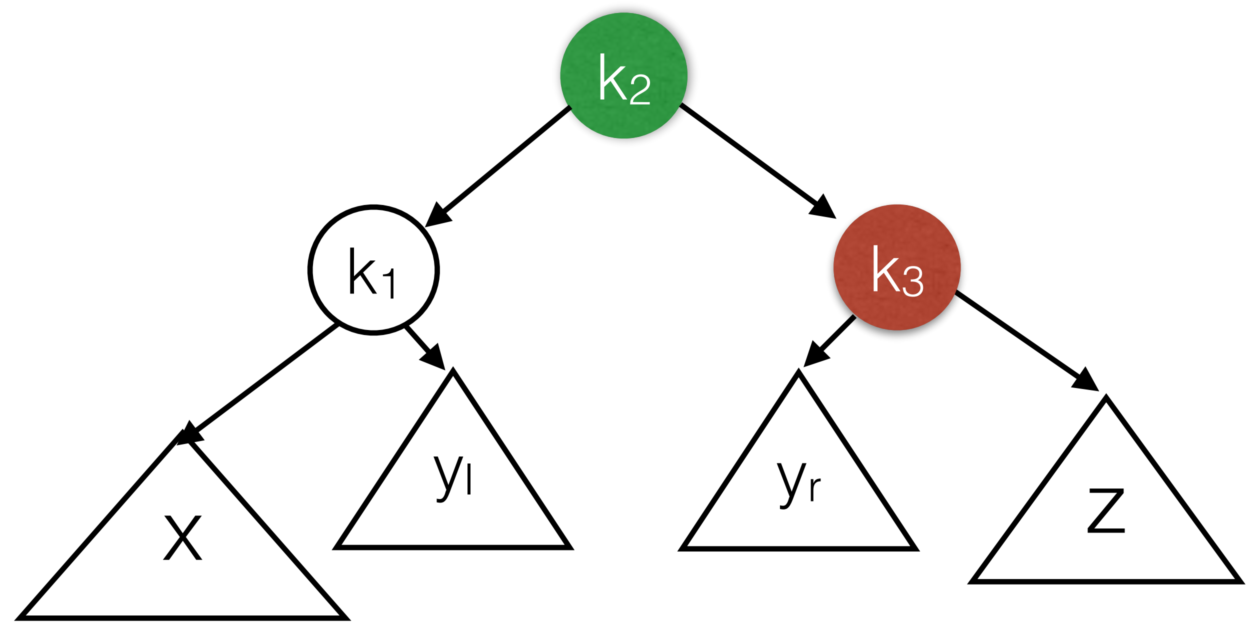
Double Rotation (2)



Maintain BST property:

- x is still left subtree of k_1.
- z is still right subtree of k_3.
- For all values v in y_l: $k_1 < v < k_2$
  so y_l becomes new right subtree of k_1.
- For all values w in y_r: $k_2 < w < k_3$
  so y_r becomes new left subtree of k_3.

Double Rotation (2)



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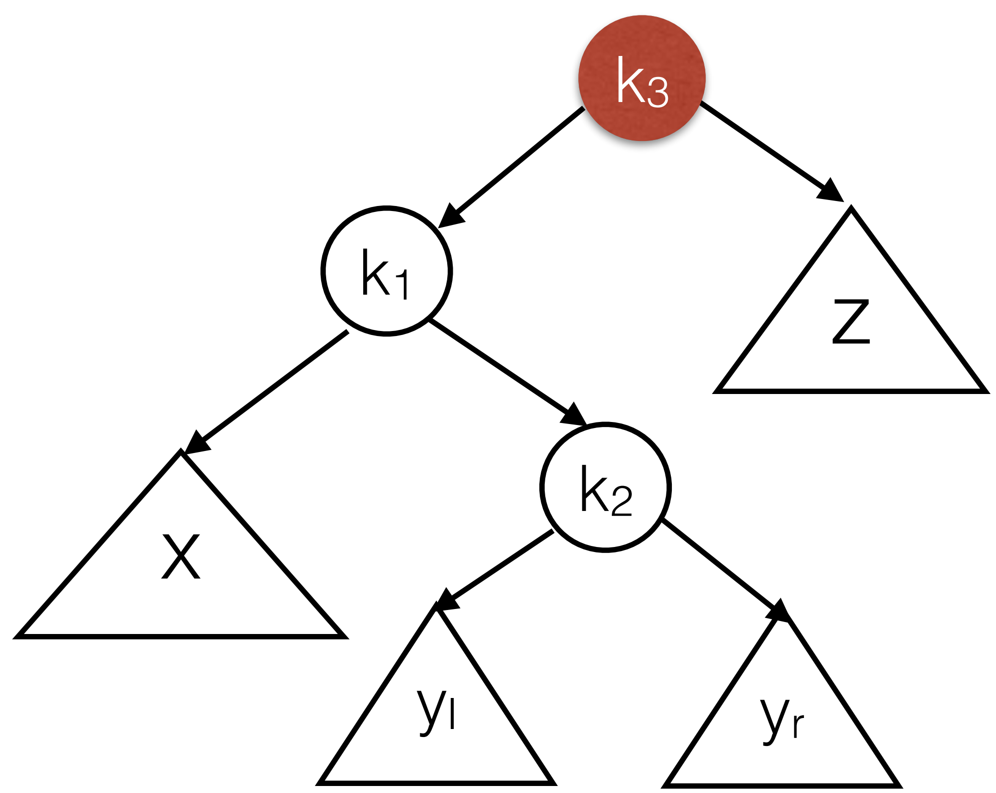
- For all values w in y_r: $k_2 < w < k_3$
  so y_r becomes new left subtree of k_3.

Double Rotation (2)

These are actually two single rotations:
First at k1, then at k3.



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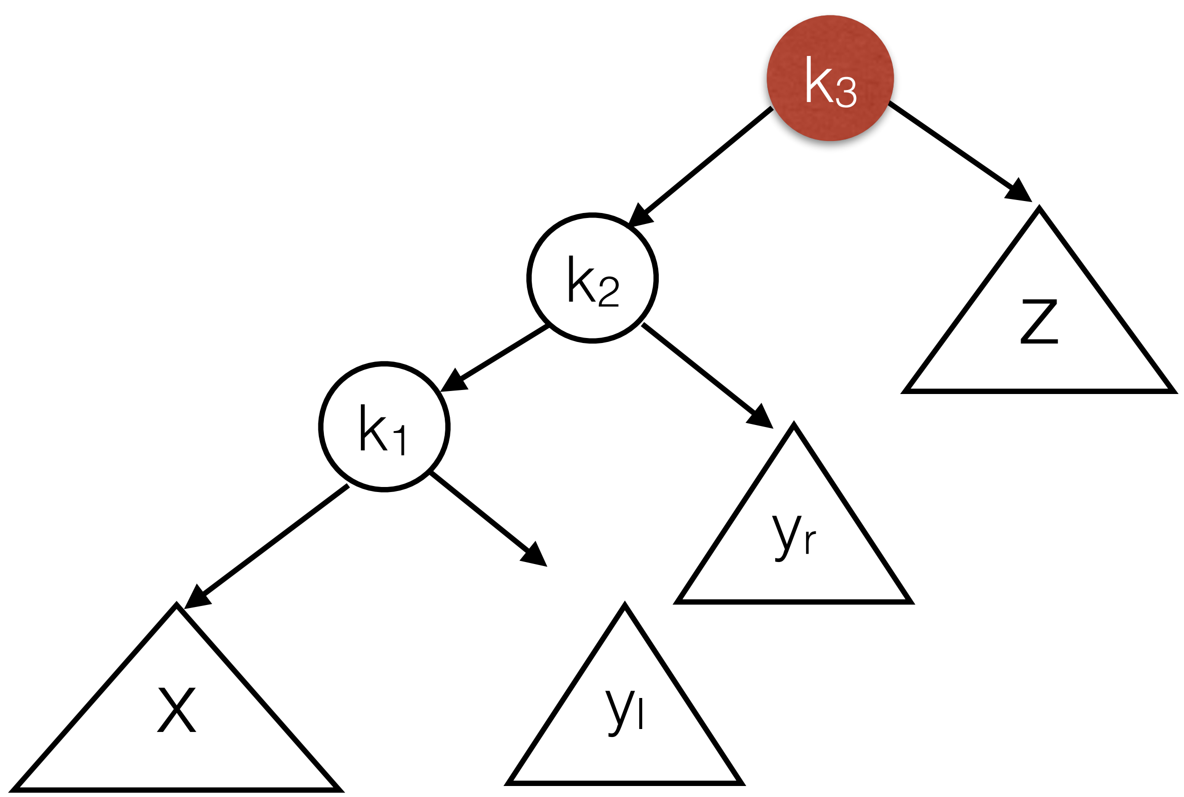

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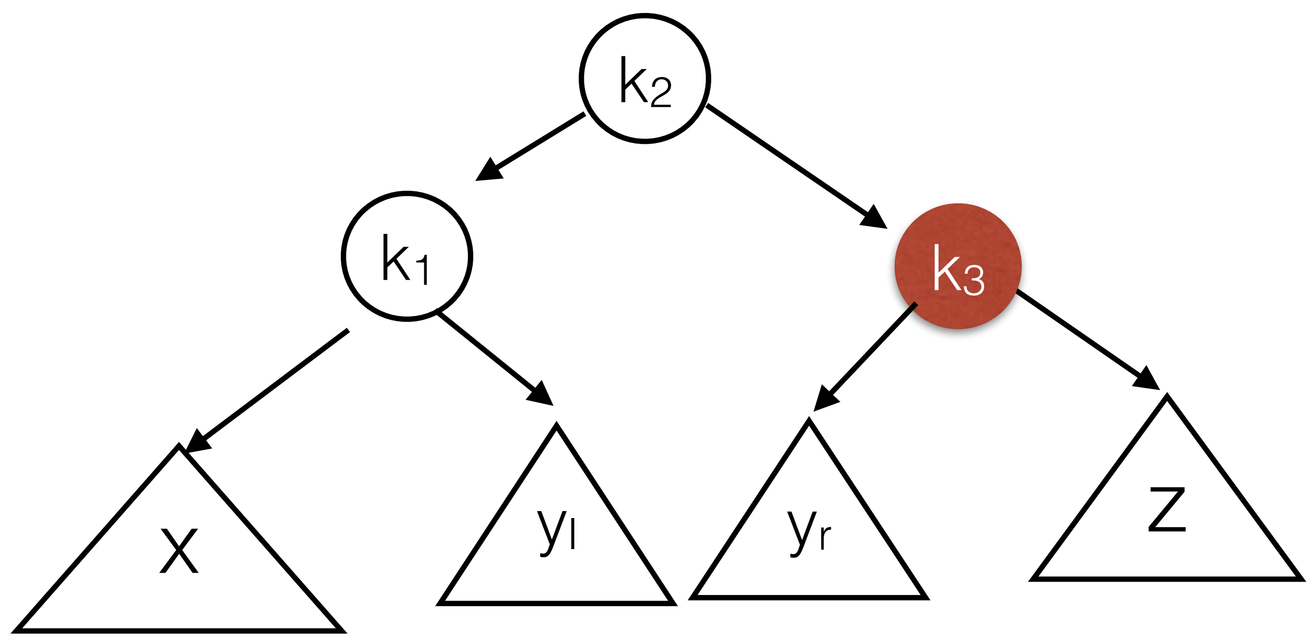
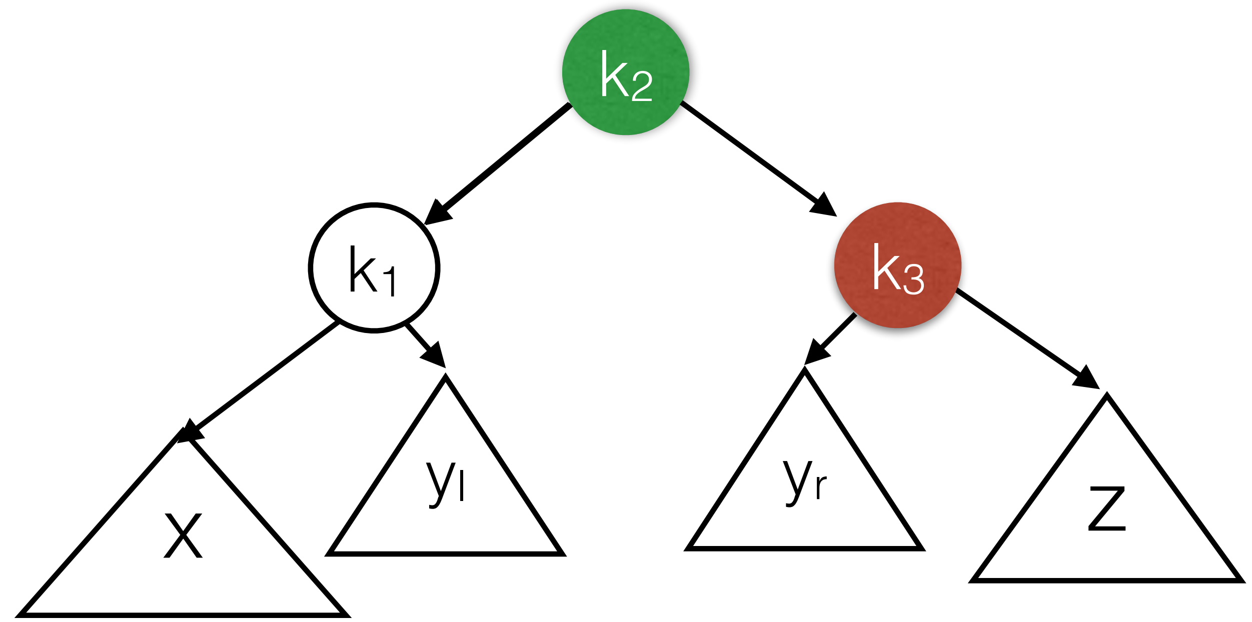


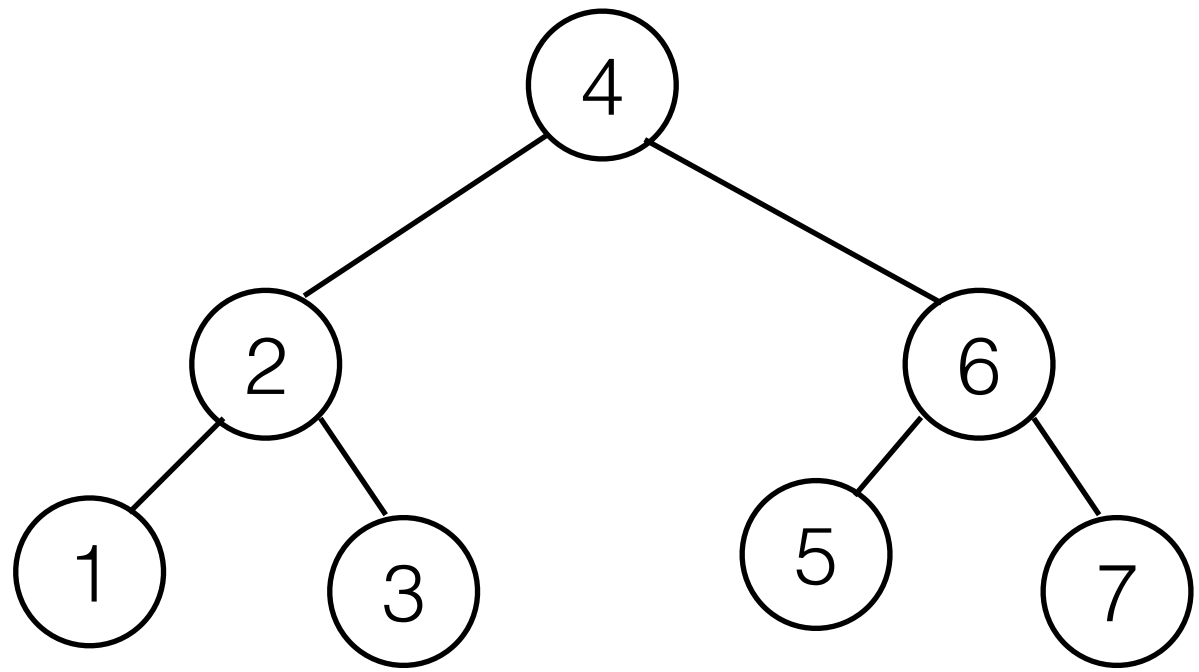
Double Rotation (3)

Modify 5 references:

- parent(k_3).left = k_2 or parent(k_3).right = k_2
- k_2.left = k_1
- k_2.right = k_3
- k_1.right = root(y_l)
- k_3.left = root(y_r)

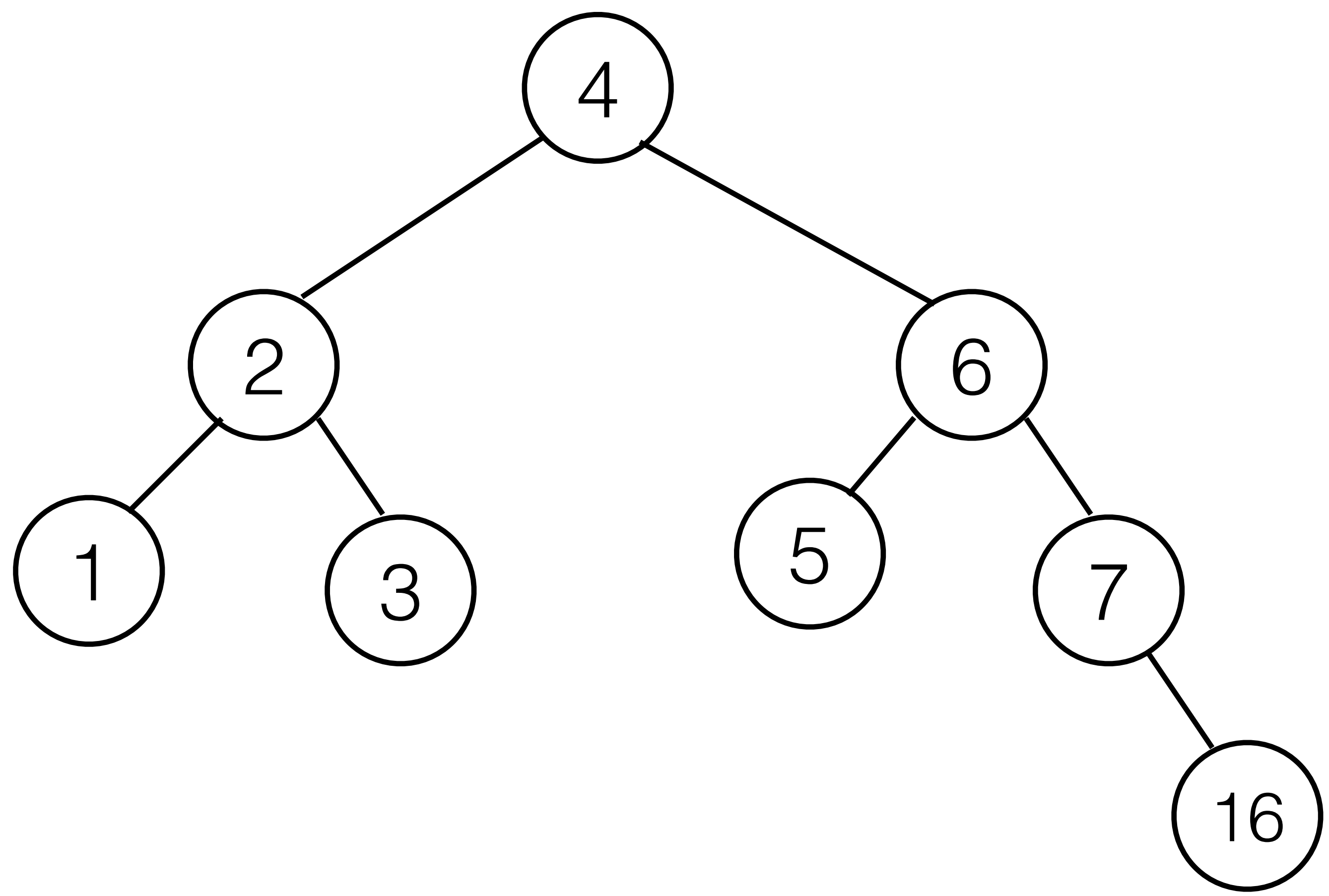


Double Rotation Example



Double Rotation Example

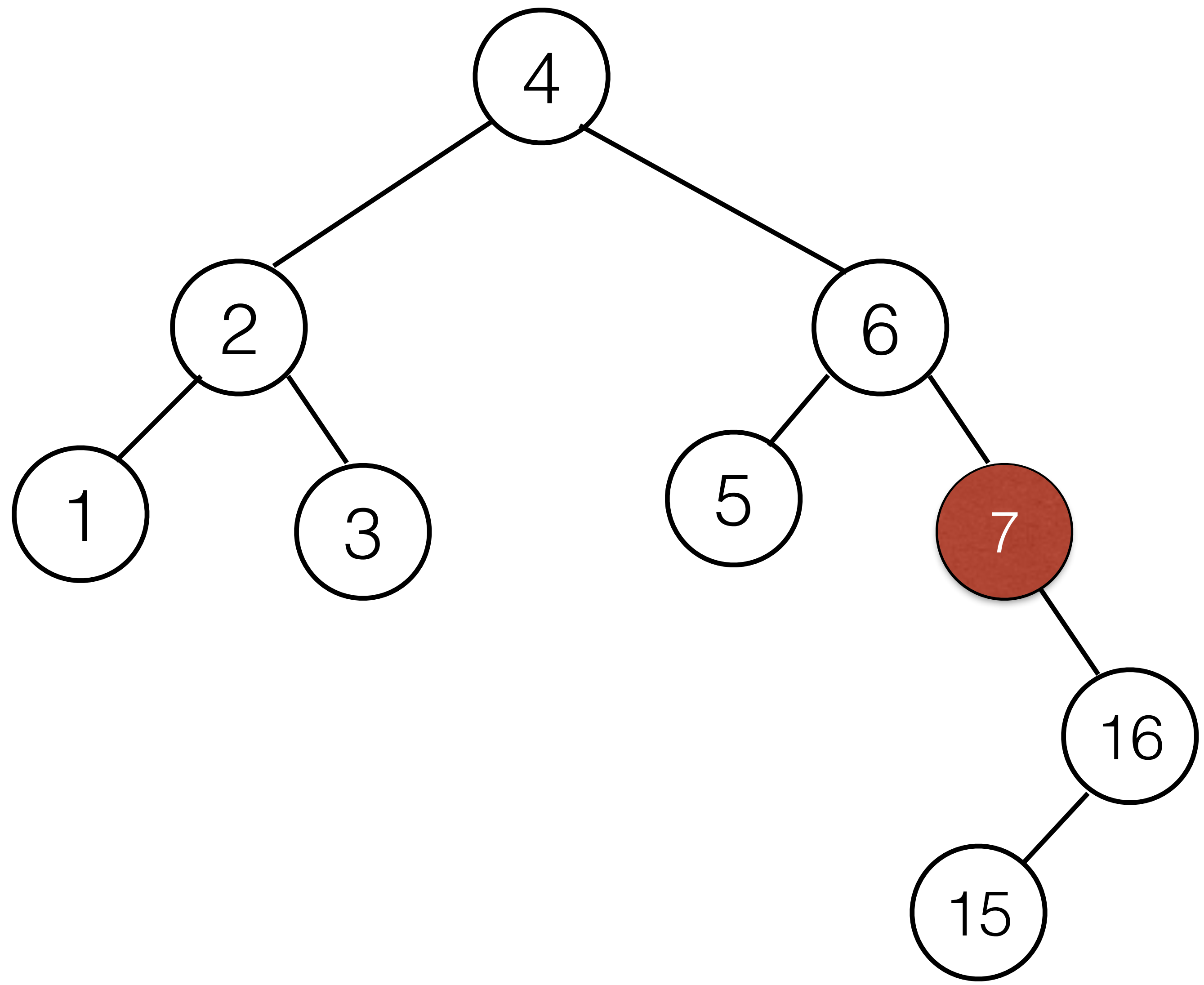
insert(16)



Double Rotation Example

insert(16)

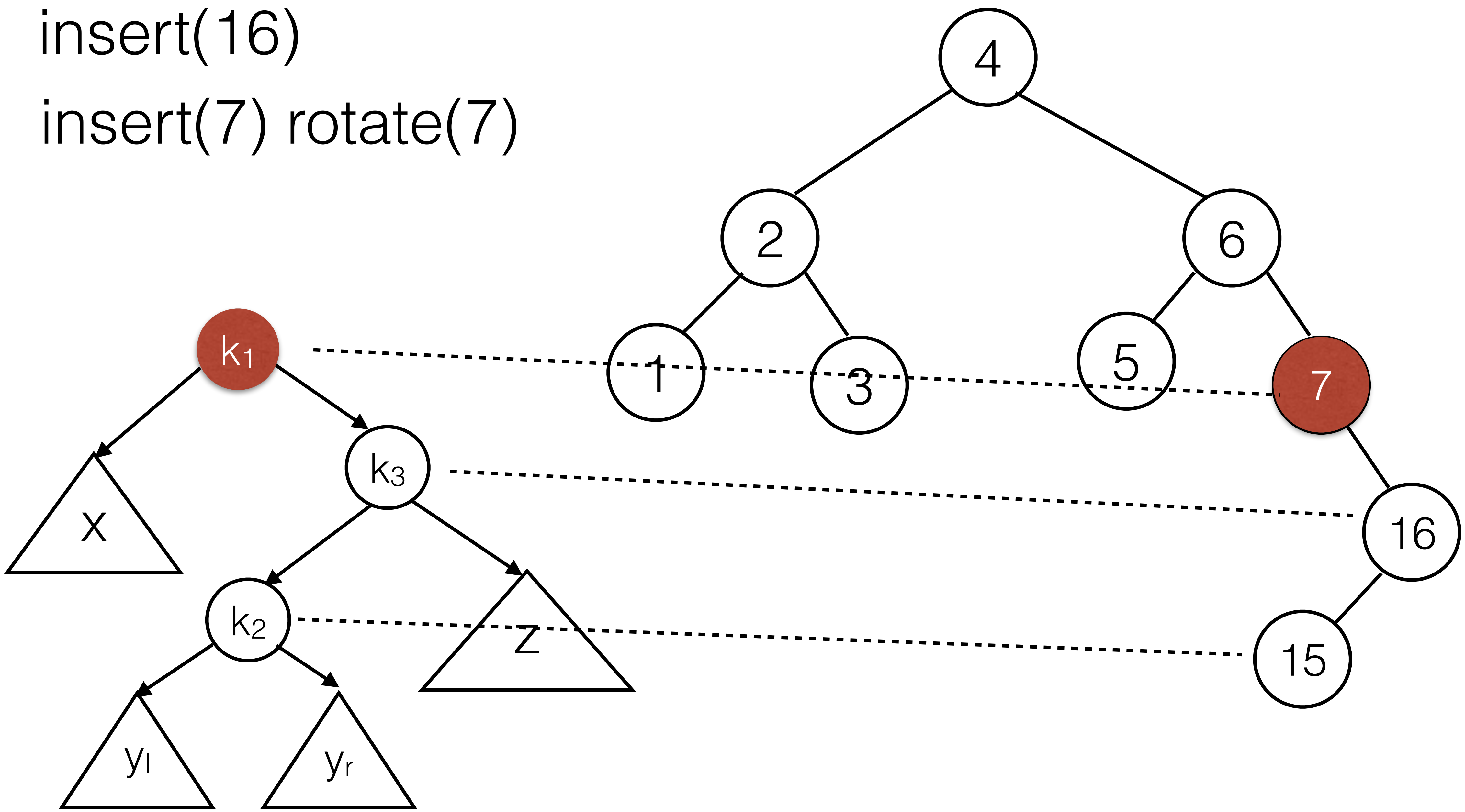
insert(7) rotate(7)



Double Rotation Example

insert(16)

insert(7) rotate(7)

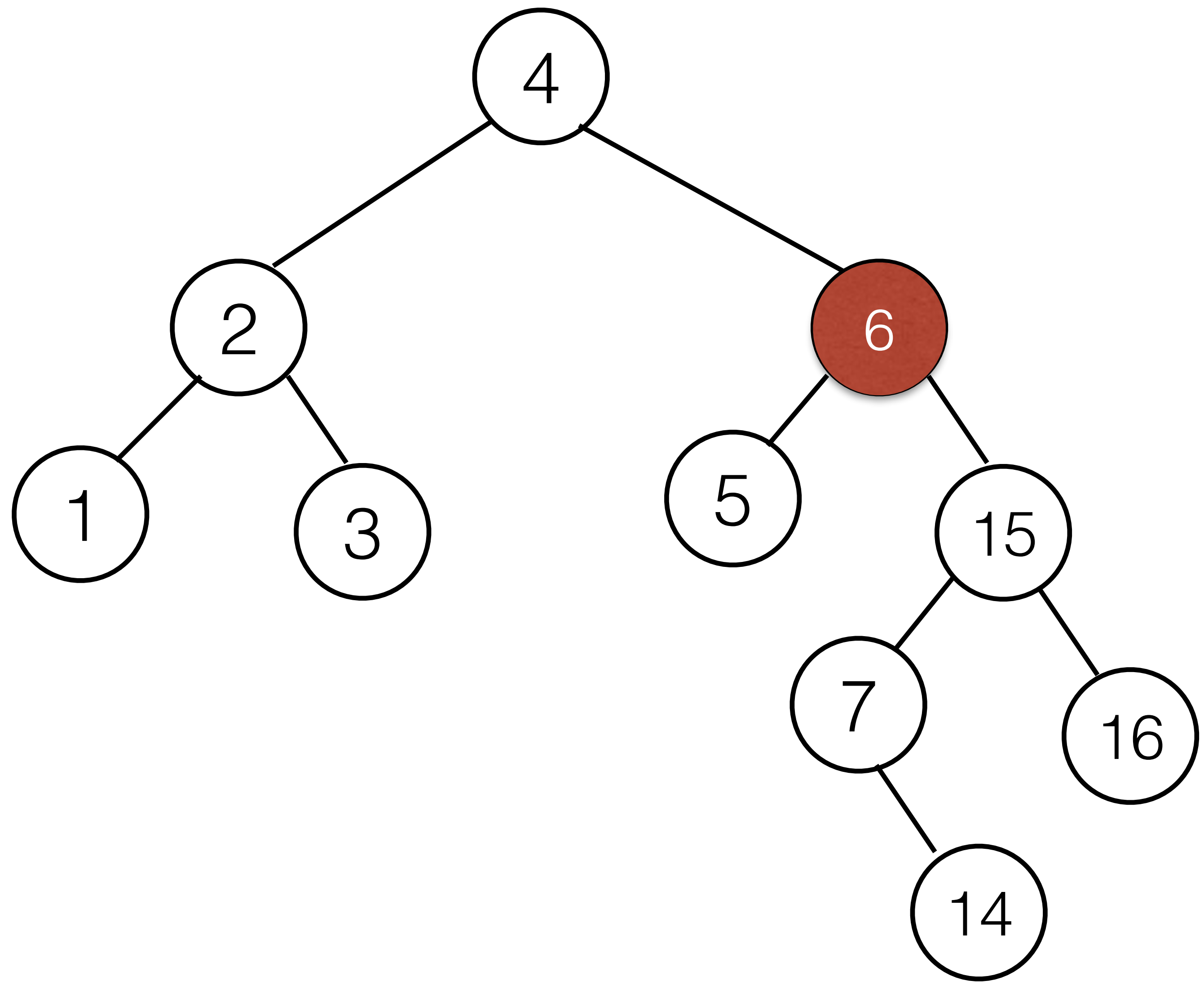


Double Rotation Example

insert(16)

insert(7) rotate(7)

insert(14) rotate(6)



Double Rotation Example

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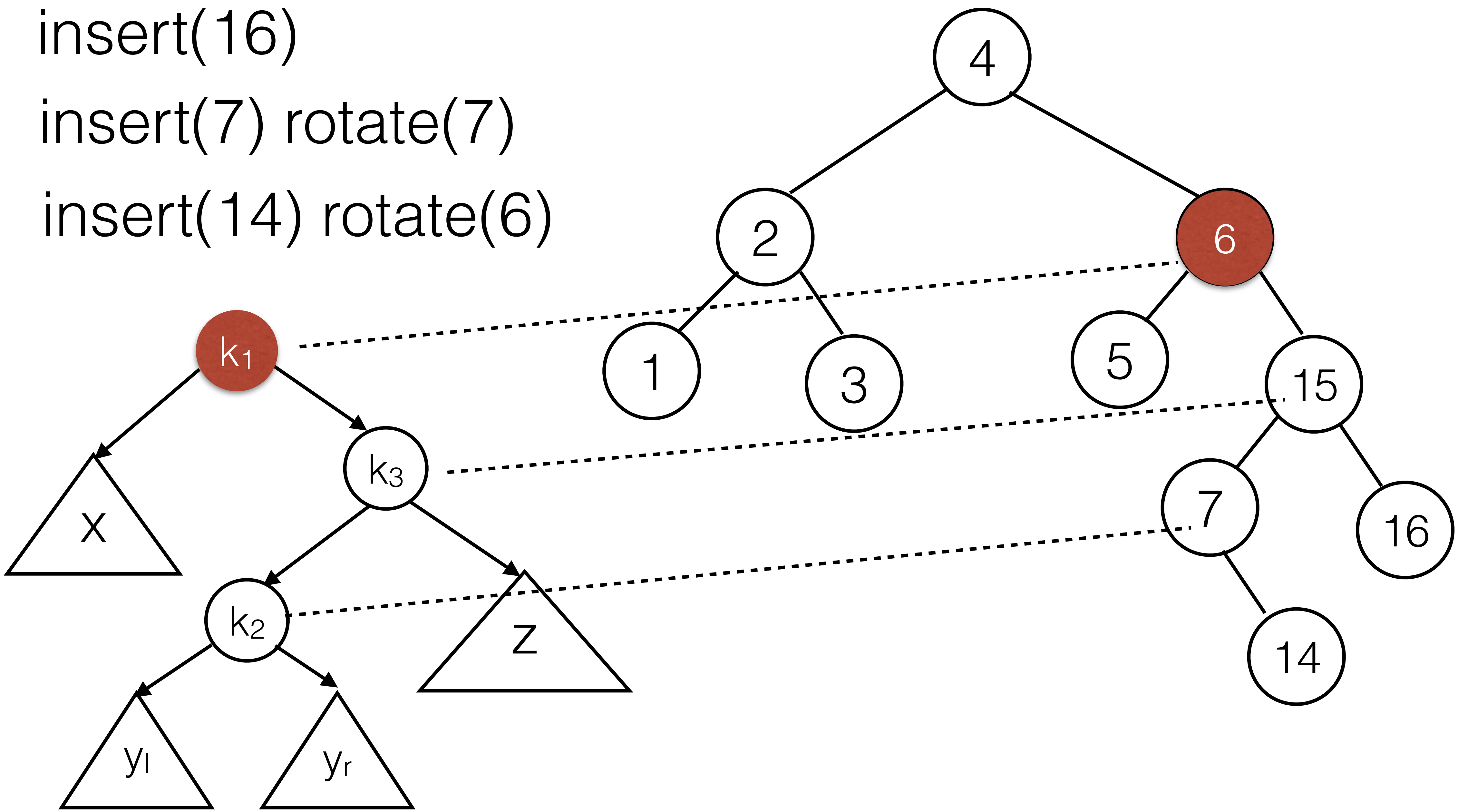

Double Rotation Example

insert(16)

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insert(14) rotate(6)