Data Structures in Java

Lecture 8: Trees and Tree Traversals.

10/5/2015

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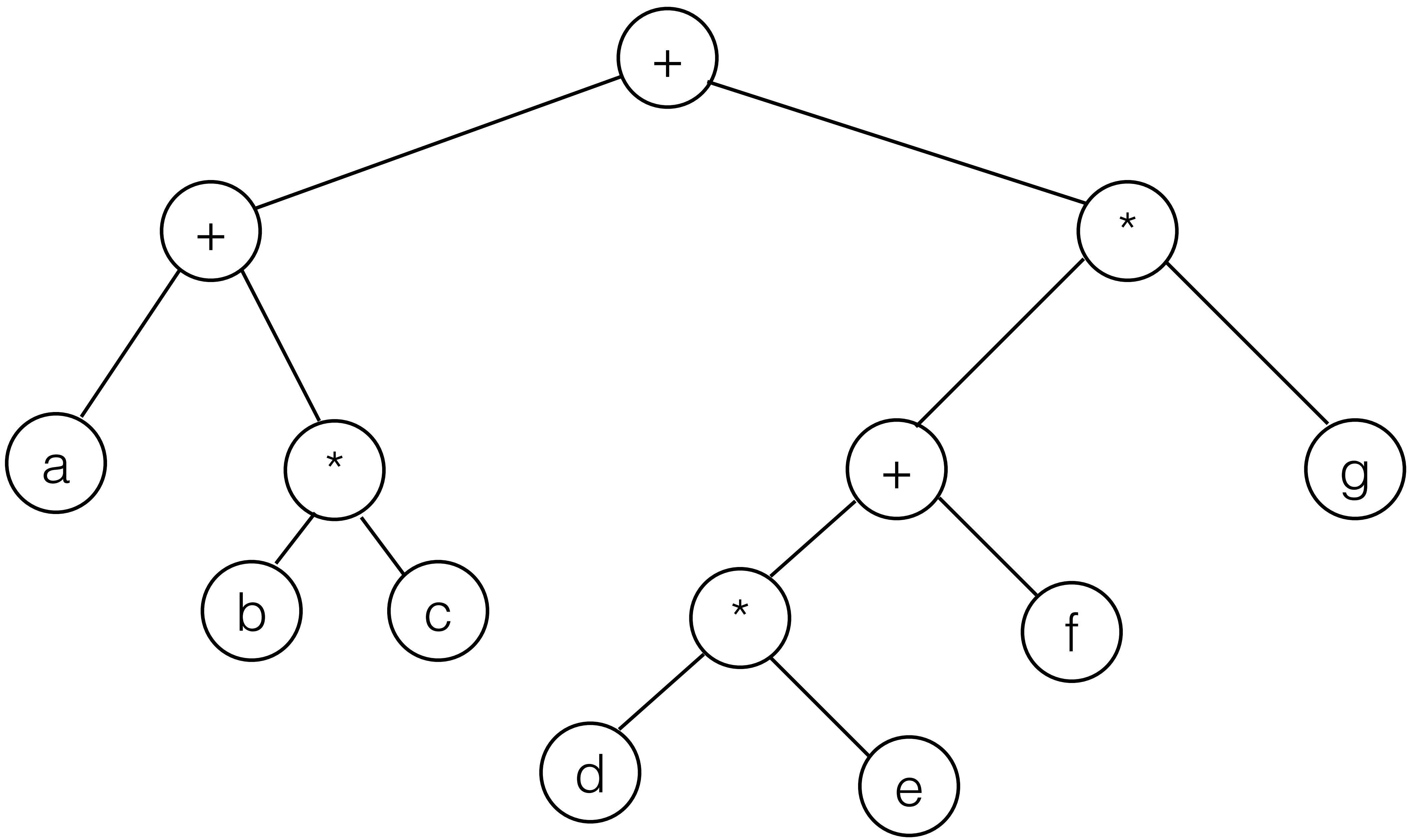
Trees in Computer Science

- A lot of data comes in a *hierarchical/nested structure.*
  - Mathematical expressions.
  - Program structure.
  - File systems.
  - Decision trees.
  - Natural Language Syntax, Taxonomies, Family Trees, …

Example: Expression Trees

(a + b * c) + (d * e + f) * g

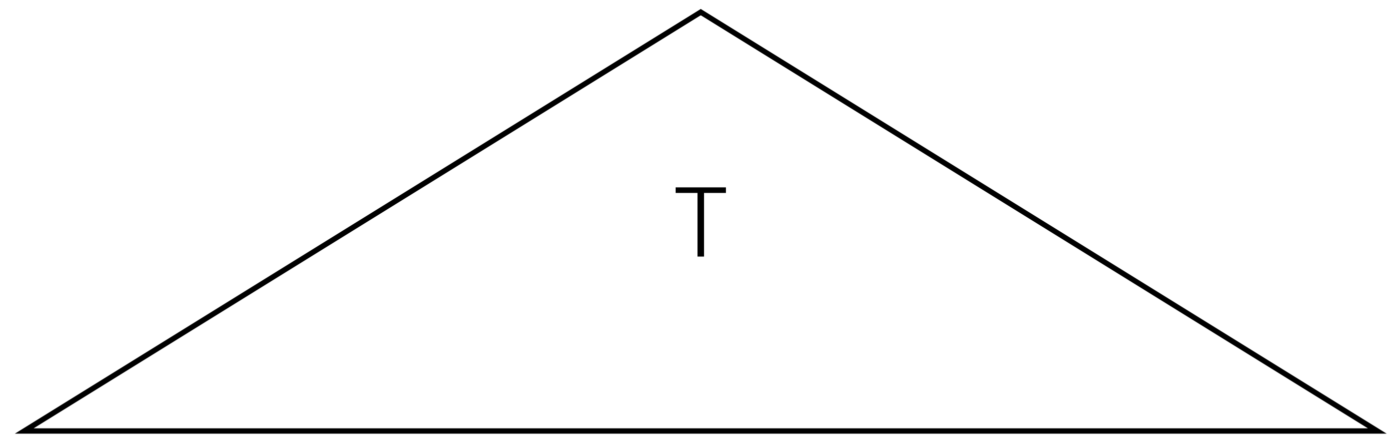


More Efficient Algorithms with Trees

- Sometimes we can represent data in a tree to speed up algorithms.
- Only need to consider part of the tree to solve certain problems:
  - Searching, Sorting,…
- Can often speed up O(N) algorithms to O(log N) once data is represented as a tree.

Tree ADT

- A tree *T* consists of
  - A root node *r*.
  - zero or more nonempty subtrees T_1, T_2, … $T_{N,}$
    - each connected by a directed edge from r.
  - Support typical collection operations: size, get, set, add, remove, find, …



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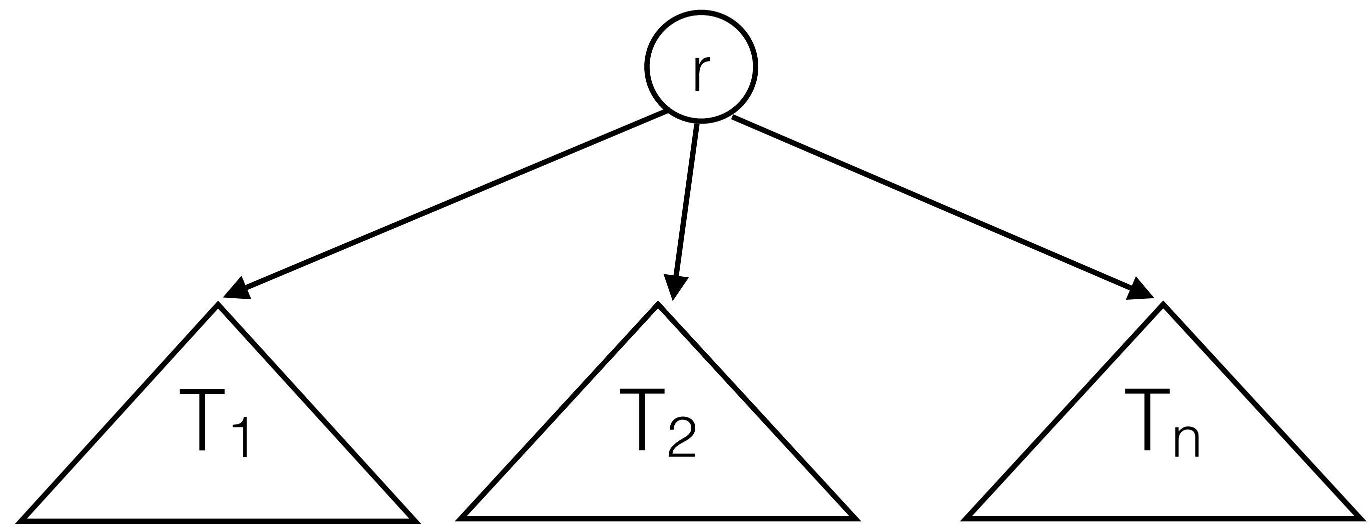

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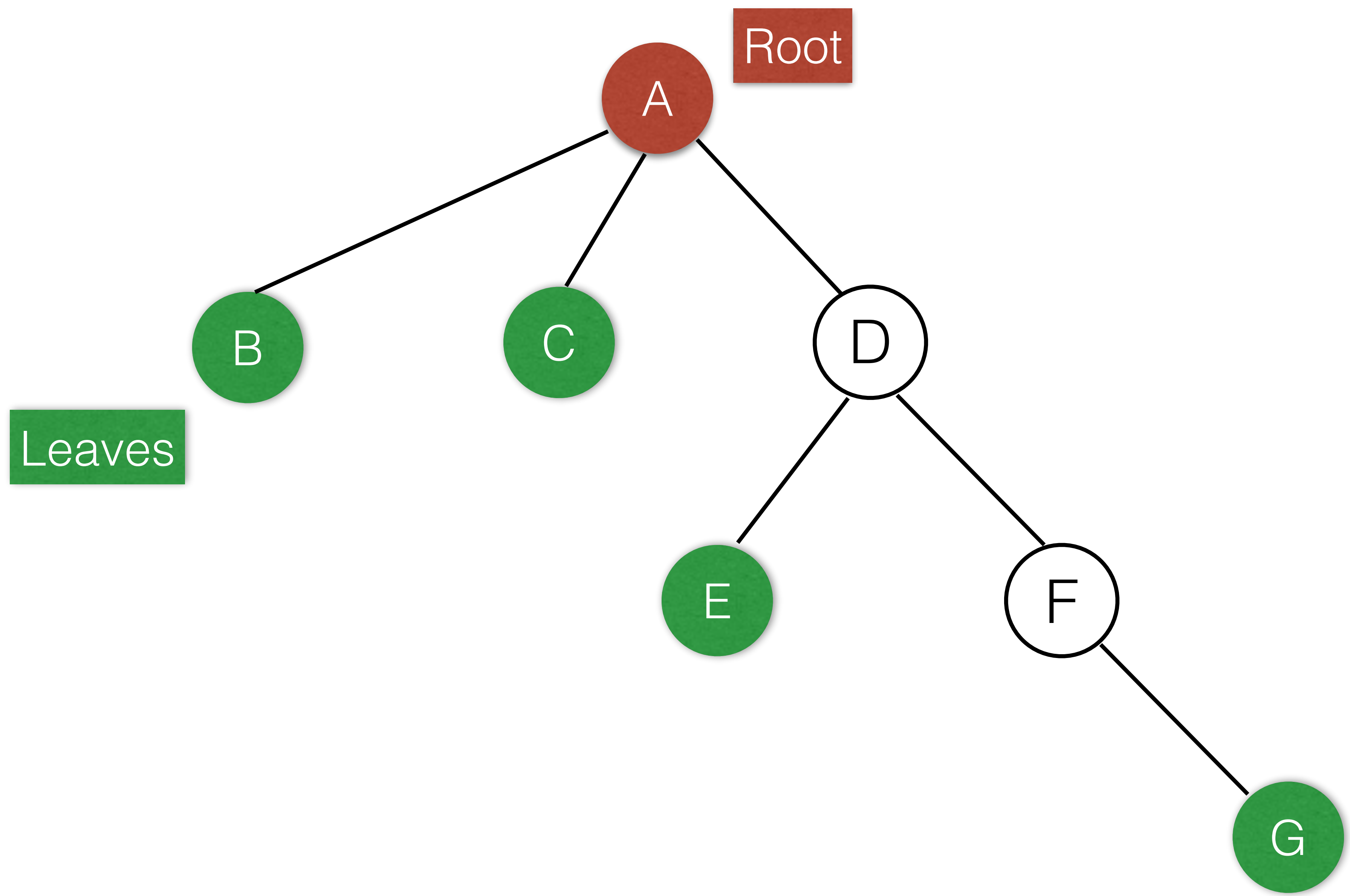
Tree Terminology



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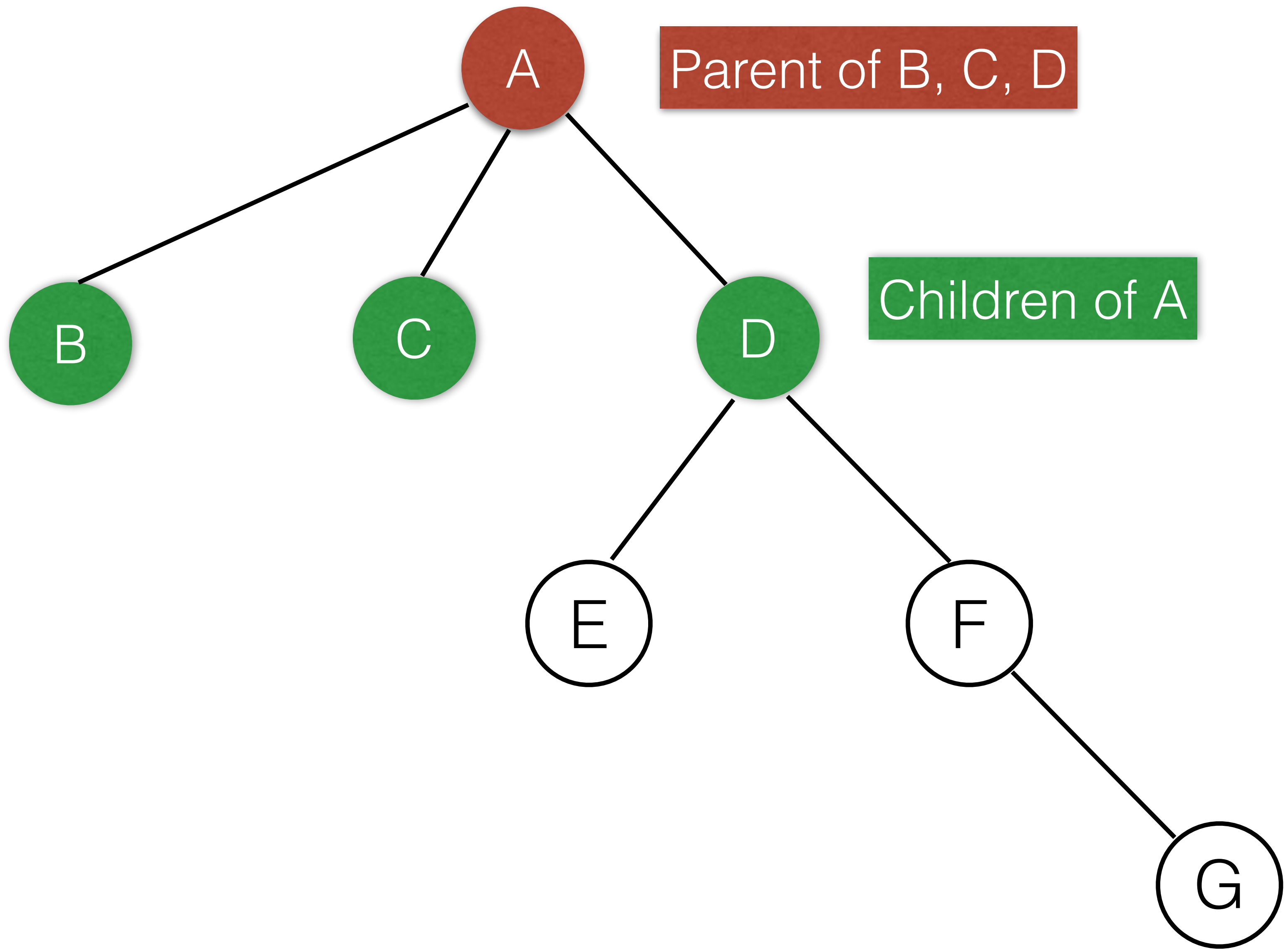

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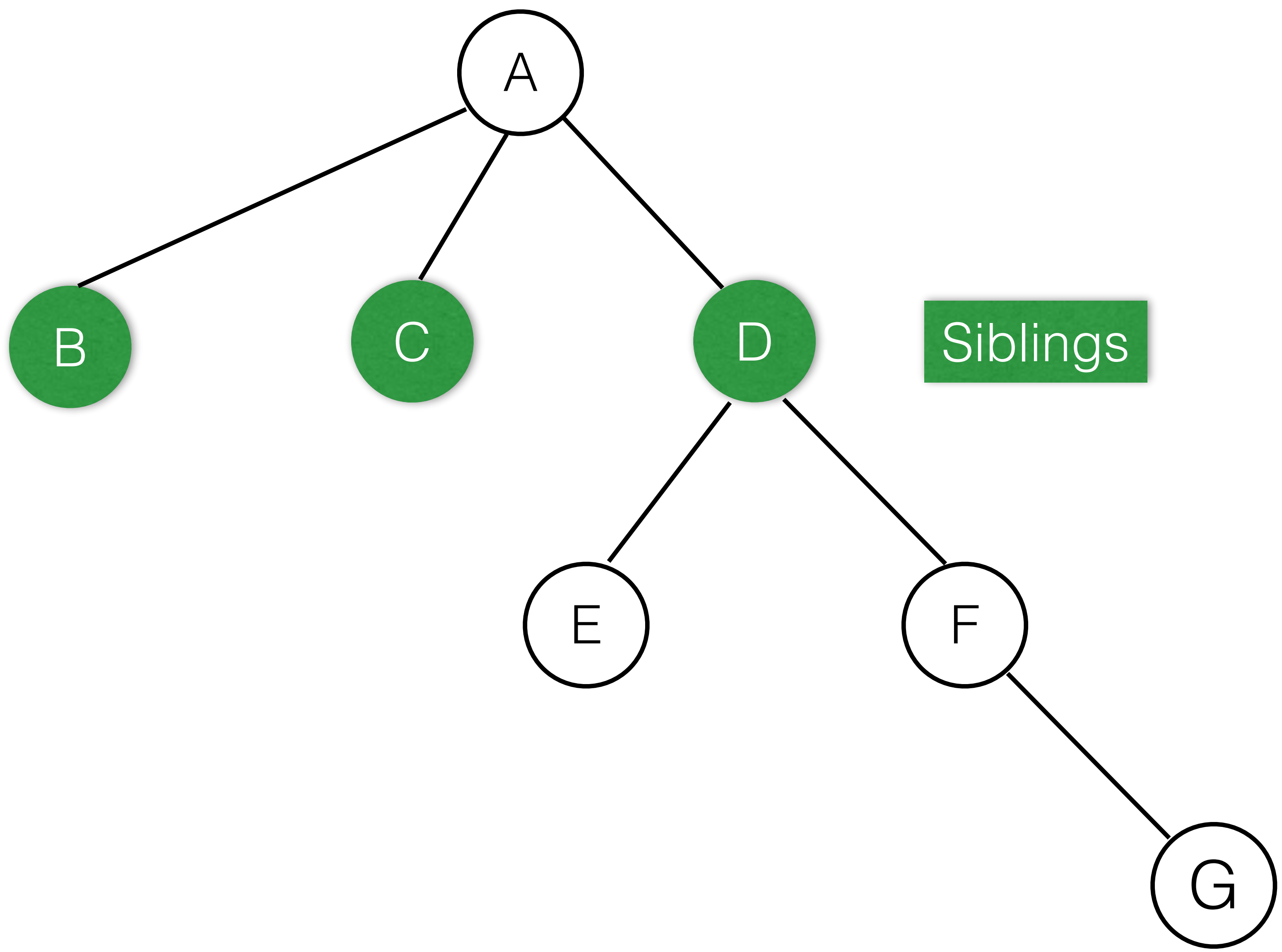

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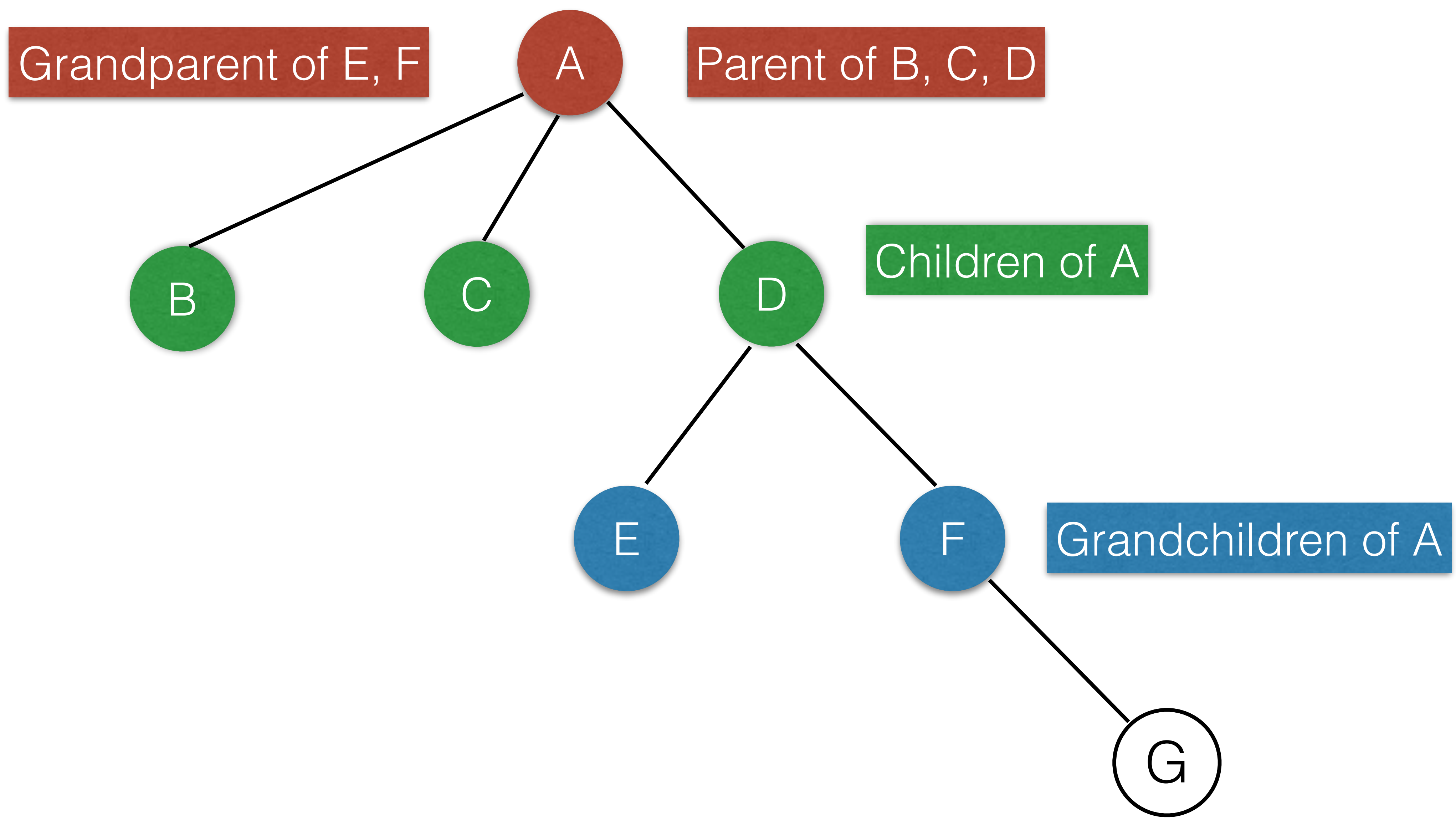

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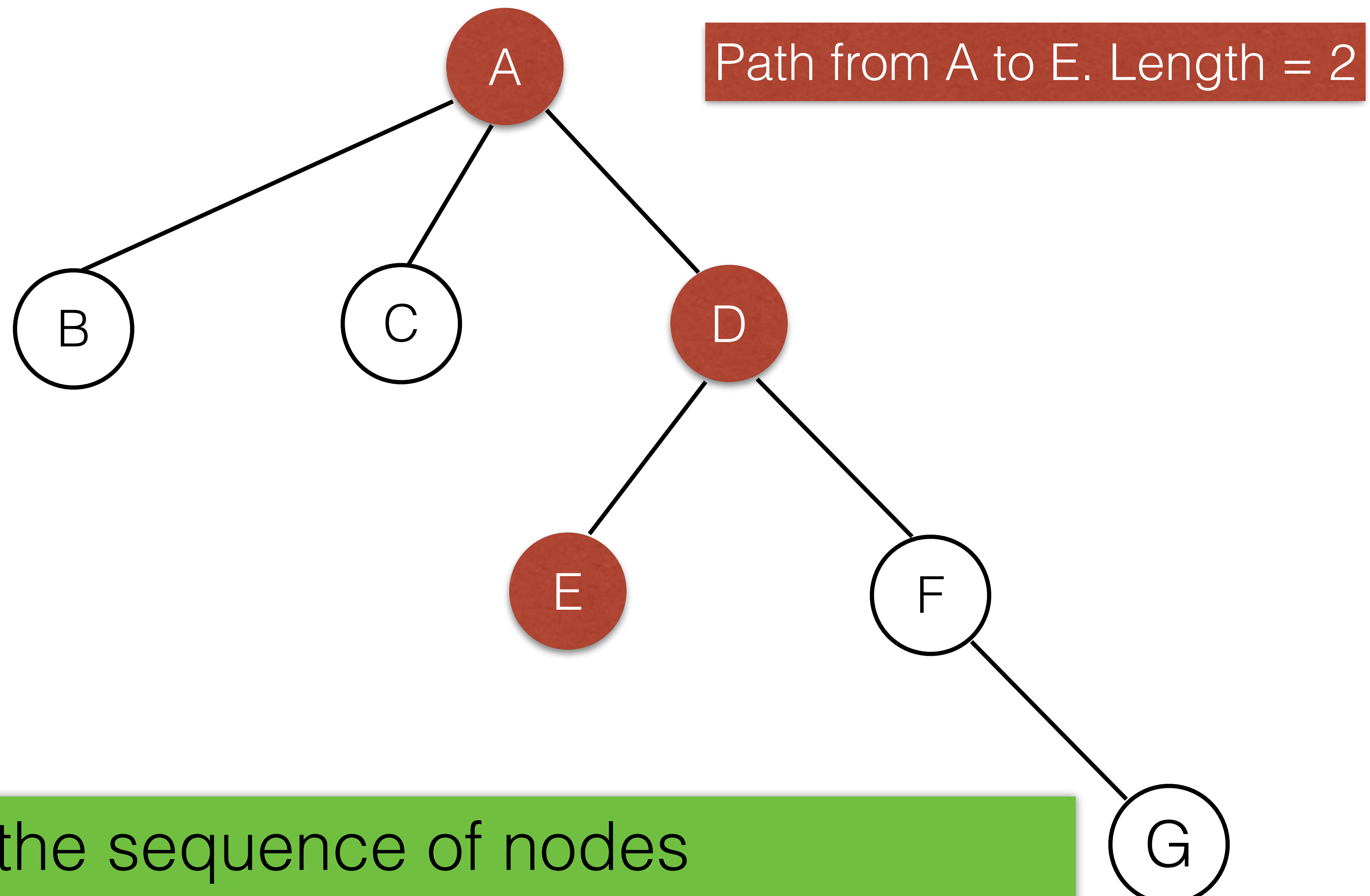
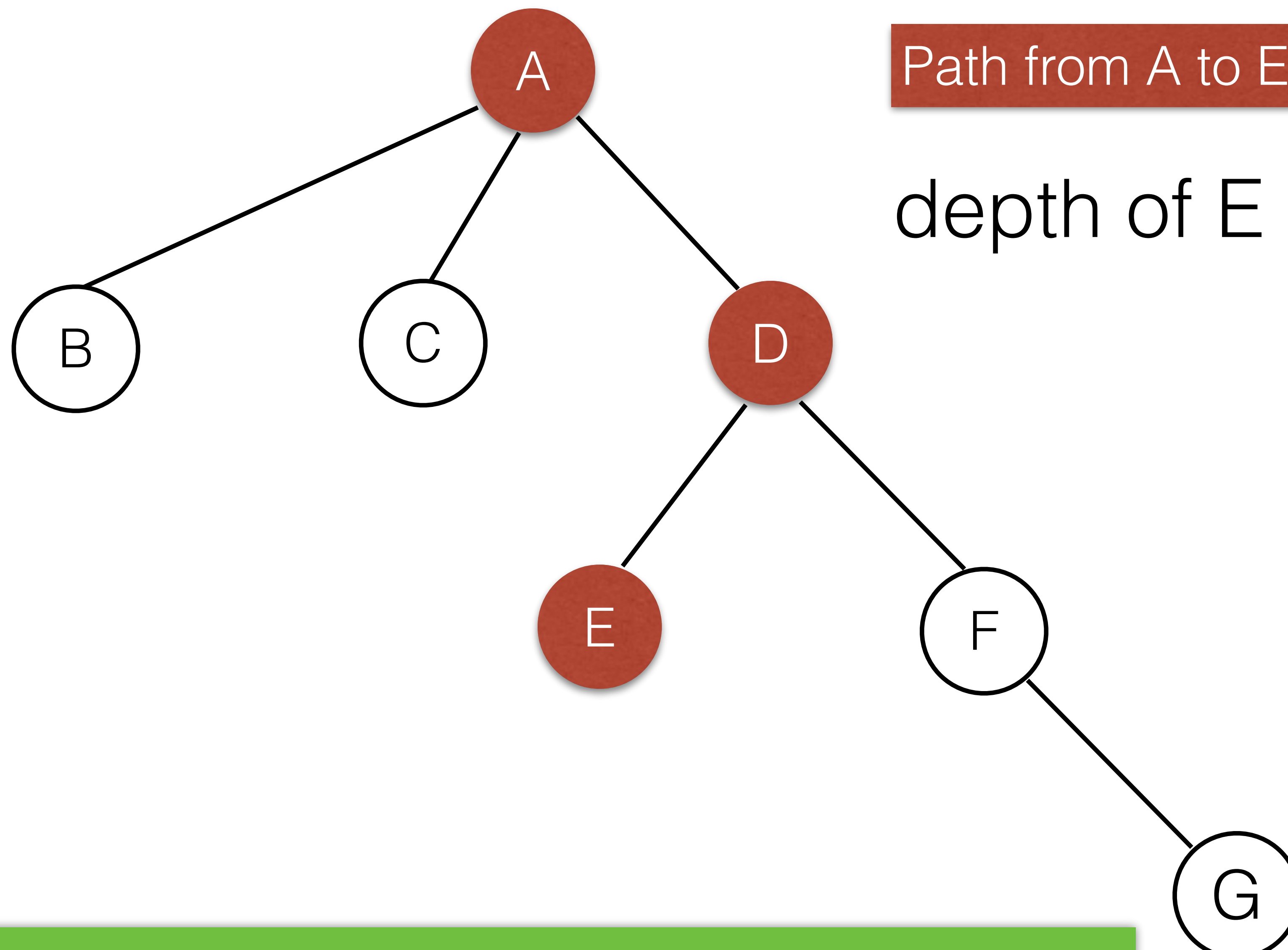


Path from n_1 to n_k : the sequence of nodes $n_k, n_2, \ldots, n_k$, such that n_i is the parent of n_{i+1} for $1 \leq i < k$.

Length of a path: $k-1$ = number of edges on the path

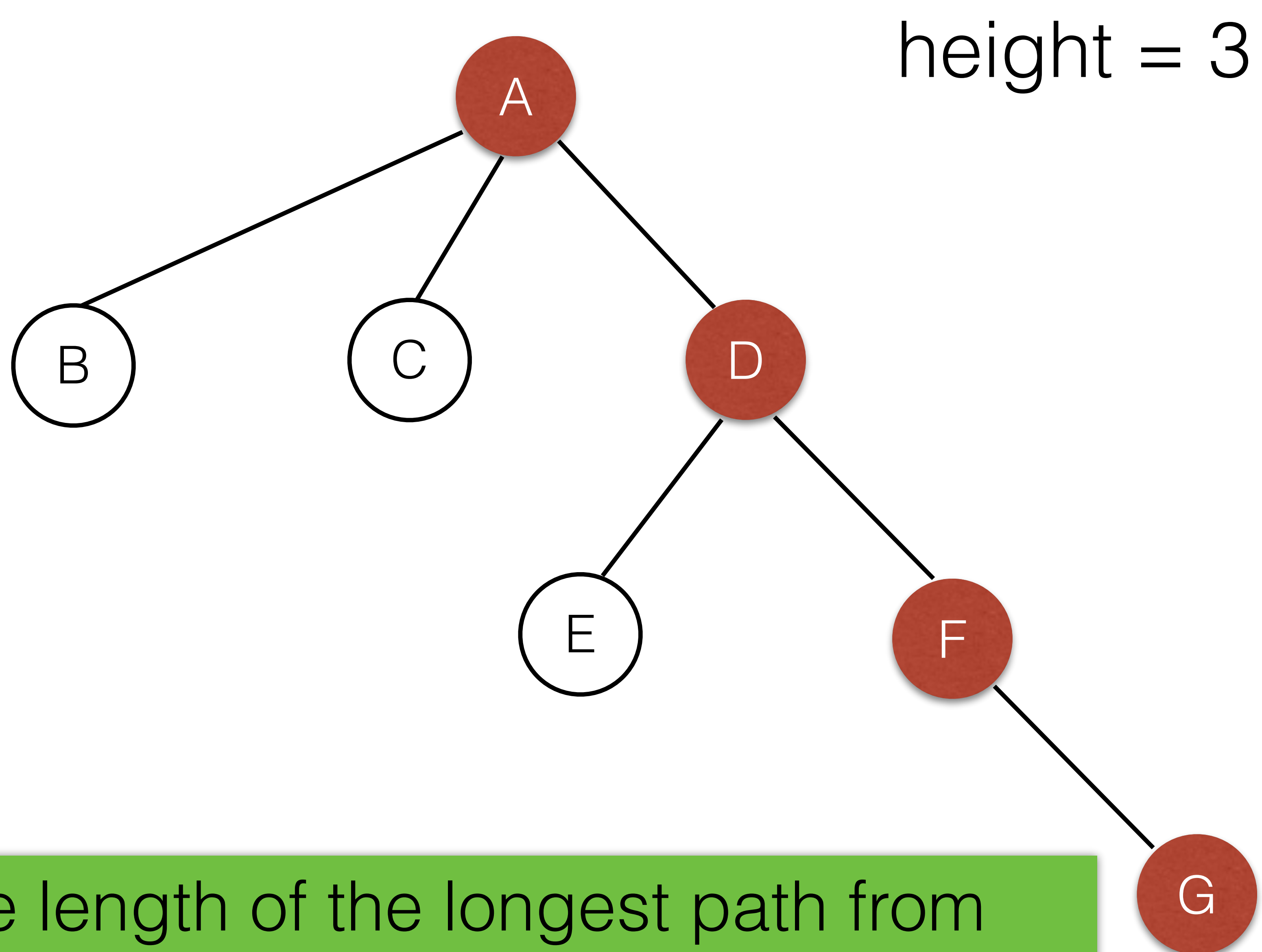
Tree Terminology



depth of E = 2

Depth of n_k: the length of the path from root to n_k.

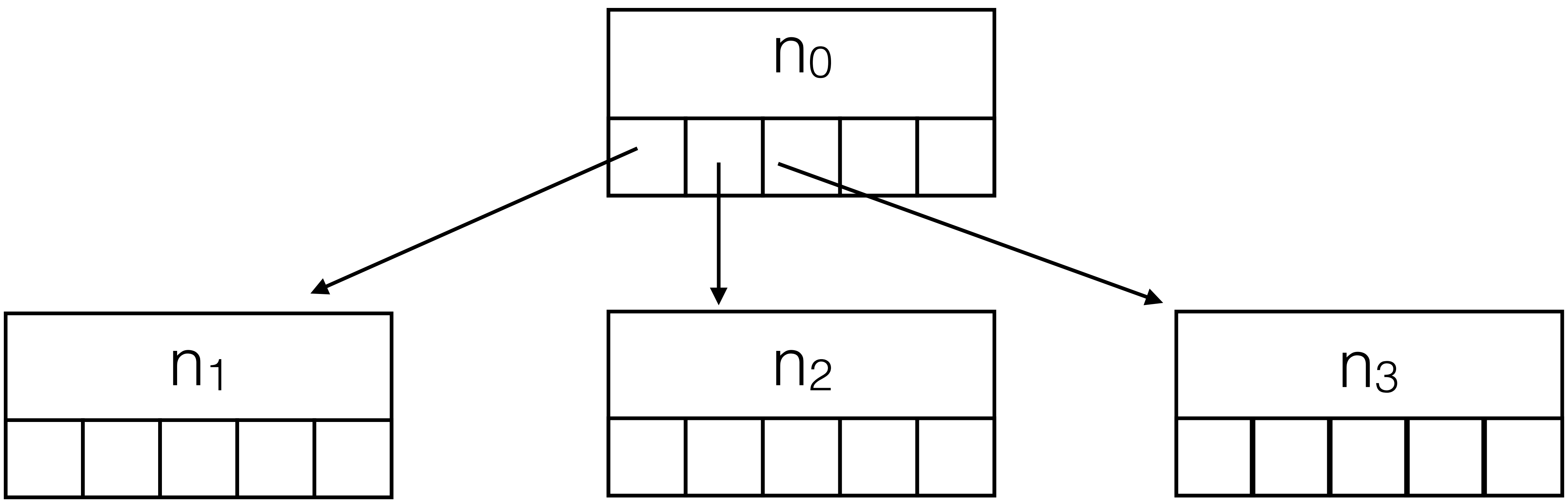
Tree Terminology



Height of tree T: the length of the longest path from root to a leaf.

Representing Trees

- Option 1: Every node has fixed number of references to children.



- Problem: Only reasonable for small or constant number of children.

Binary Trees

- For binary trees, the number of children is at most two.
- Binary trees are very common in data structures and algorithms.
- Binary tree algorithms are convenient to analyze.

Implementing Binary Trees

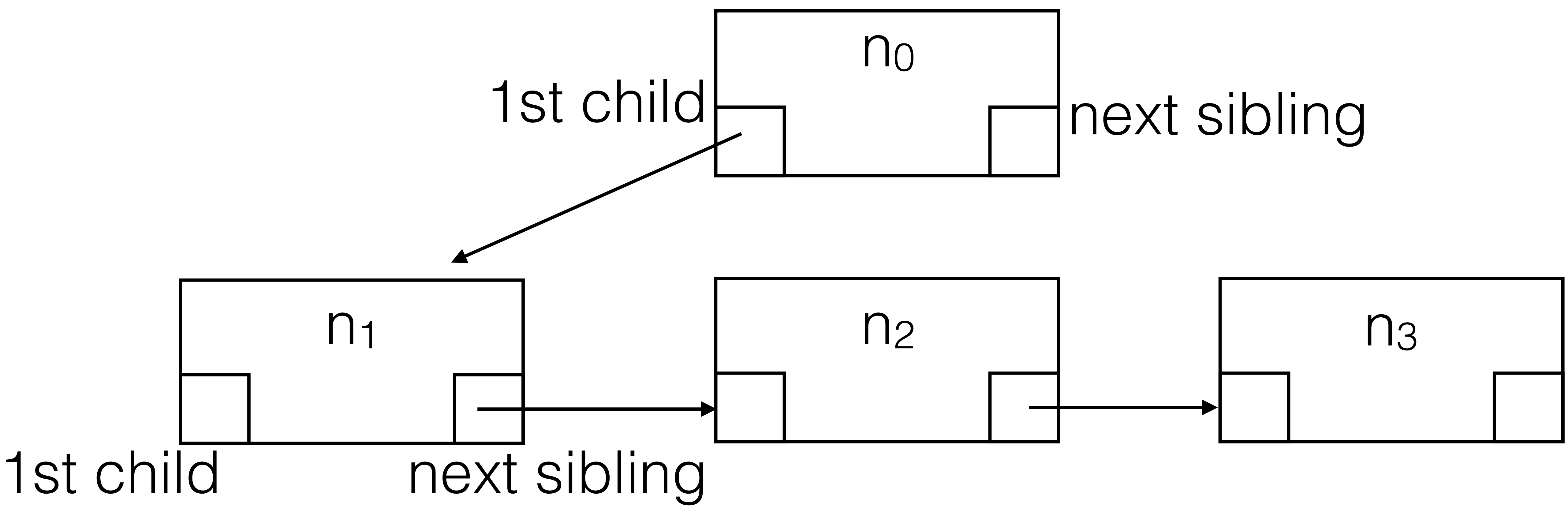
```
public class BinaryTree<T> {

    // The BinaryTree is essentially just a wrapper around the
    // linked structure of BinaryNodes, rooted in root.
    private BinaryNode<T> root;

    /**
     * Represent a binary subtree.
     */
    private static class BinaryNode<T>{
        public T              data;
        public BinaryNode<T> left;
        public BinaryNode<T> right;
        …
    }
    …
}
```

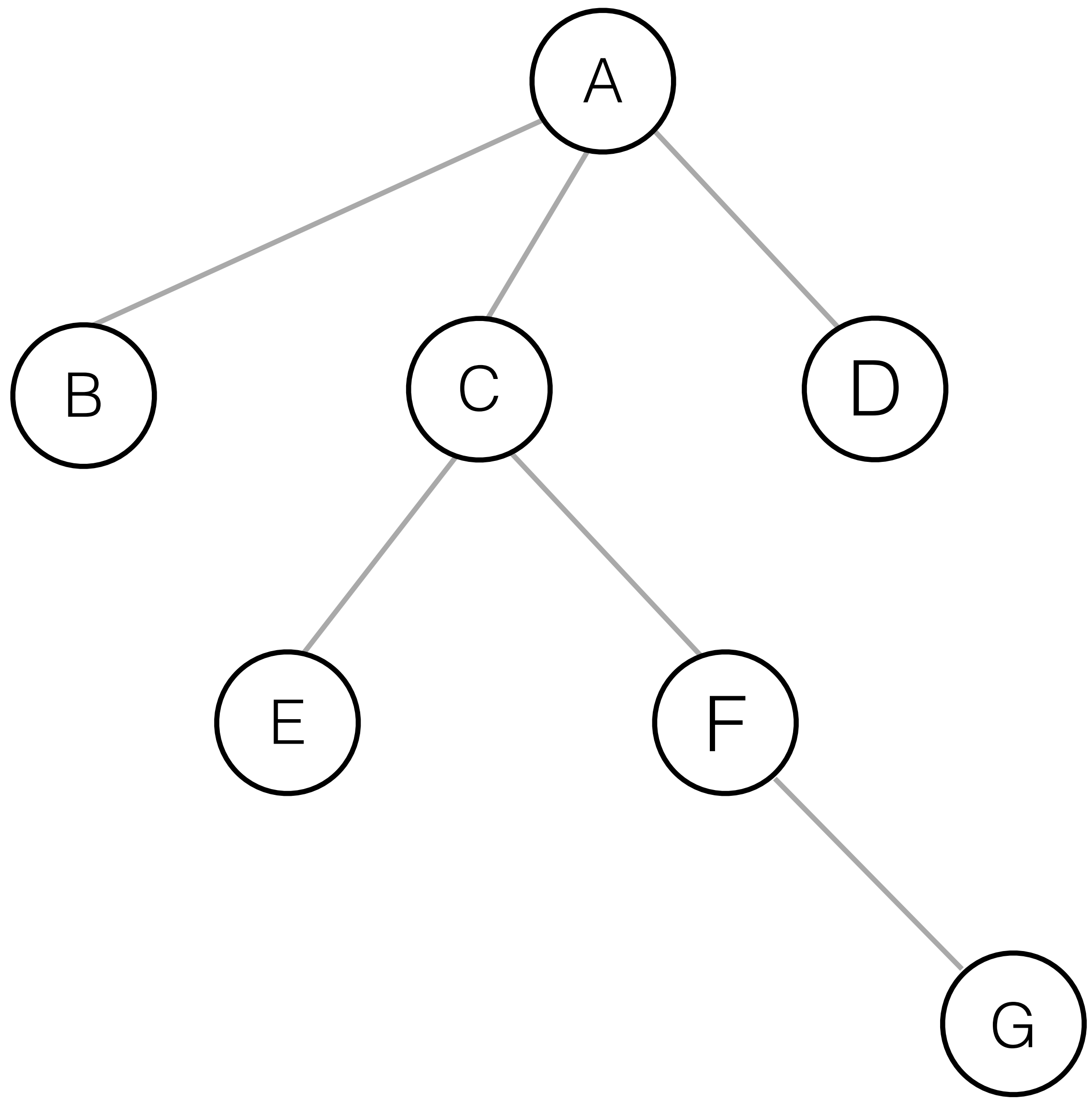
Representing Trees

- Option 2: Organize siblings as a linked list.



- Problem: Takes longer to find a node from the root.

Siblings as Linked List



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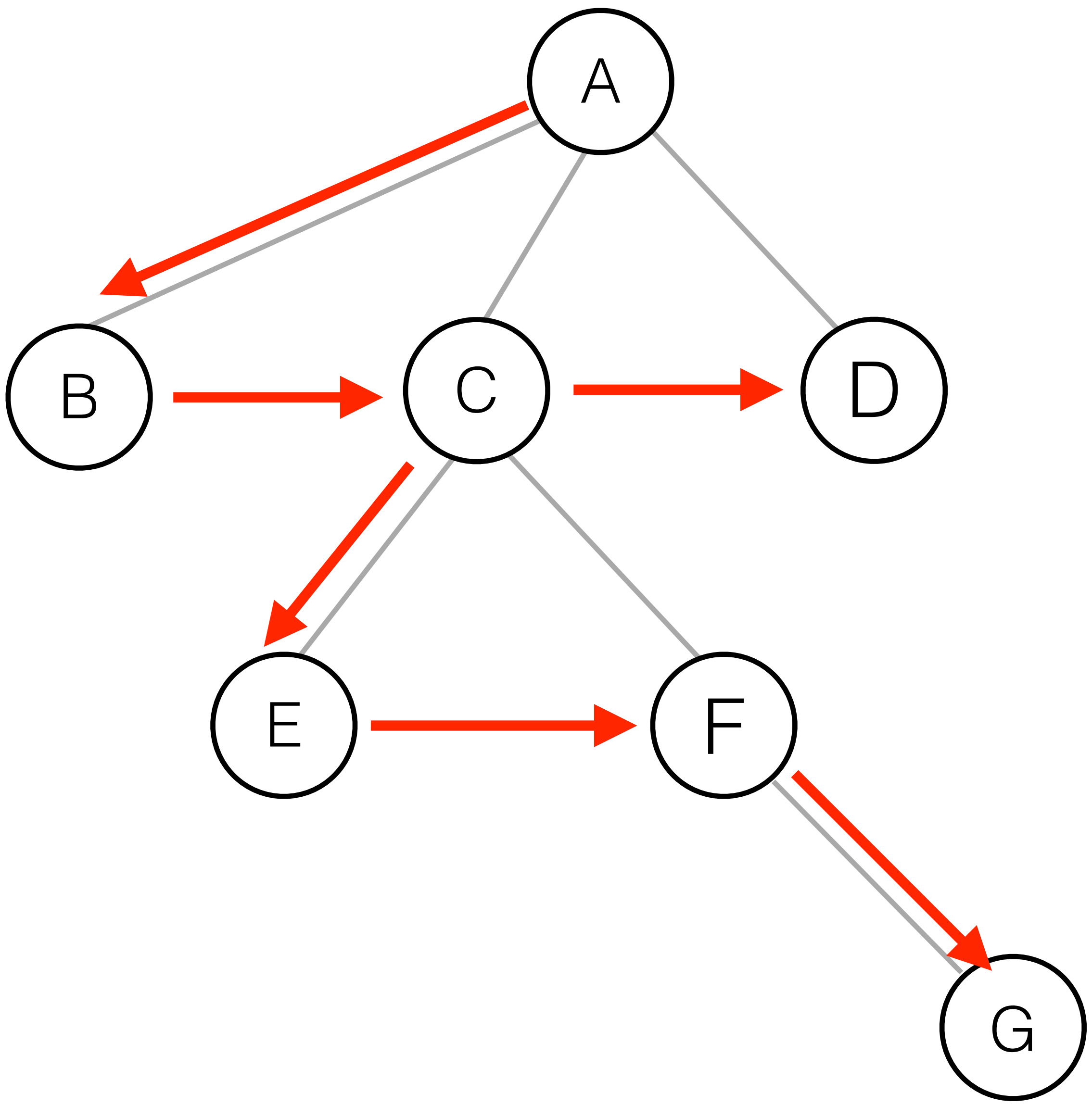
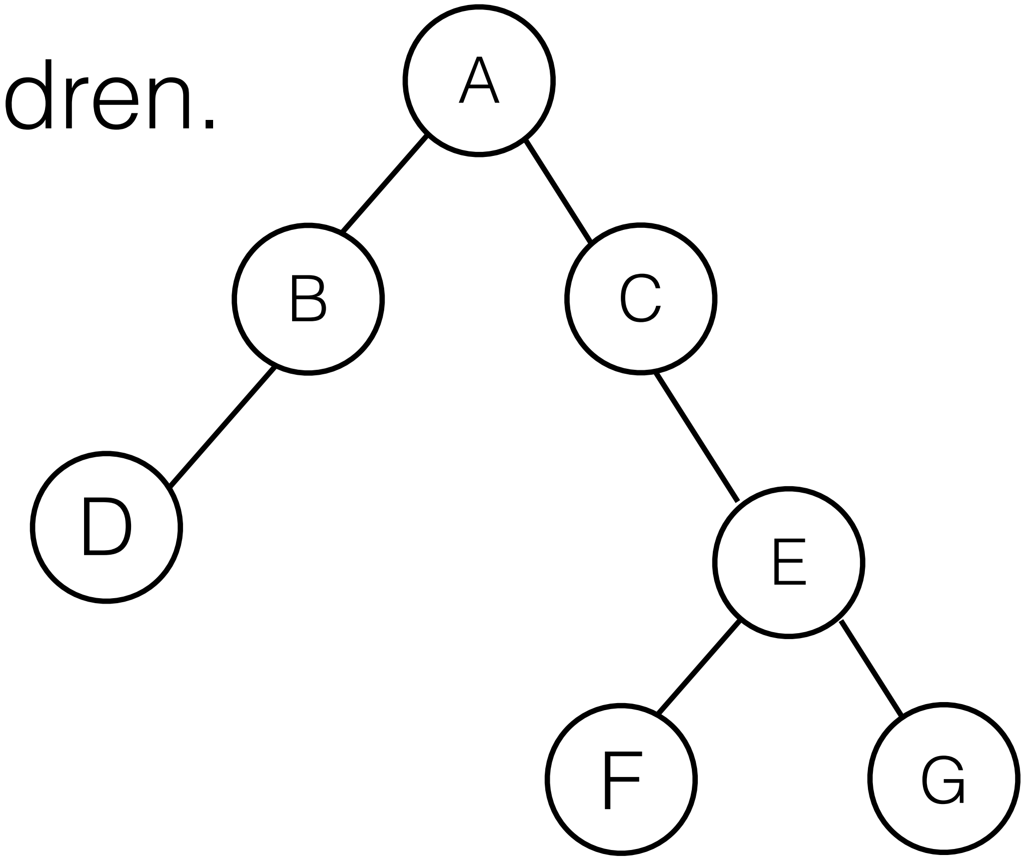


Implementing Siblings as Linked List

```
public class LinkedSiblingTree<E>  {

    private TreeNode<E> root;

    private static class TreeNode<E> {
        E element;
        TreeNode<E> firstChild;
        TreeNode<E> nextSibling;
        …
    }

    ...
}
```

Full Binary Trees

- In a full binary tree every node
  - is either a leaf.
  - or has exactly two children.



not full

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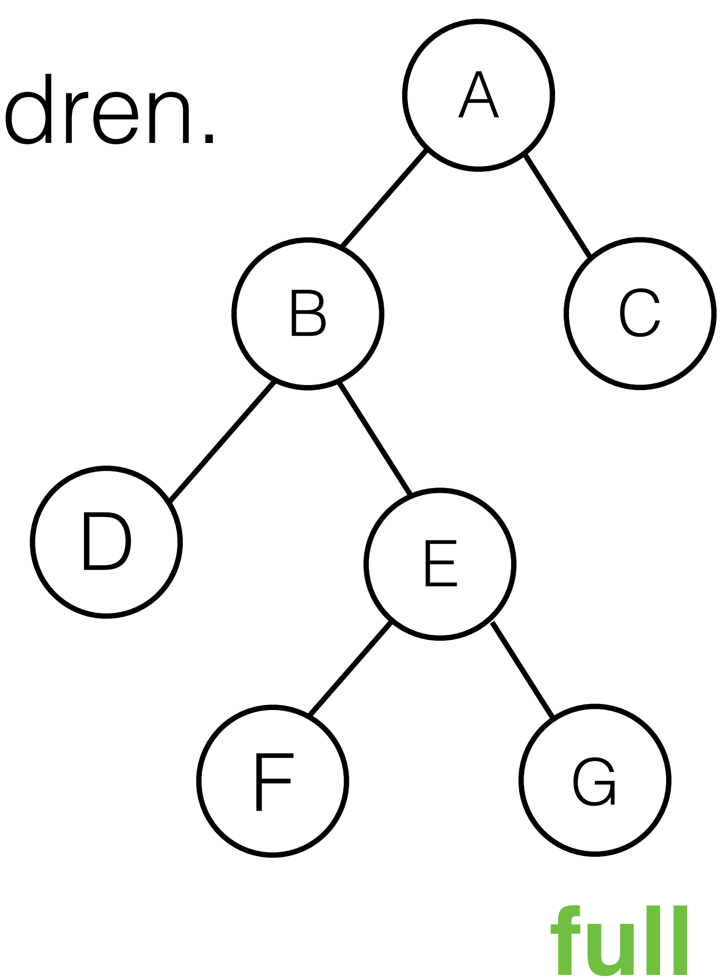
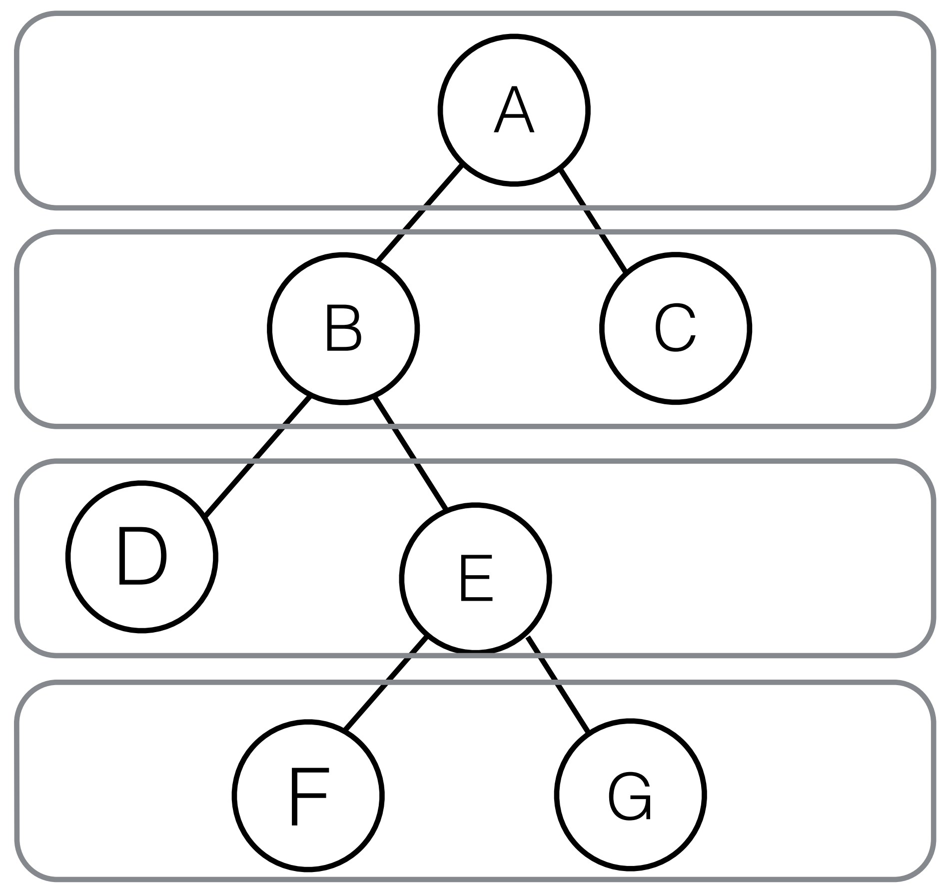


Complete Binary Trees

- A complete binary tree is a full binary tree in which all levels (except possibly the last) are completely filled.



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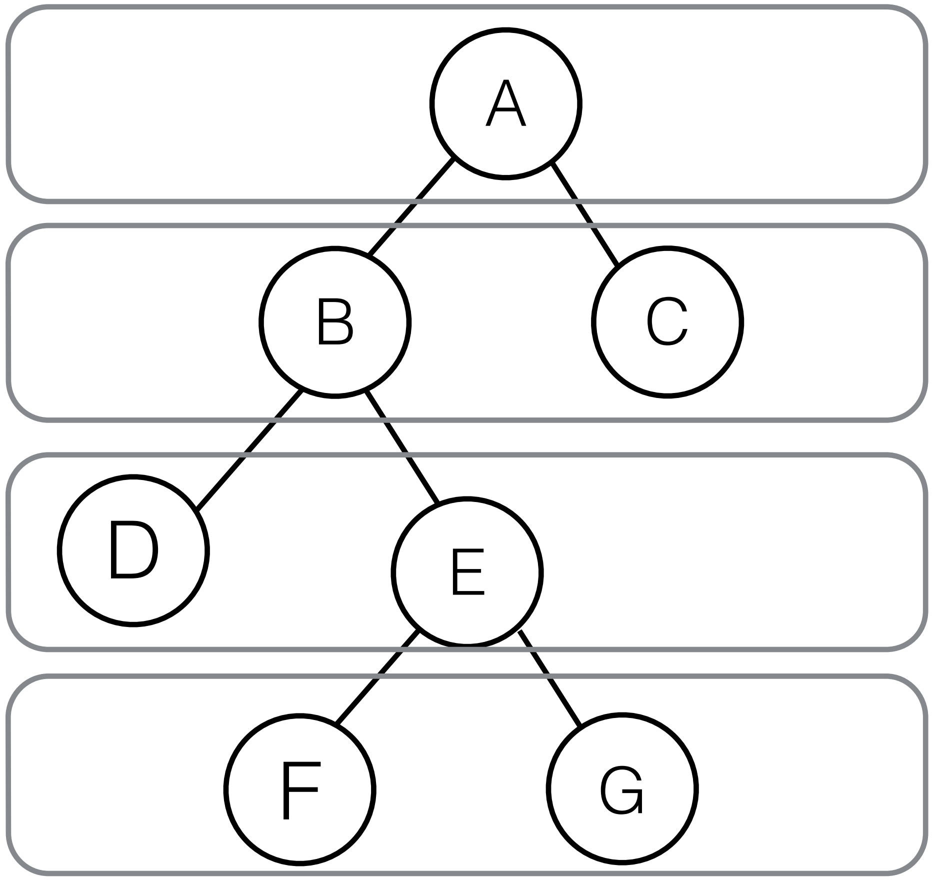

full, but not complete

Complete Binary Trees

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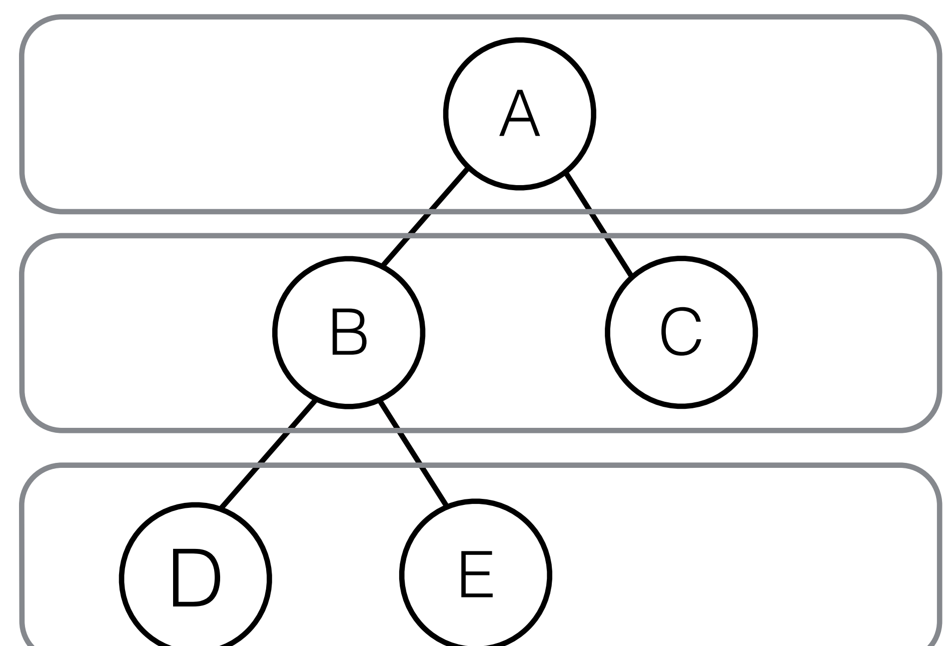

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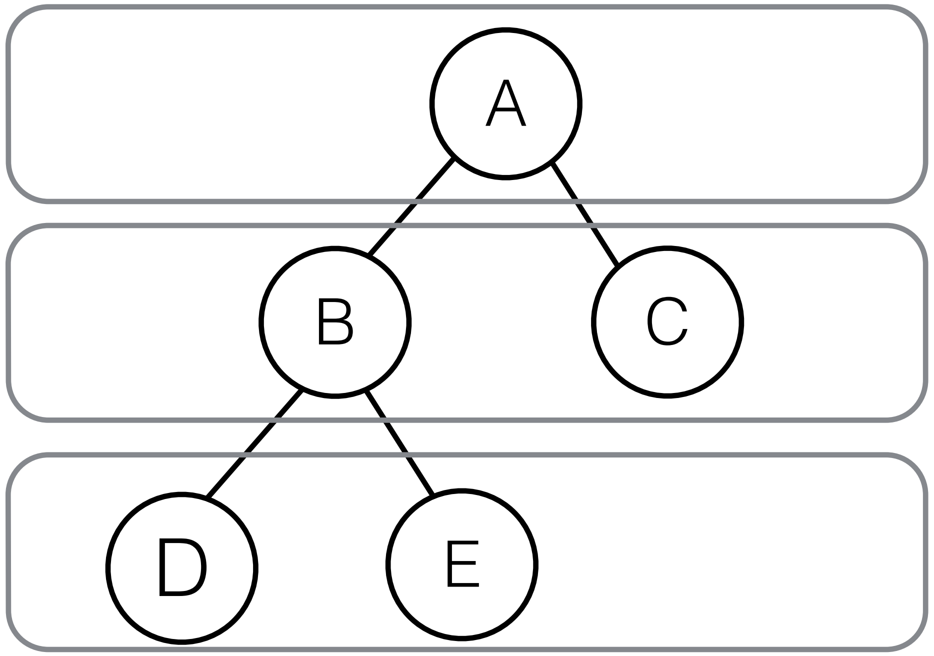

complete

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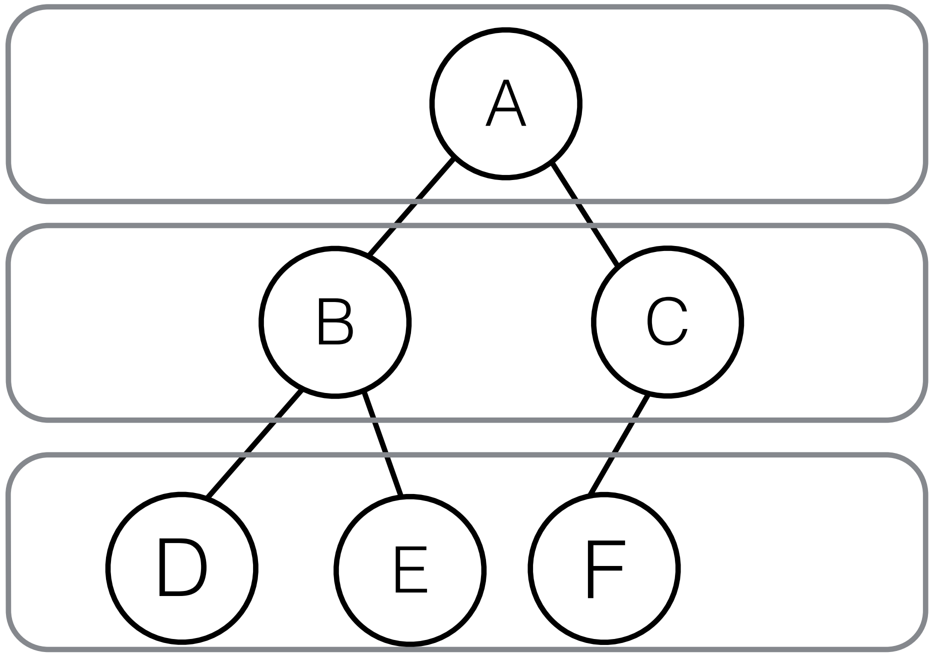
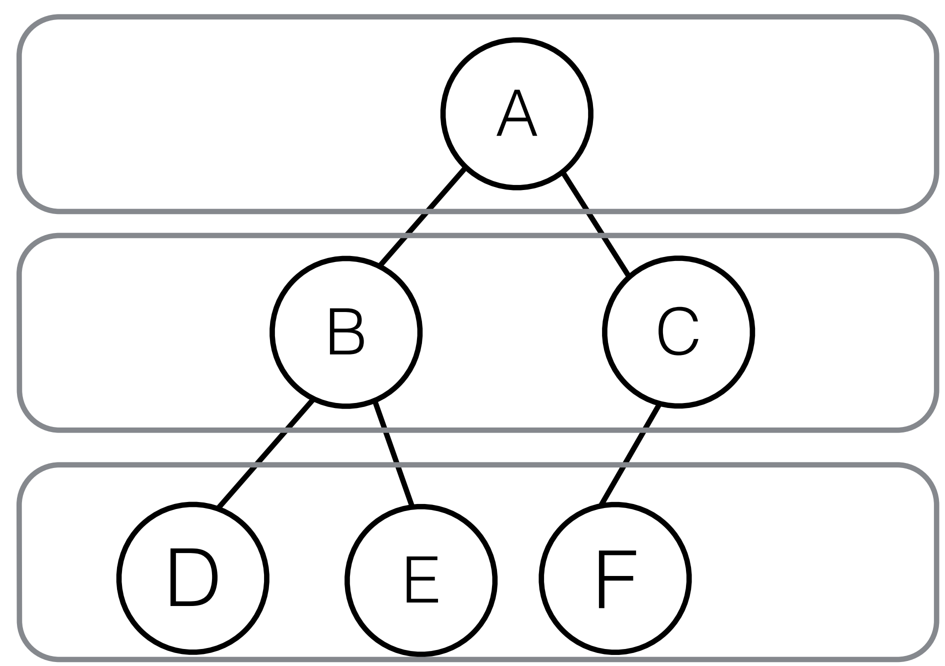


complete but not full

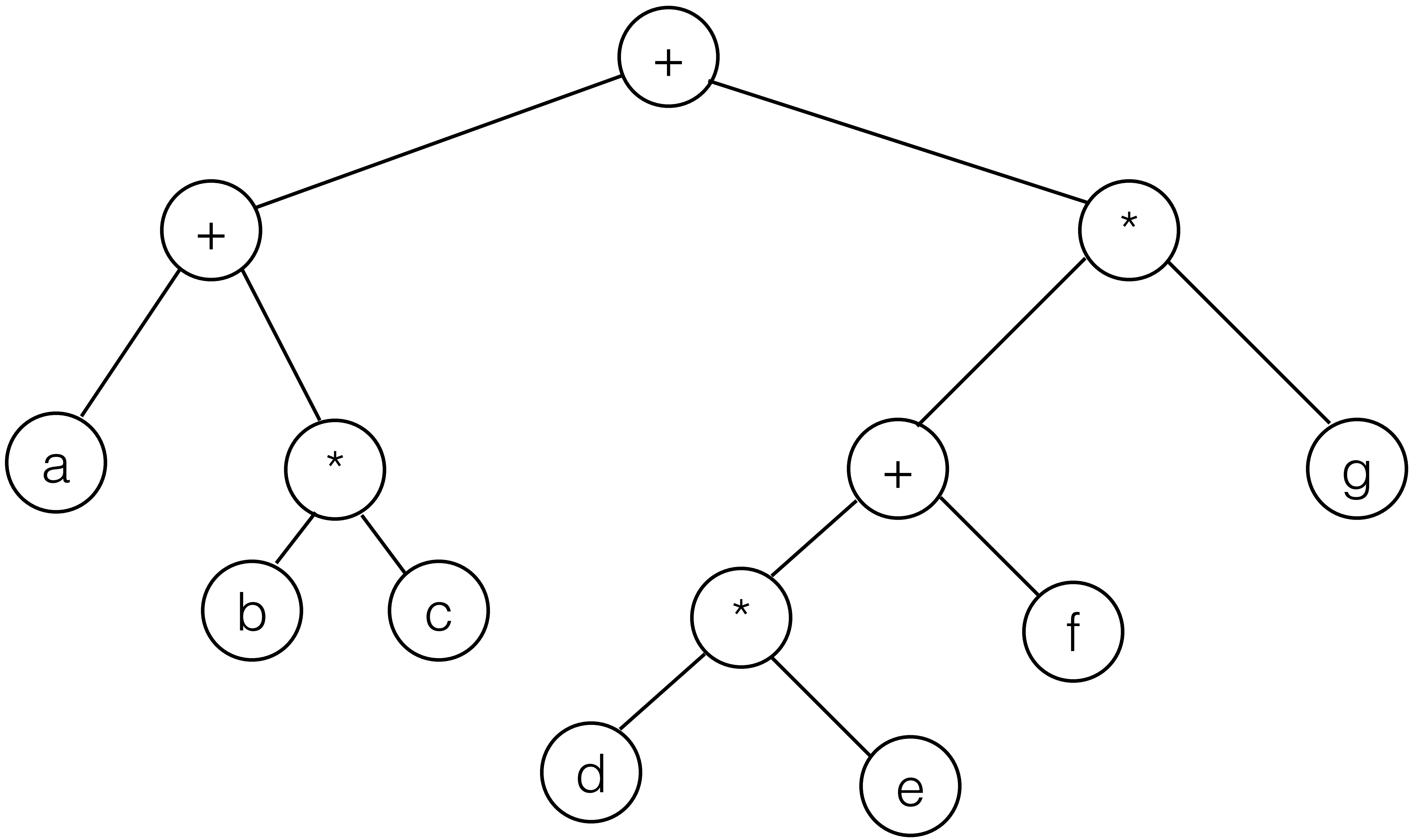
Storing Complete Binary Trees in Arrays

A	B	C	D	E	F		
0	1	2	3	4	5		



Structure of the tree only depends on the number of nodes.

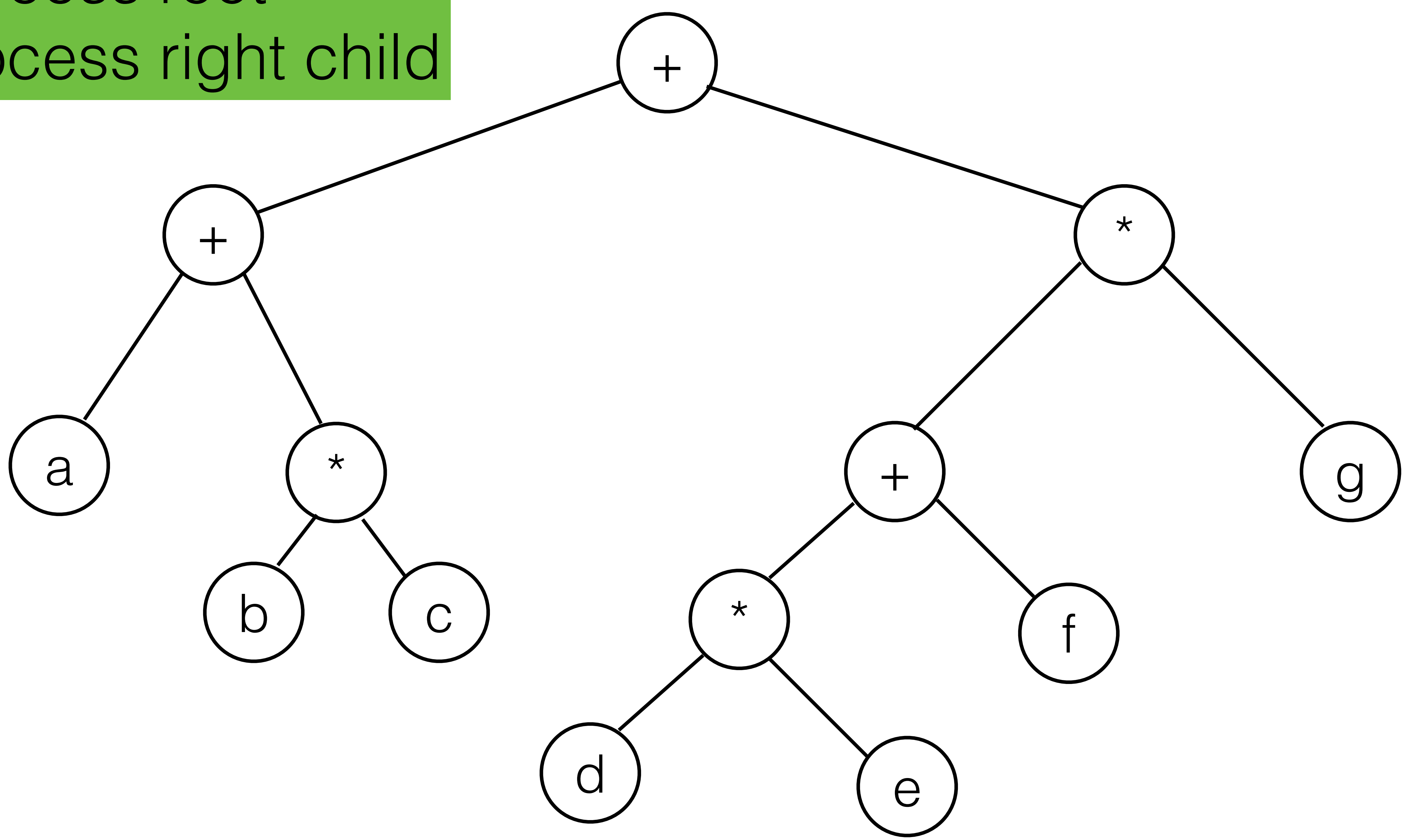
Example Binary Tree: Expression Trees



Tree Traversals: In-order

1. Process left child
2. Process root
3. Process right child

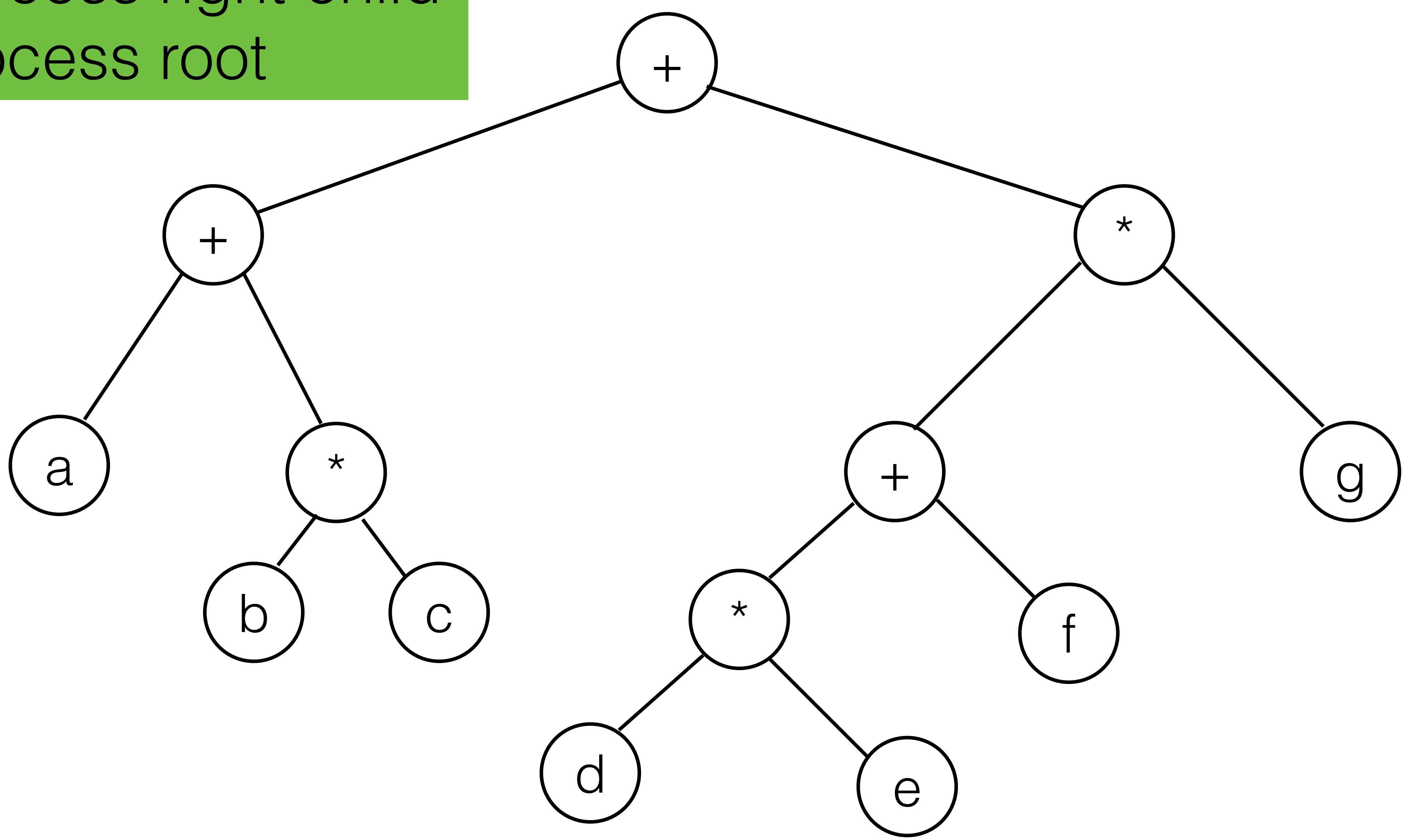
(a + b * c) + (d * e + f) * g



Tree Traversals: Post-order

1. Process left child
2. Process right child
3. Process root

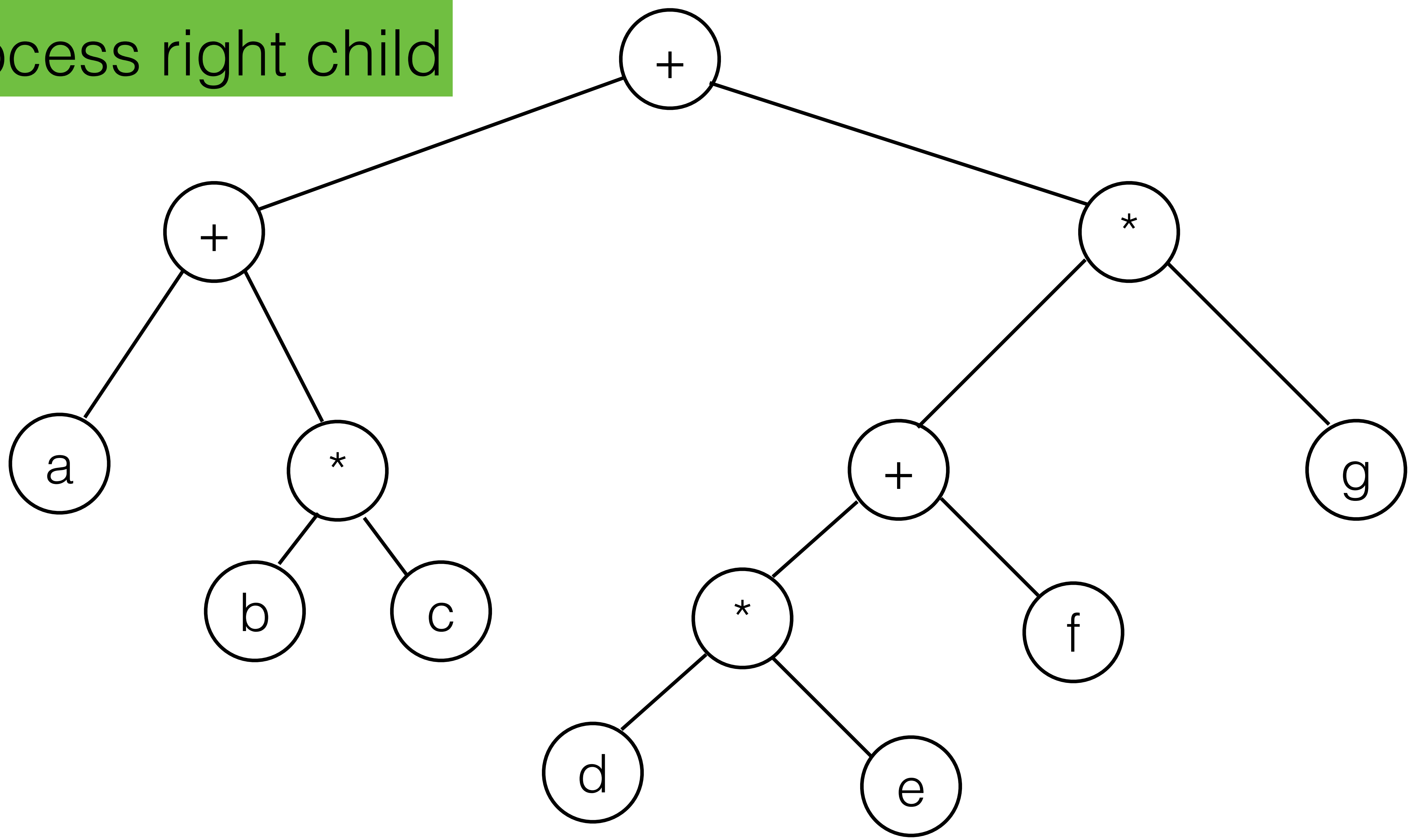
a b c * + d e * f + g * +



Tree Traversals: Pre-order

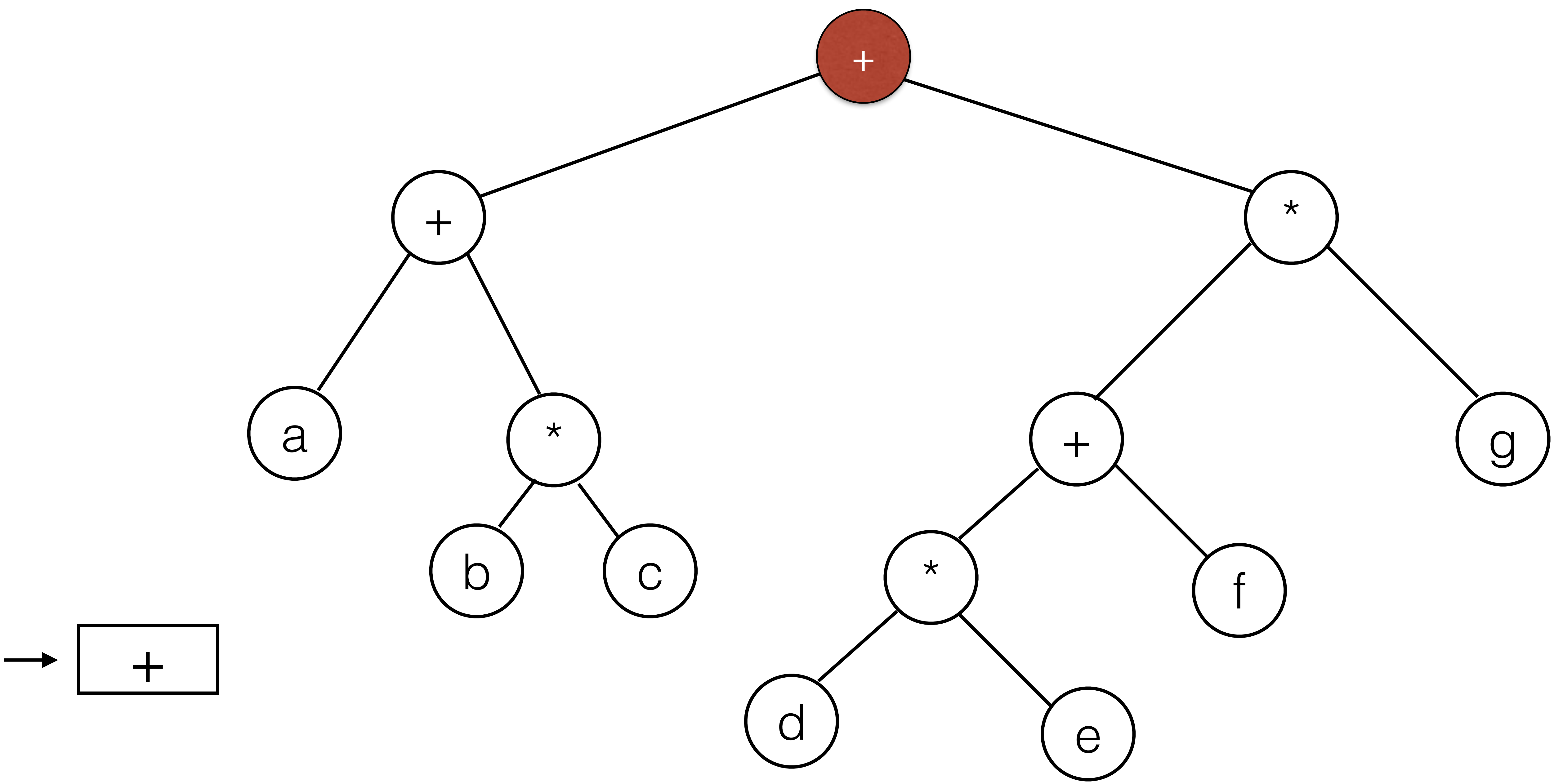
1. Process root
2. Process left child
3. Process right child

+ + a * b c * + * d e f g



Tree Traversals and Stacks

- Keep nodes that still need to be processed on a stack.



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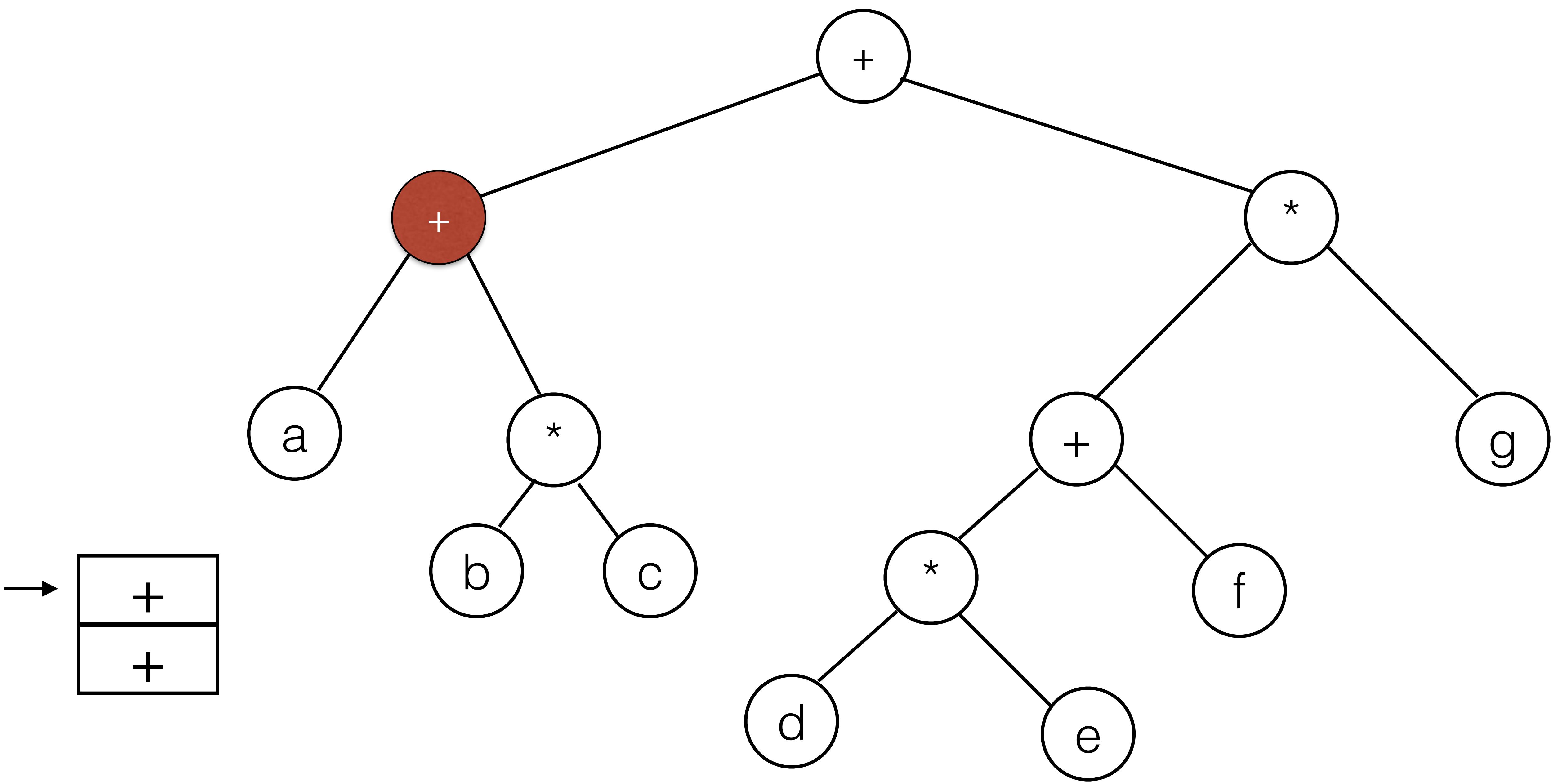

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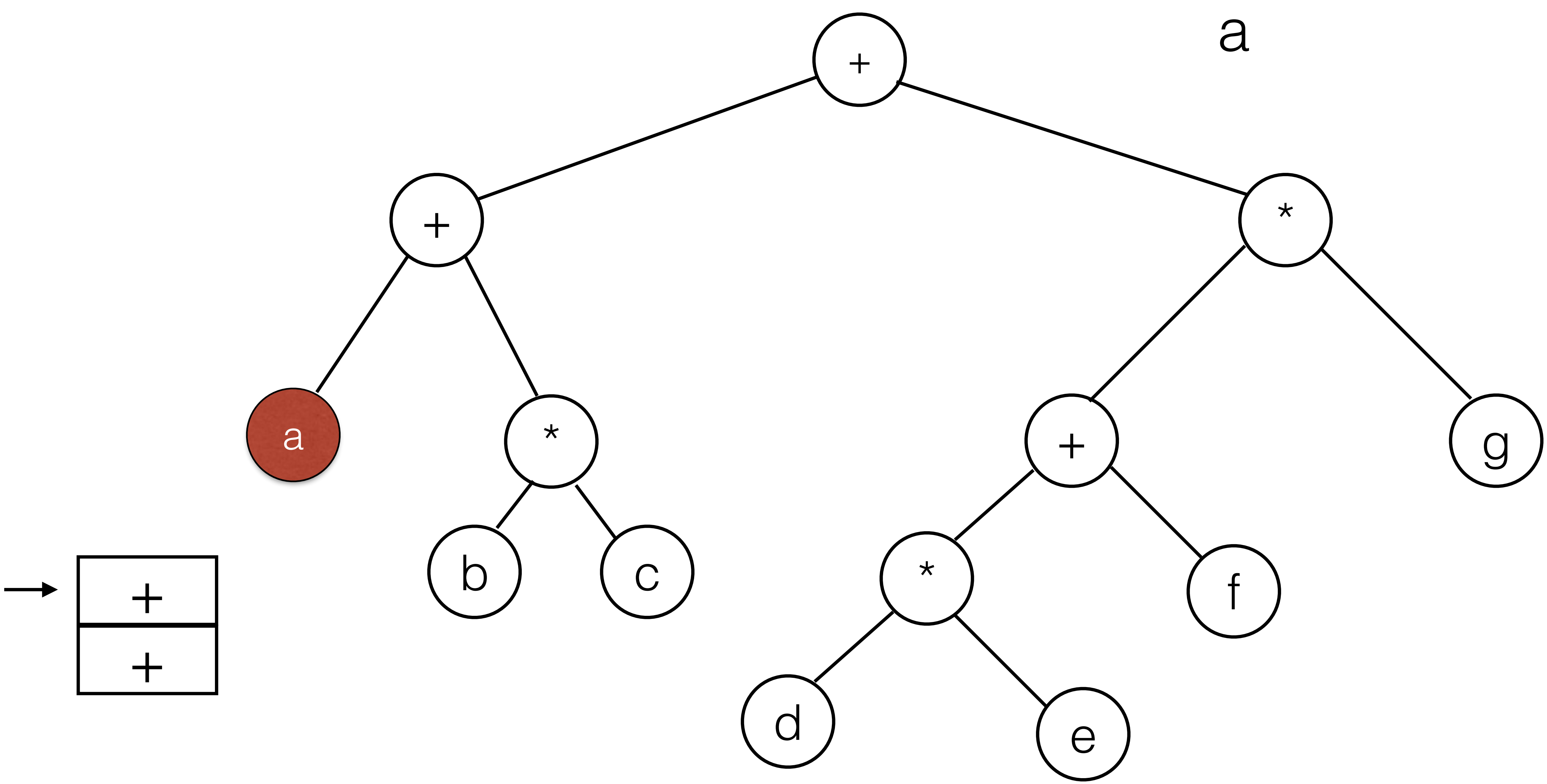

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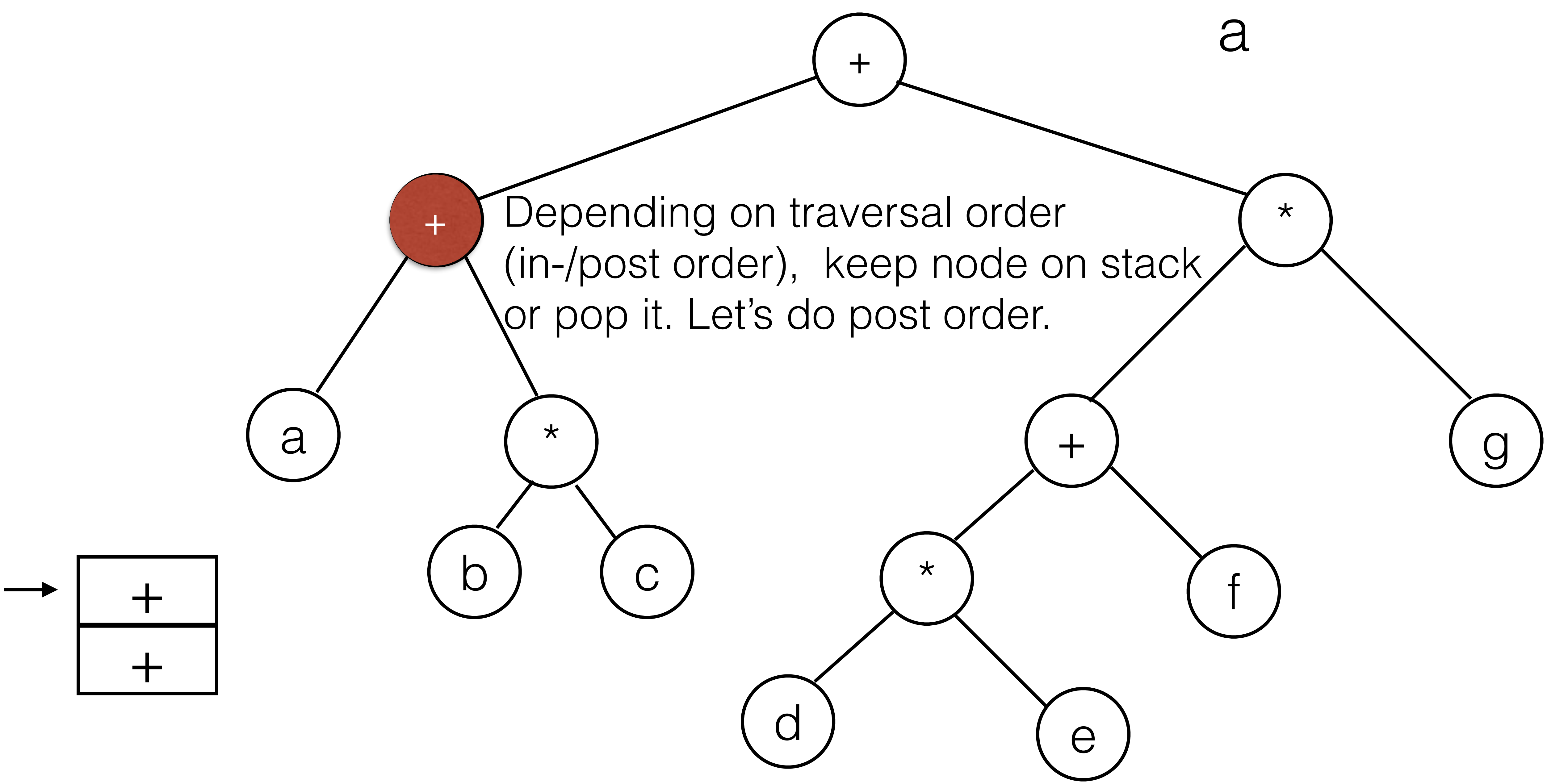

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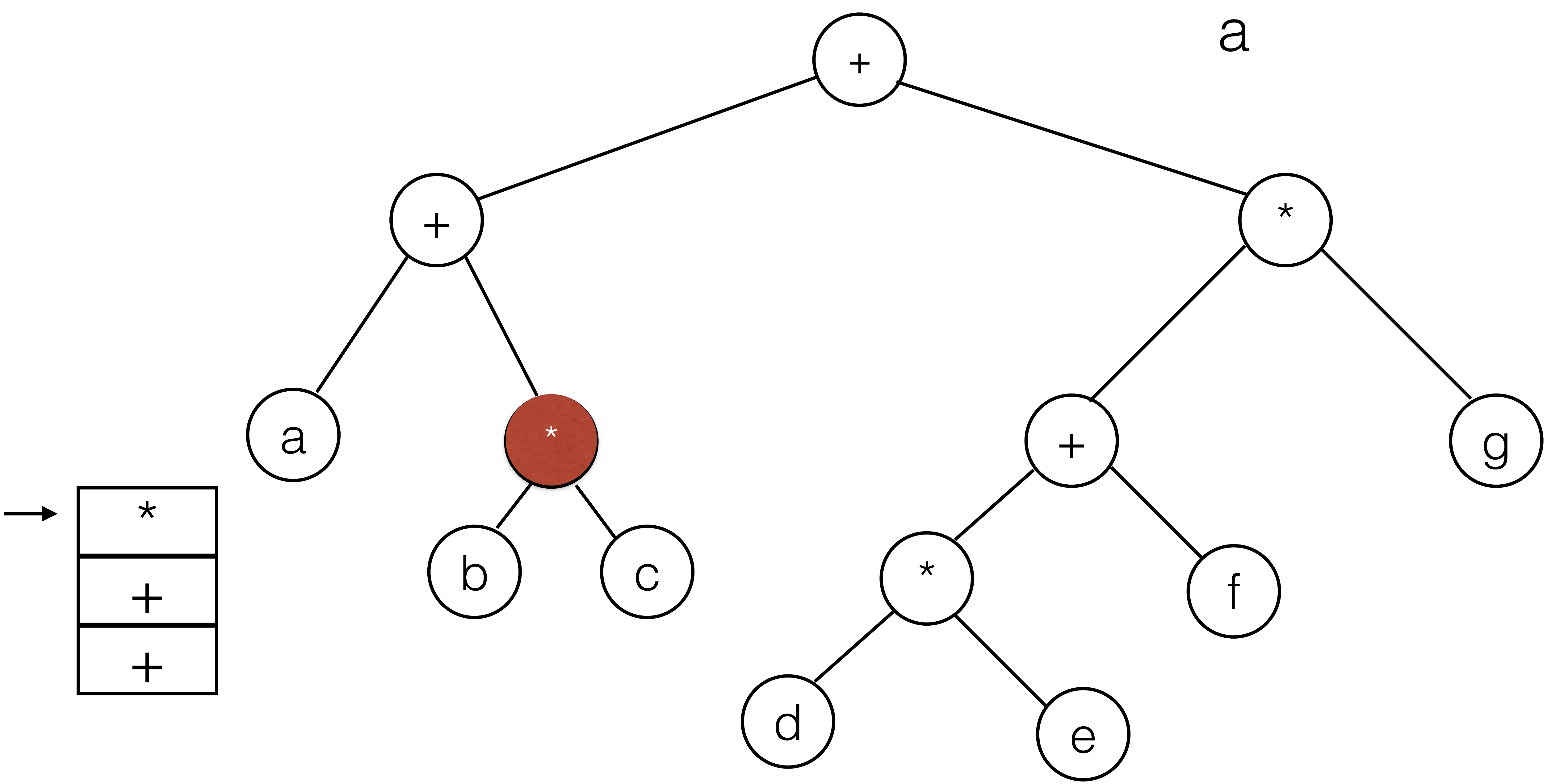

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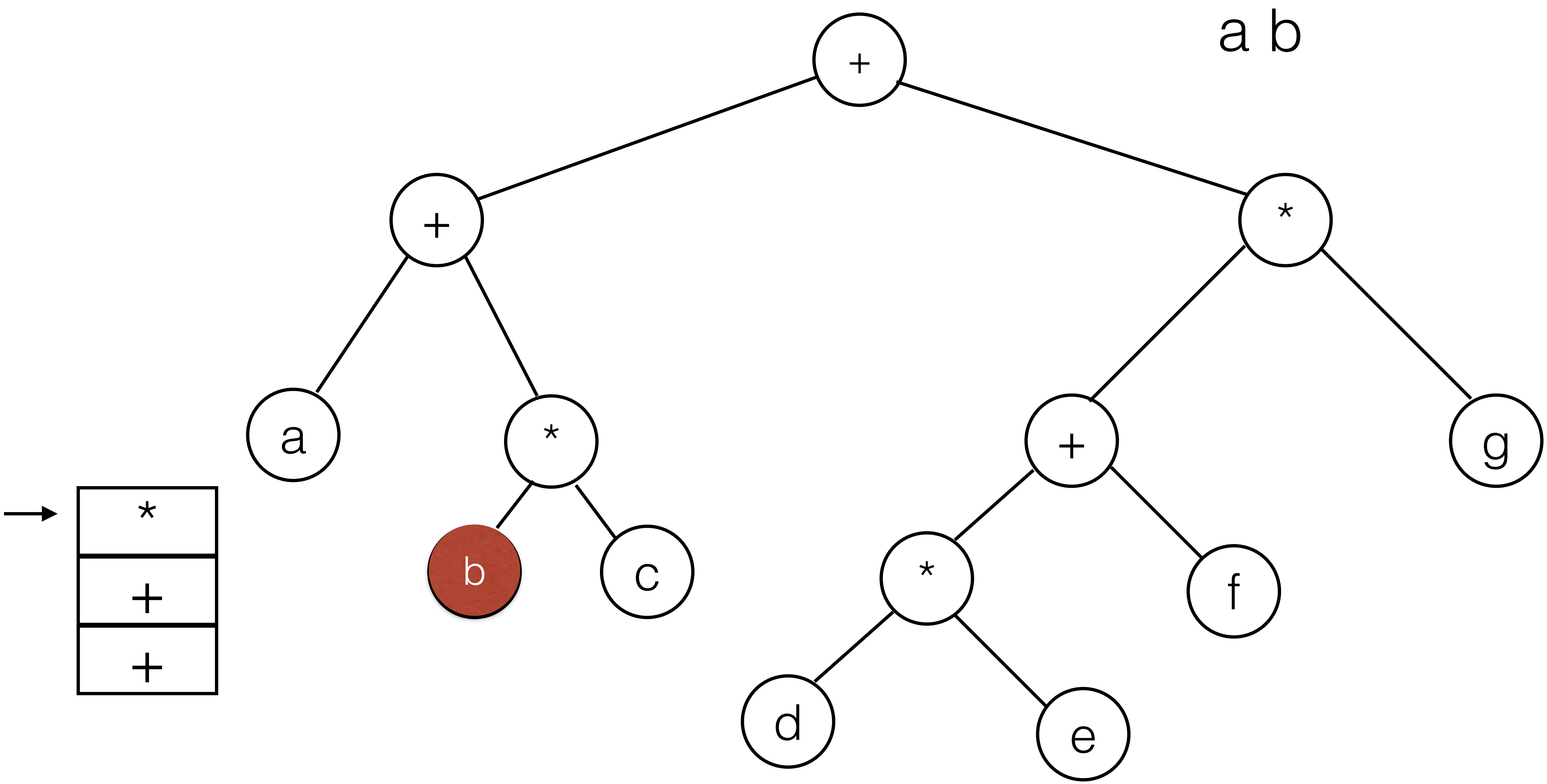

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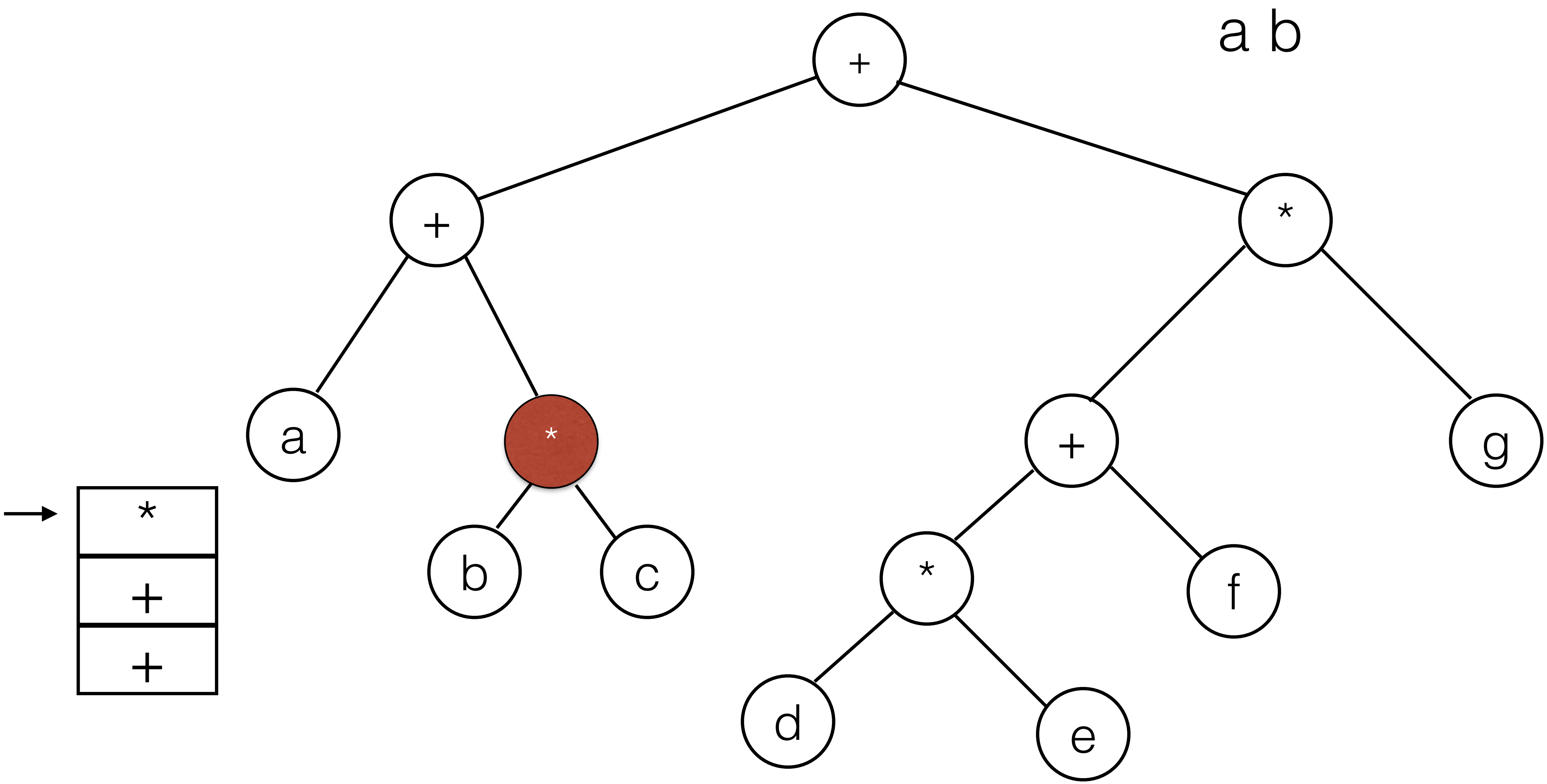

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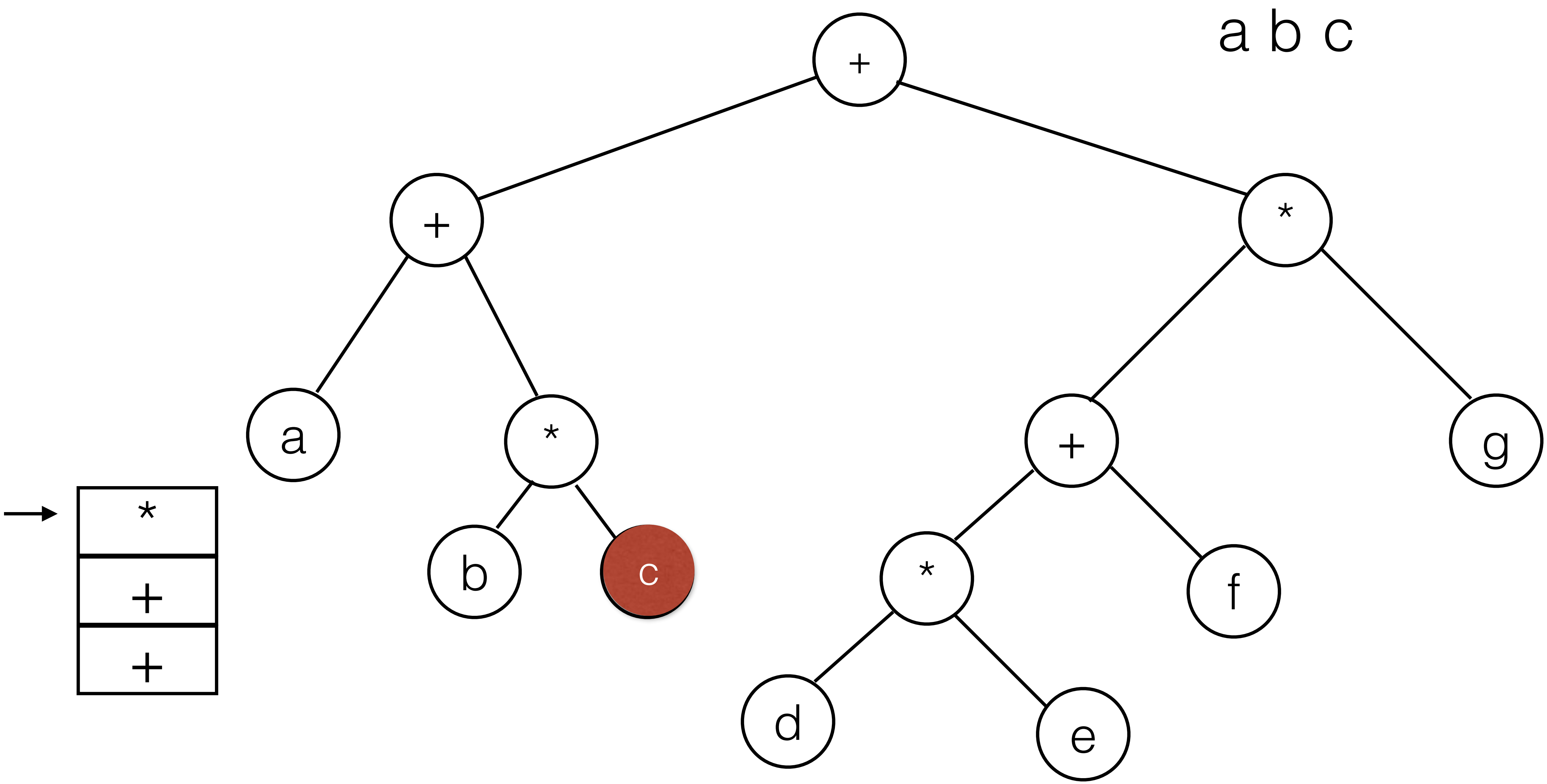

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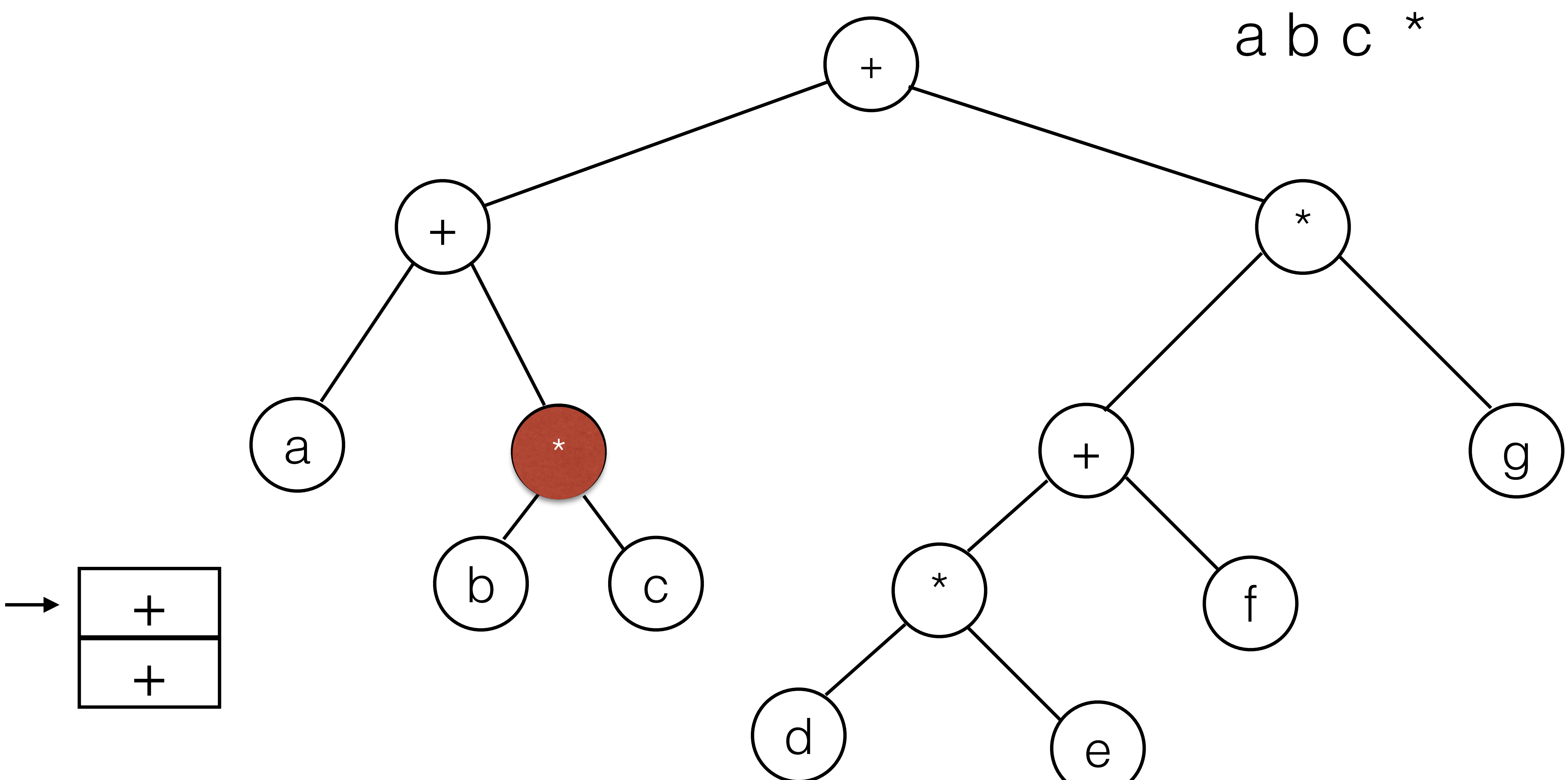

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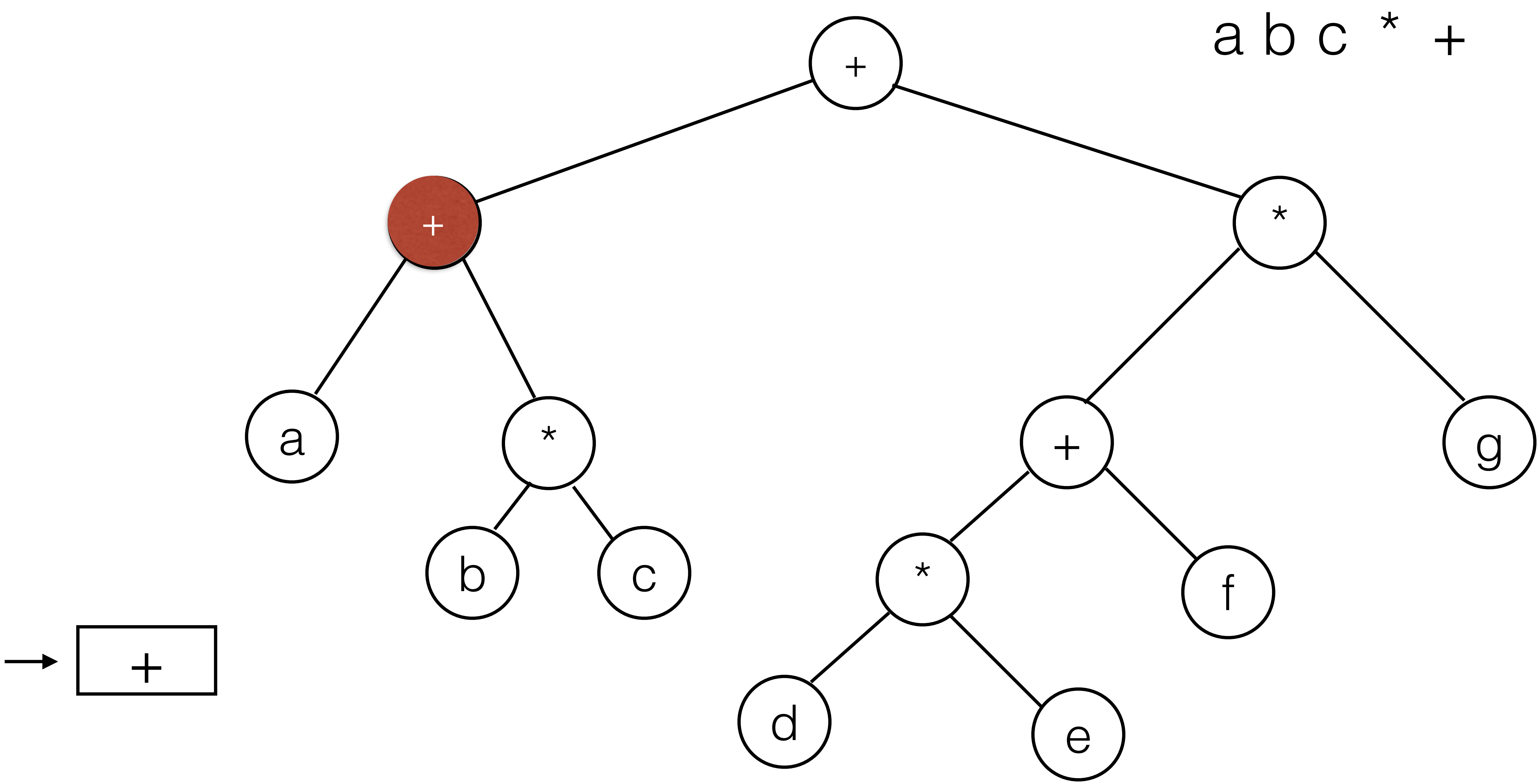


Tree Traversal using Recursion

- We often use recursion to traverse trees (making use of Java's method call stack implicitly).

```
public void printTree(int indent ) {
    for (i=0;i<indent;i++)
        System.out.print(" ");

    System.out.println( data);          // Node
    if( left != null )
        left.printTree(indent + 1);   // Left
    if( right != null )
        right.printTree(indent + 1); // Right
}
```

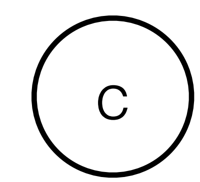
Bare-bones Implementation of a Binary Tree

- Public methods in BinaryTree usually call recursive methods, implementation either in BinaryNode or in BinaryTree.
- (sample code)

Constructing Expression Trees using a Stack

5 27 2 3 * / +

- for c in input
  - if c is an operand, push a tree



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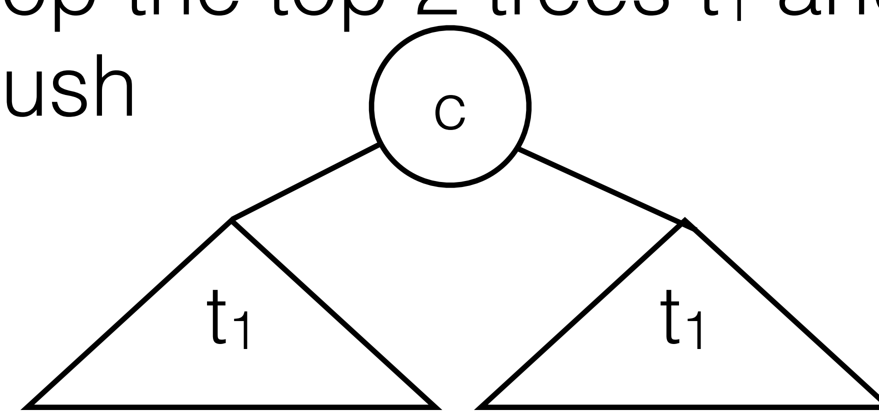
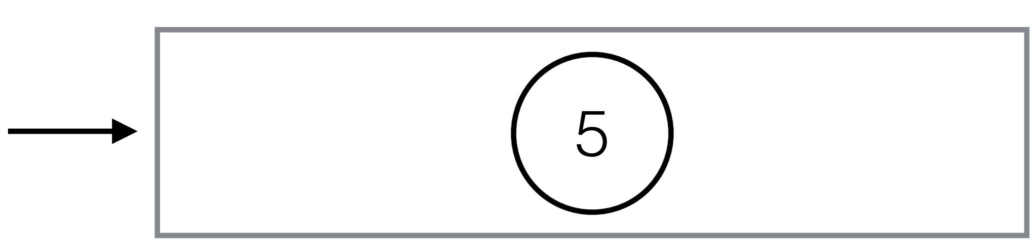


- pop the result.



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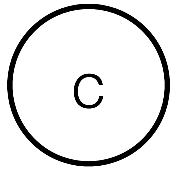

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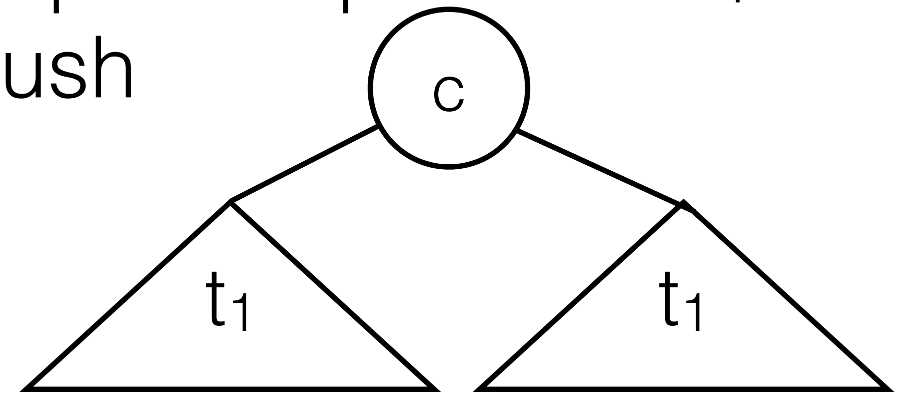

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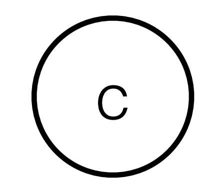

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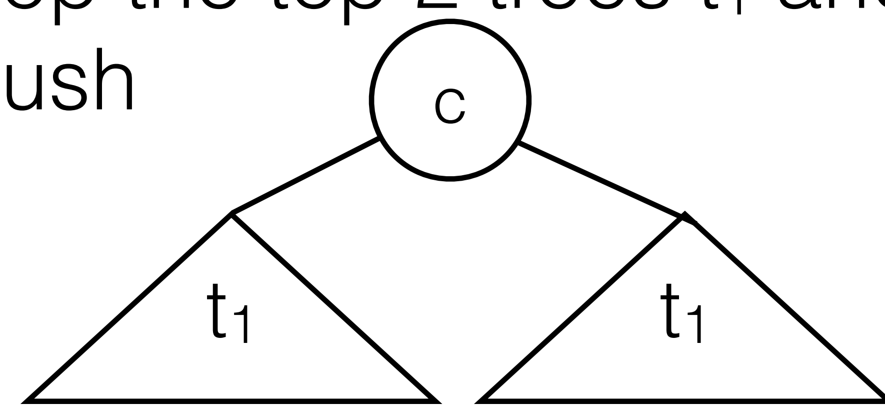

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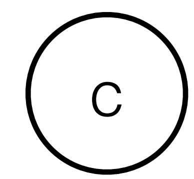

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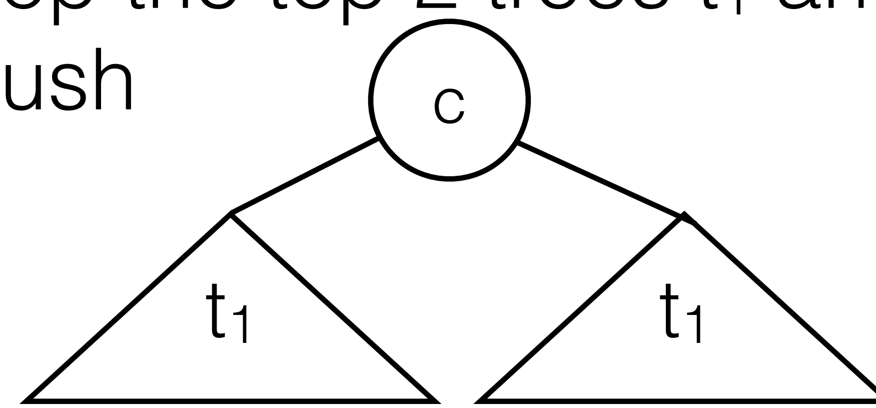

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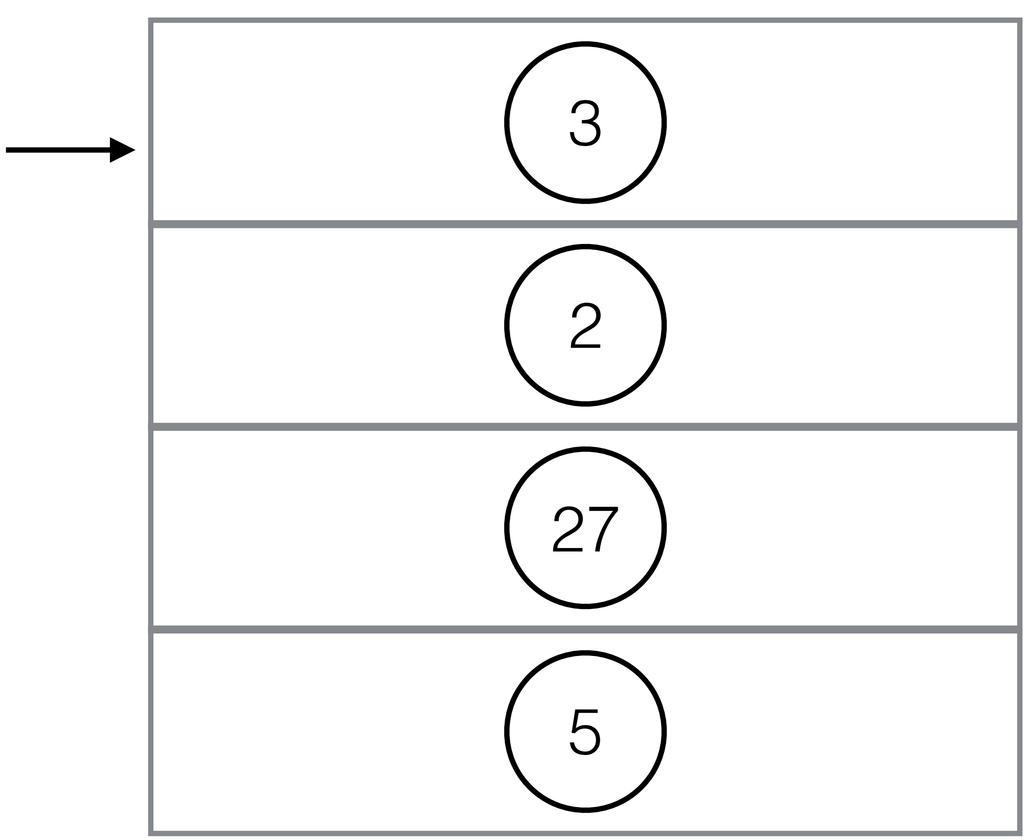

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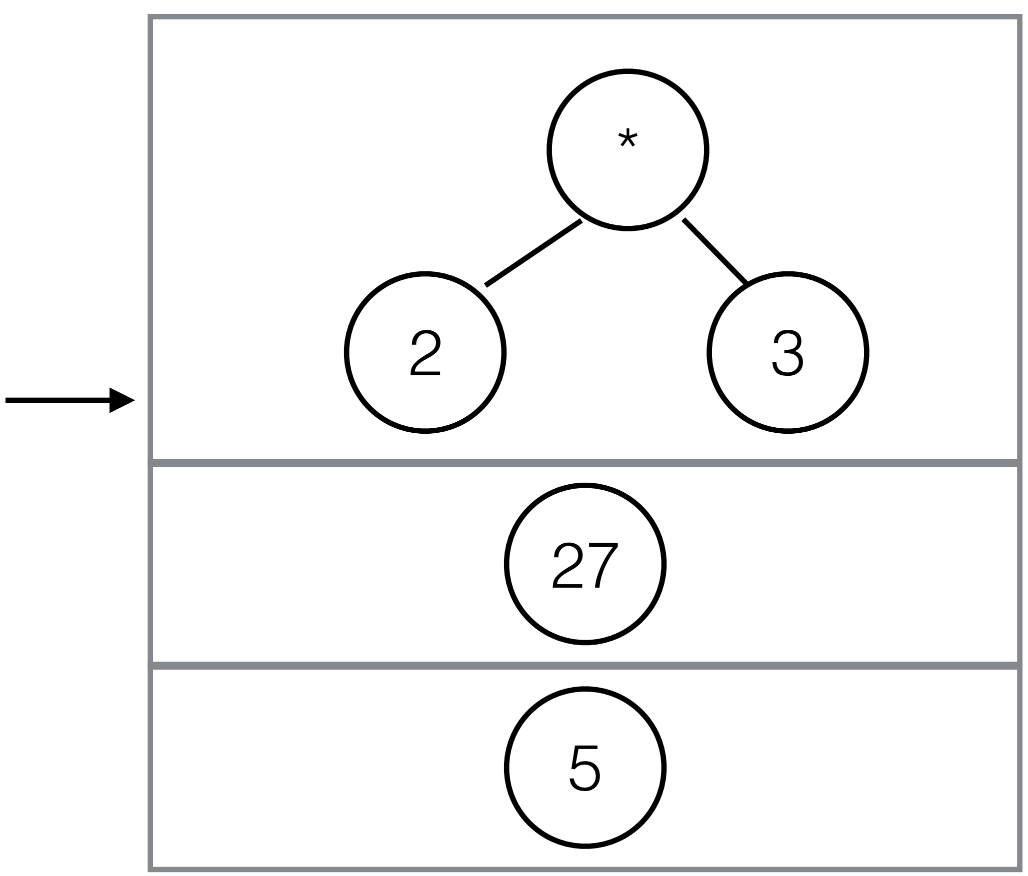

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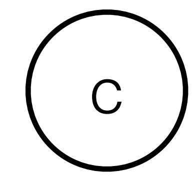

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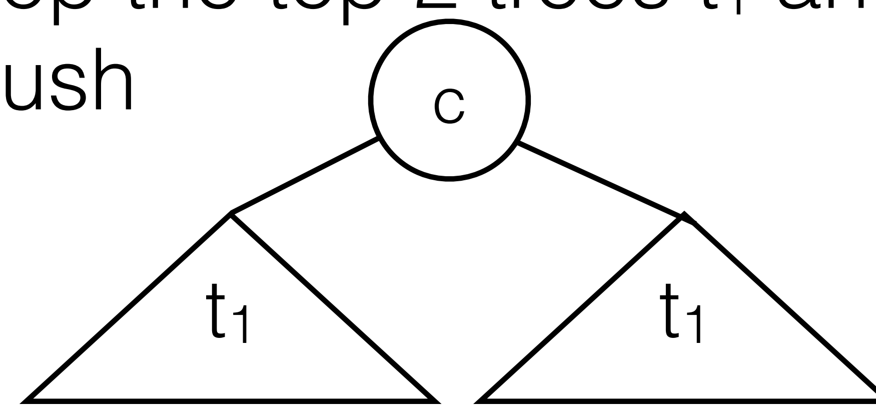

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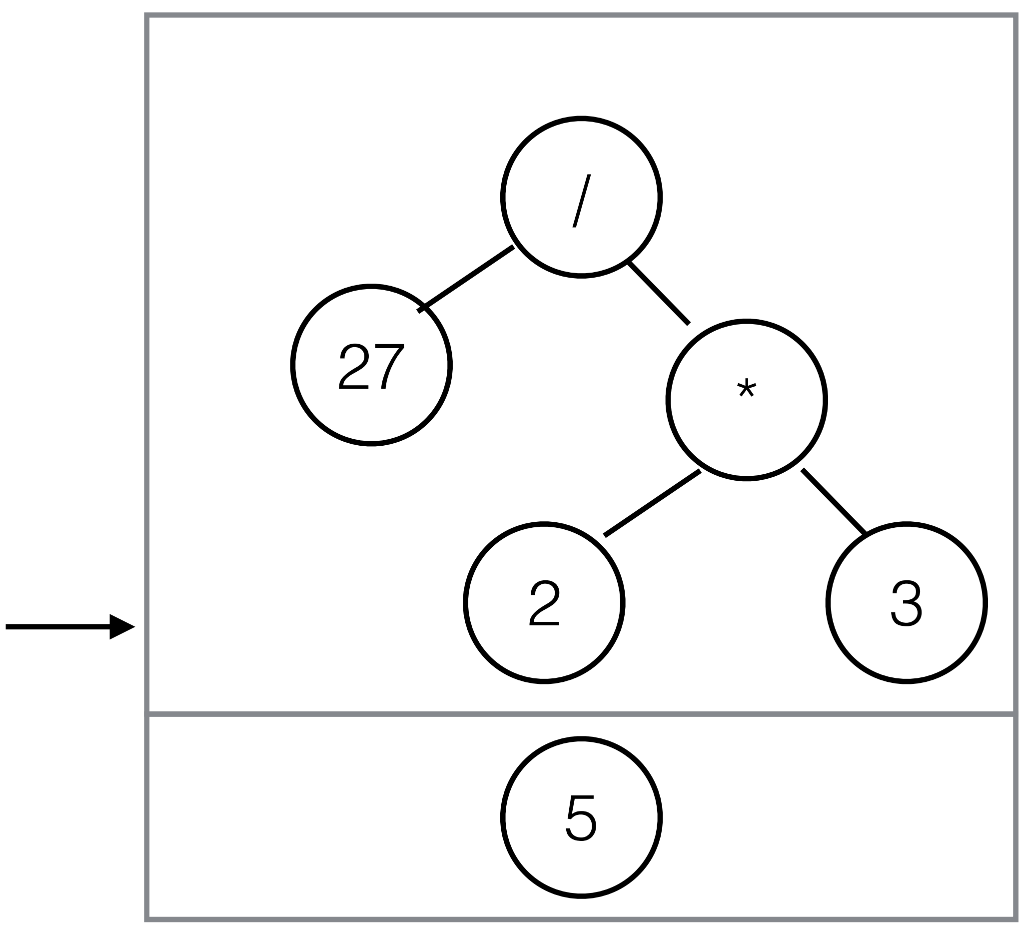

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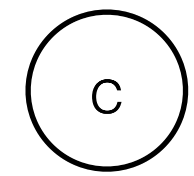

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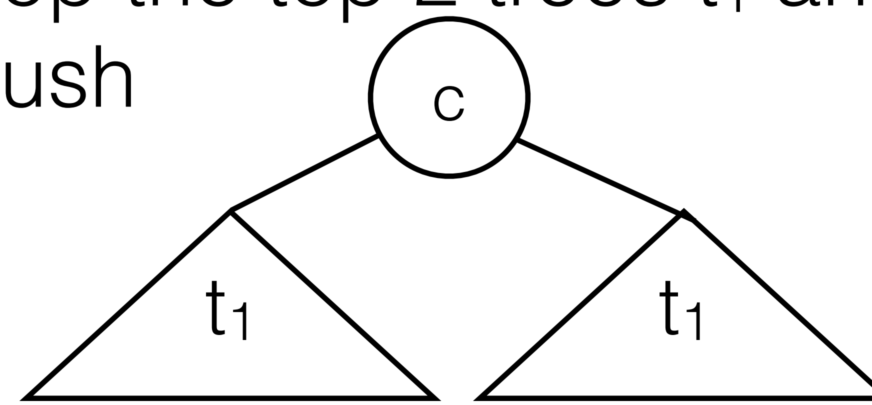

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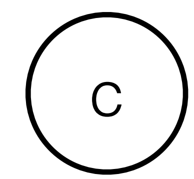

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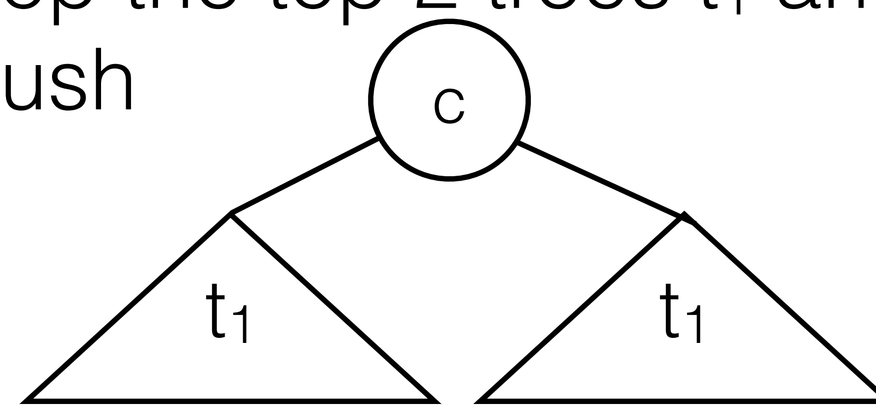

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