Data Structures in Java

Lecture 6: Stacks.

9/28/2015

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Homework

- Thank you for submitting homework 1!
- Homework 2 out tonight.

Reminder: Recitation Session tonight

- Thursday session permanently moved to Monday.
- 7:35 - Schermerhorn 614
- This week: Homework 1 review.

The Stack ADT

- A Stack *S* is a sequence of *N* objects $A_0, A_1, A_2, \ldots, A_{N-1}$ with three operations:
  - **`void push(x)`** - append element *x* to the end (on “top”) of S.
  - **`Object top() / peek()`** = returns the last element of S.
  - **`Object pop()`** - remove and return the last element from S.
- Stacks are also known as **L**ast **I**n **F**irst **O**ut (LIFO) storage.

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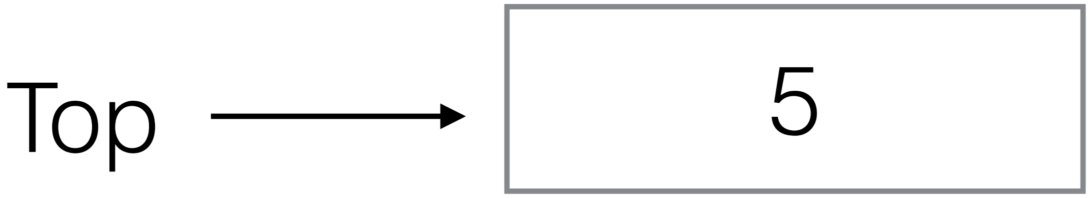
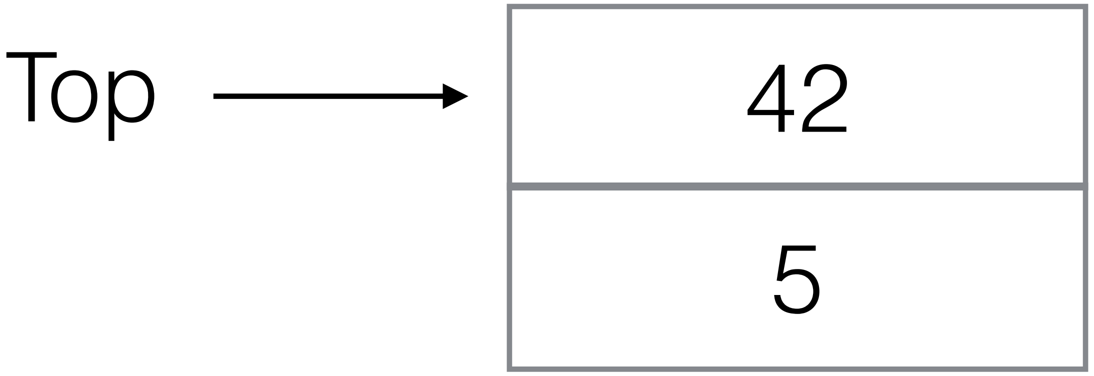
Stack Example



Stack Example

`push(42)`



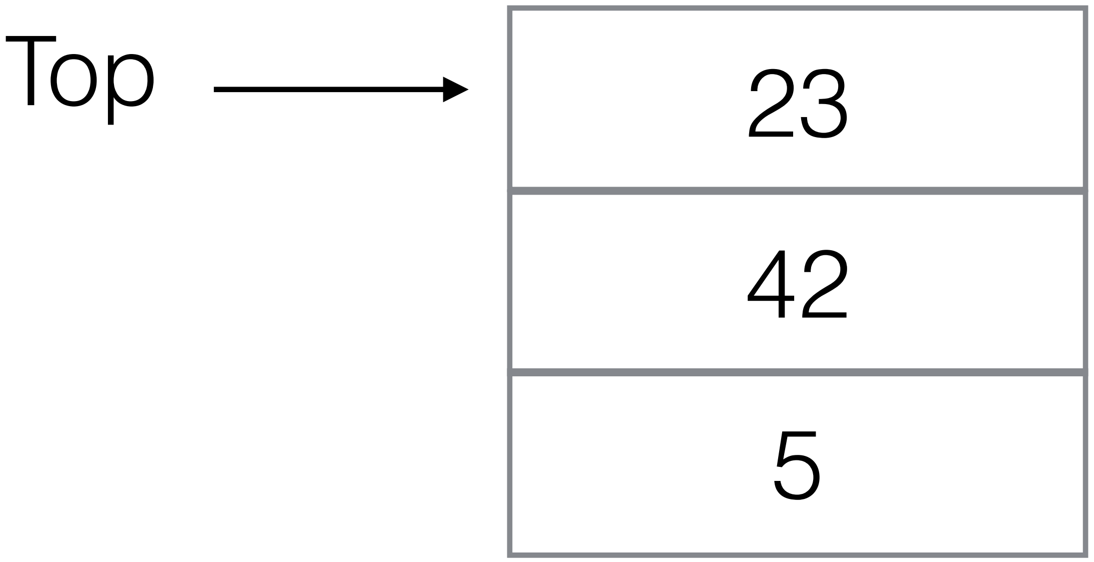
Top

42

5

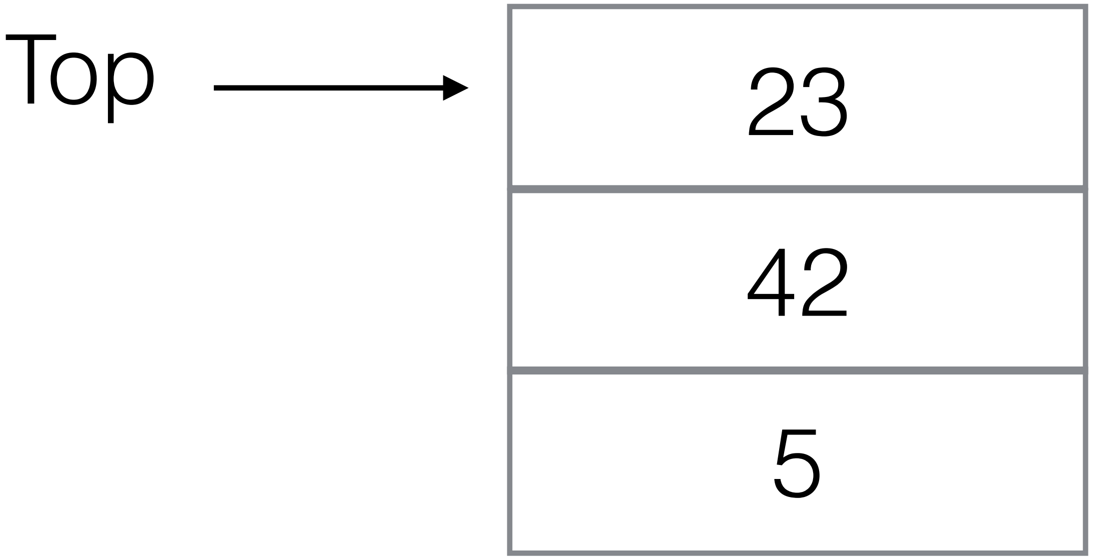
Stack Example

`push(42)` `push(23)`



Stack Example

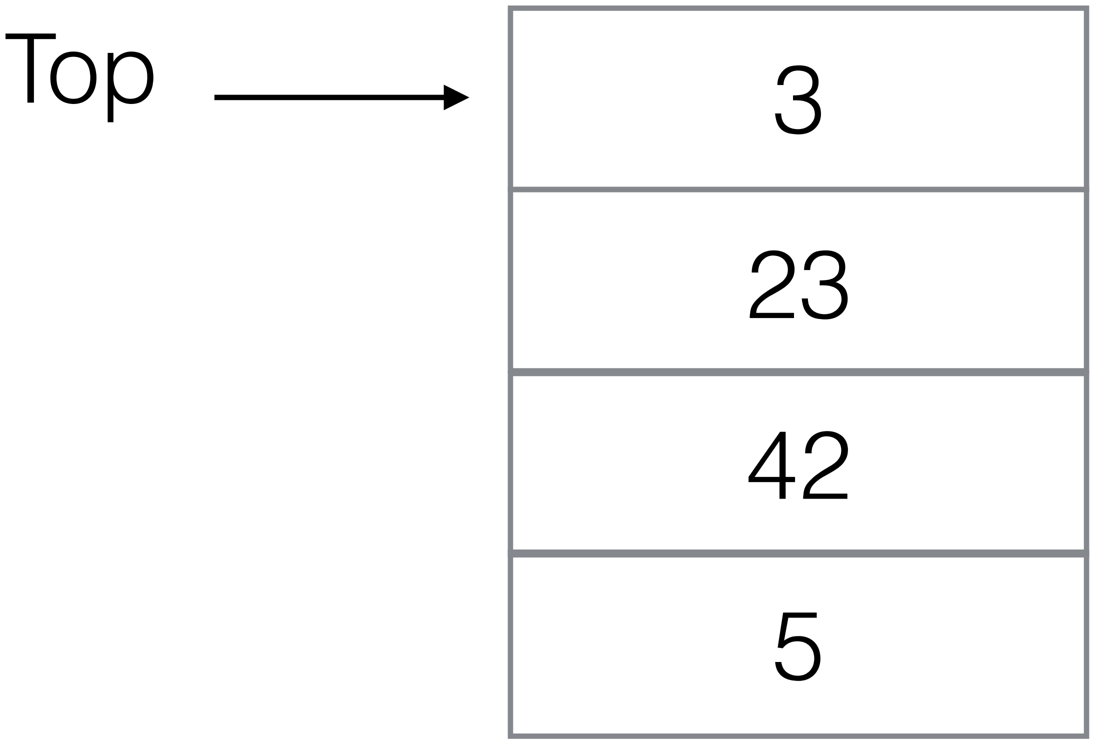
`push(42)` `push(23)`



`top()` → 23

Stack Example

`push(42)` `push(23)` `push(3)`



Top

3

23

42

5

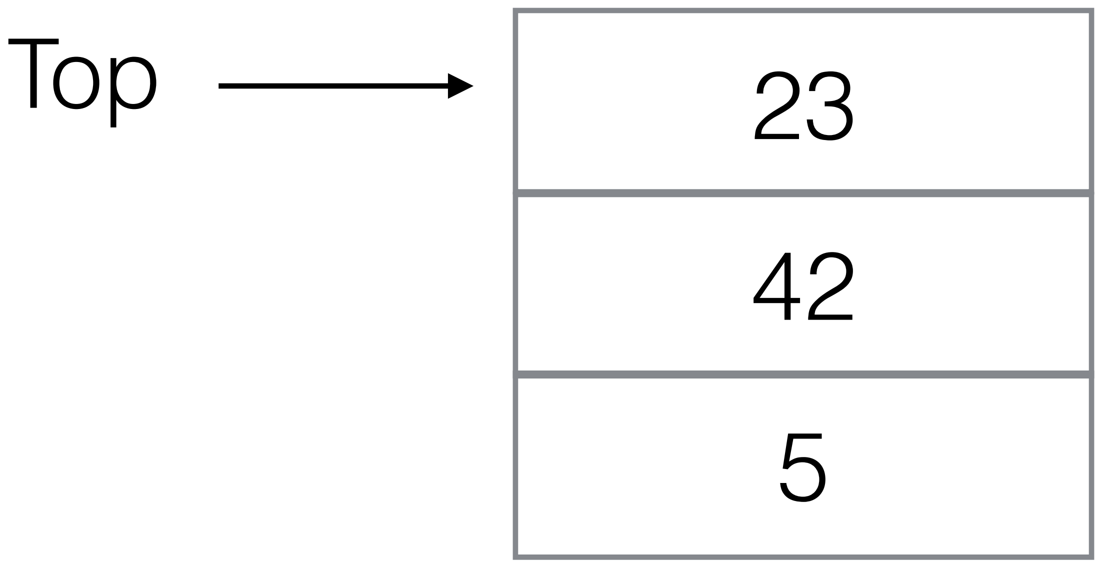
`top()` → 23

Stack Example

`push(42)` `push(23)` `push(3)`

`pop()` → 3

`top()` → 23


Top →

23

42

5

Implementing Stacks

- Think of a Stack as a specialized List:
  - push: Inserts only allowed at the end of the list.
  - pop: Remove only allowed at the end of the list.
- Can implement Stack using any List implementation.

Implementing Stacks

- Think of a Stack as a specialized List:
  - push: Inserts only allowed at the end of the list.
  - pop: Remove only allowed at the end of the list.
- Can implement Stack using any List implementation.
- push and pop run in O(1) time with ArrayList or LinkedList.

A Stack Interface

```
interface Stack<T> {
    /* Push a new item x on top of the stack */
    public void push(T x);
    /* Remove and return the top item of the stack */
    public T pop();
    /* Return the top item of the stack without removing it */
    public T top();
}
```

Using MyLinkedList to implement Stack

```
public class LinkedListStack<T> extends MyLinkedList<T>
                               implements Stack<T> {

    public void push(T x) {
        add(size(), x);
    }

    public T pop() {
        return remove(size()-1);
    }

    public T top() {
        return get(size()-1);
    }
}
```

Direct Implementation Using an Array

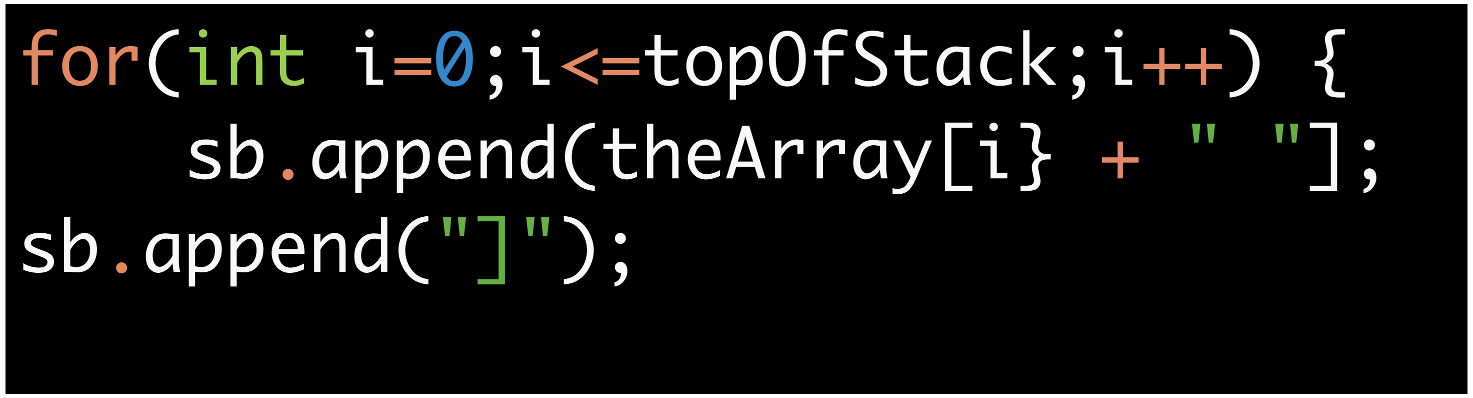
(sample code)

Application: Balancing Symbols

- Compilers need to check for syntax errors.

- Need to make sure braces, brackets, parentheses are well nested.

- What’s wrong with this code:



```
for(int i=0;i<=topOfStack;i++) {
    sb.append(theArray[i} + " "];
sb.append("]");
```

Balancing Symbols

```
for(int i=0;i<=topOfStack;i++) {
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push(“(“)

Balancing Symbols

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push(“(“) pop(“(“)

Balancing Symbols

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for(int i=0;i<=topOfStack;i++) {
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push(“(“) pop(“(“) push(“{“)

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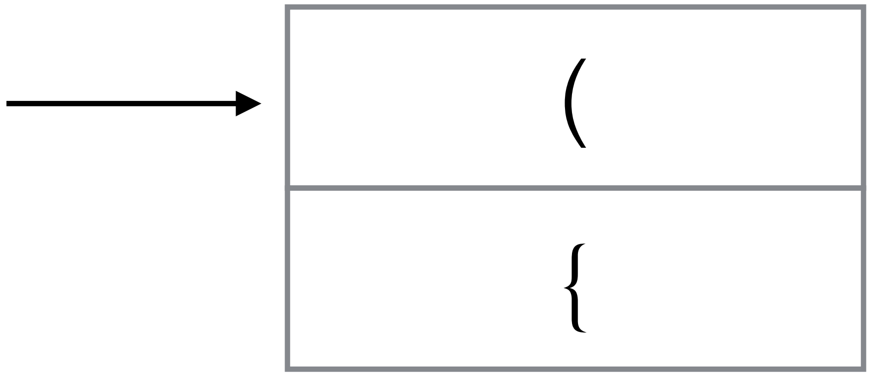

Balancing Symbols

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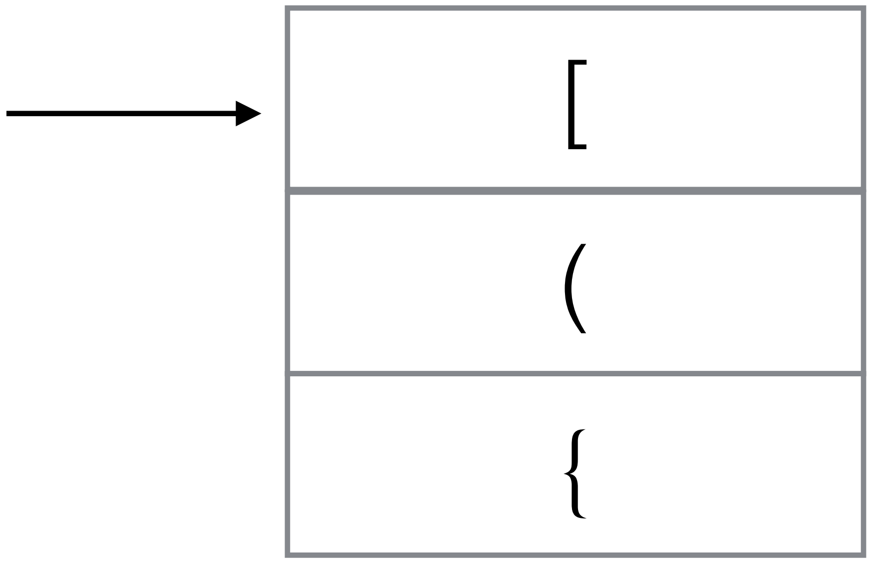

Balancing Symbols

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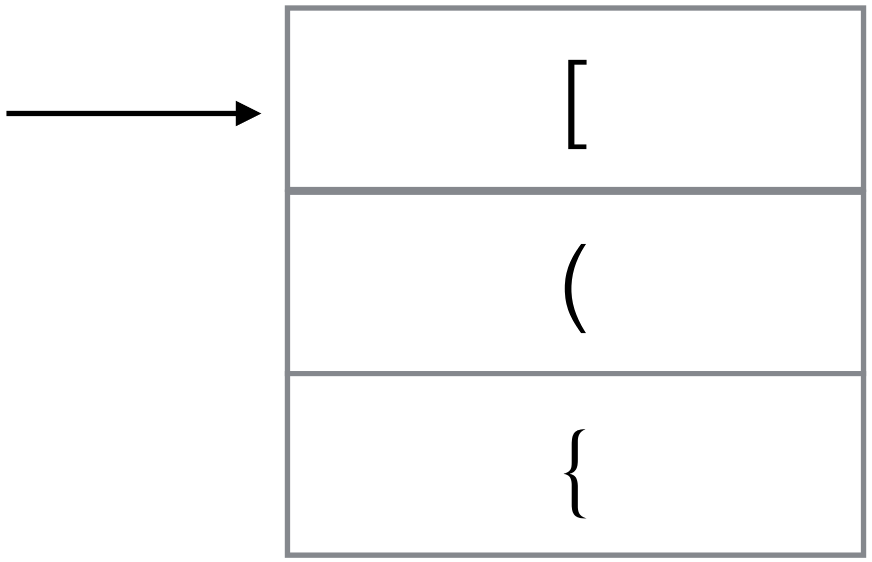

Postfix Expressions

- How would you do the following calculation using a **simple** calculator:

5 + 27 / (2 * 3)

remember
intermediate
results

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5 + 4.5 = 9.5

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2 * 3 = 6

27 / 6 = 4.5

5 + 4.5 = 9.5

5 27 2 3 * / +

Evaluating Postfix Expressions

5 + 27 / (2 * 3)

5 27 2 3 * / +

- for c in input
  - if c is an operand, push it
  - if c is an operator x:
    - pop the top 2 operands a_1 and a_2
    - push $a_3 = a_2 \text{ x } a_1$
- pop the result.

Evaluating Postfix Expressions

5 + 27 / (2 * 3)

$\boxed{5}$ 27 2 3 * / +

push(5)

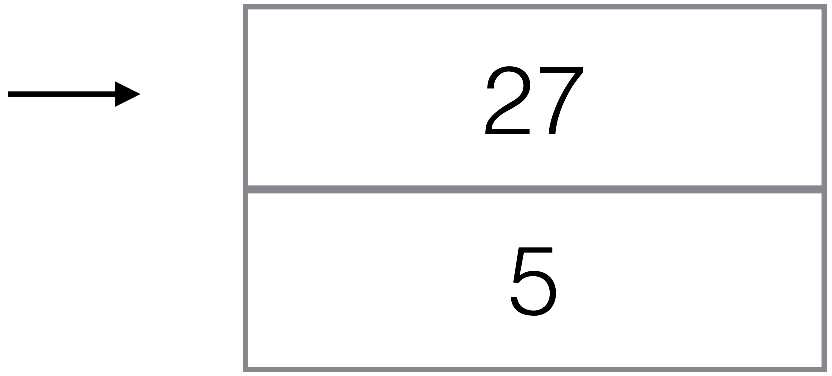
- for c in input
  - if c is an operand, push it
  - if c is an operator x:
    - pop the top 2 operands a_1 and a_2
    - push $a_3 = a_2 \text{ x } a_1$
- pop the result.

Evaluating Postfix Expressions

5 + 27 / (2 * 3)

5 27 2 3 * / +

push(27)



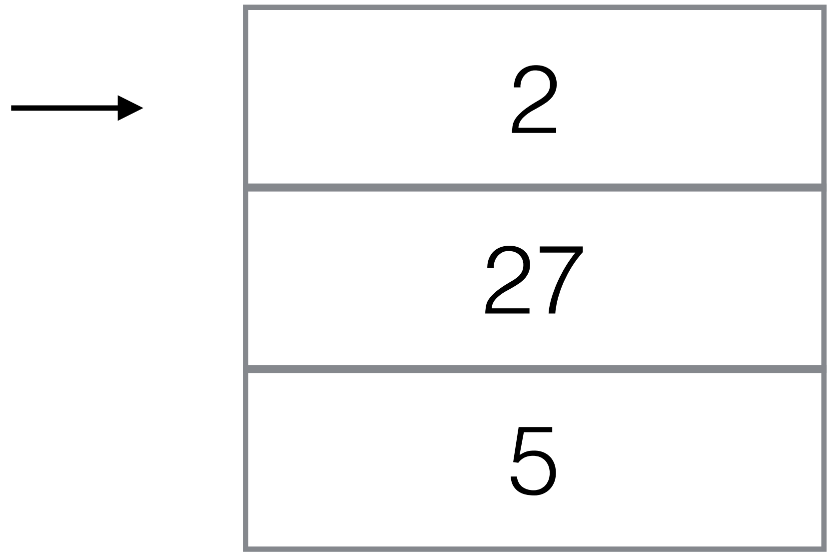
- for c in input
  - if c is an operand, push it
  - if c is an operator x:
    - pop the top 2 operands a_1 and a_2
    - push $a_3 = a_2$ x a_1
- pop the result.

Evaluating Postfix Expressions

5 + 27 / (2 * 3)

5 27 $\boxed{2}$ 3 * / +

push(2)



2

27

5

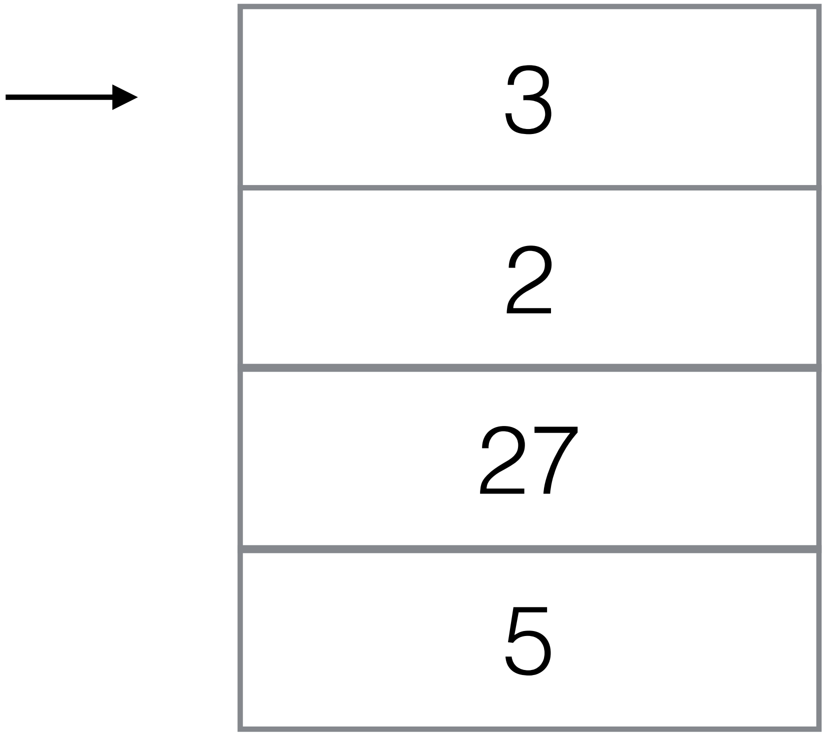
```
• for c in input
    • if c is an operand, push it
    • if c is an operator x:
        • pop the top 2 operands $a_1$ and $a_2$
        • push $a_3 = a_2$ x $a_1$
• pop the result.
```

Evaluating Postfix Expressions

5 + 27 / (2 * 3)

5 27 2 3 * / +

push(3)



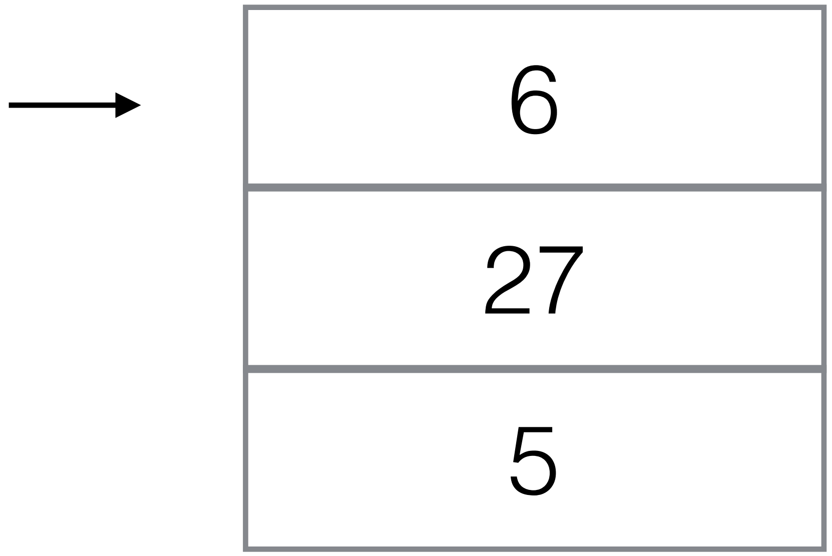
```
• for c in input
    • if c is an operand, push it
    • if c is an operator x:
        • pop the top 2 operands $a_1$ and $a_2$
        • push $a_3 = a_2 \times a_1$
• pop the result.
```

Evaluating Postfix Expressions

5 + 27 / (2 * 3)

5 27 2 3 $\boxed{*}$ / +

pop() -> 3
pop() -> 2
push(2*3)



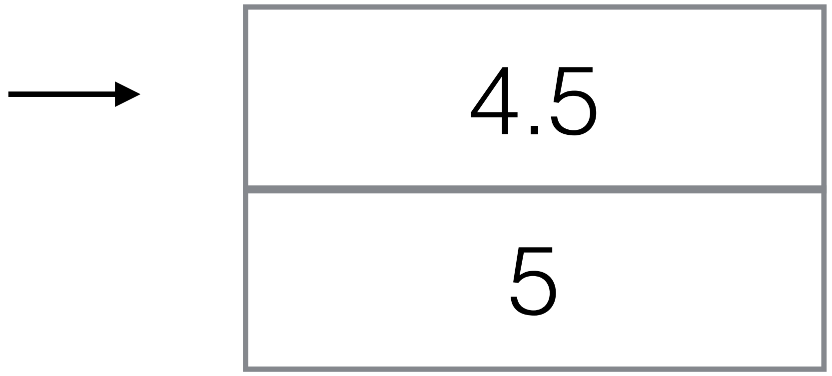
- for c in input
  - if c is an operand, push it
  - if c is an operator x:
    - pop the top 2 operands a_1 and a_2
    - push $a_3 = a_2 \times a_1$
- pop the result.

Evaluating Postfix Expressions

5 + 27 / (2 * 3)

pop() -> 6
pop() -> 27
push(27/6)

5 27 2 3 * / +



- for c in input
    - if c is an operand, push it
    - if c is an operator x:
        - pop the top 2 operands a_1 and a_2
        - push $a_3 = a_2 \times a_1$
- pop the result.

Evaluating Postfix Expressions

5 + 27 / (2 * 3)

pop() -> 4.5
pop() -> 5
push(5 + 4.5)

5 27 2 3 * / +

- for c in input
  - if c is an operand, push it
  - if c is an operator x:
    - pop the top 2 operands a_1 and a_2
    - push $a_3 = a_2 \text{ x } a_1$
- pop the result.

Converting Infix to Postfix Notation

Input : a + b * c + (d * e + f) * g

Output :

Converting Infix to Postfix Notation

Input : a + b * c + (d * e + f) * g

Output : a b c * + d e * f + g * +

Converting Infix to Postfix Notation

Idea: keep lower-precedence operators on the stack.

Input: a + b * c + d

Output:

Order of Precedence:
+ = 1
* = 2

Converting Infix to Postfix Notation

Idea: keep lower-precedence operators on the stack.

Input: a + b * c + d

Output: a

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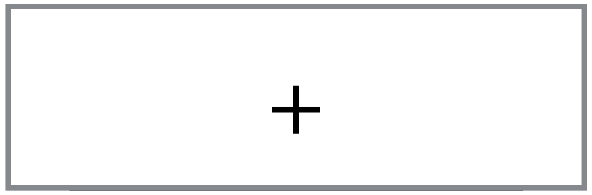

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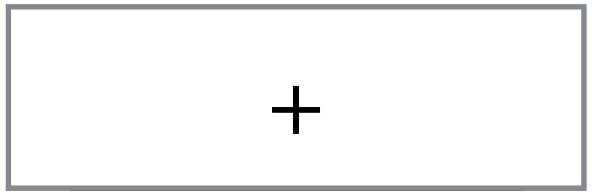
Converting Infix to Postfix Notation

Idea: keep lower-precedence operators on the stack.

Input: a + b * c + d

Output: a b



+

Order of Precedence:

+ = 1

* = 2

Converting Infix to Postfix Notation

Idea: keep lower-precedence operators on the stack.

Input: a + b * c + d

Output: a b

*

+

Order of Precedence:
+ = 1
* = 2

* has higher priority than +,
so we want * in the output first. Keep pushing.

Converting Infix to Postfix Notation

Idea: keep lower-precedence operators on the stack.

Input: a + b * c + d

Output: a b c

Order of Precedence:
+ = 1
* = 2

Converting Infix to Postfix Notation

Idea: keep lower-precedence operators on the stack.

Input: a + b * c + d

Output: a b c

*

+

Order of Precedence:

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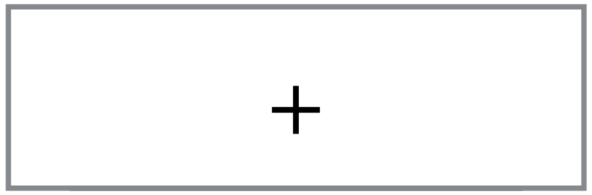
* = 2

Converting Infix to Postfix Notation

Idea: keep lower-precedence operators on the stack.

Input: a + b * c + d

Output: a b c *



Order of Precedence:

+ = 1

* = 2

+ has lower priority than *, so we need to pop * and write it to the output first.

Converting Infix to Postfix Notation

Idea: keep lower-precedence operators on the stack.

Input: a + b * c + d

Output: a b c * +

Order of Precedence:
+ = 1
* = 2

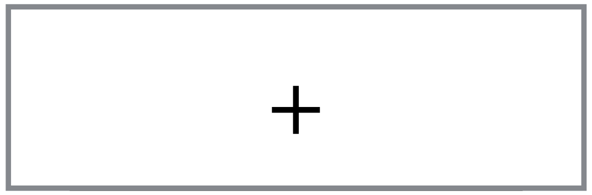
Need to pop the first + too to keep sequential order.

Converting Infix to Postfix Notation

Idea: keep lower-precedence operators on the stack.

Input: a + b * c + d

Output: a b c * +



Order of Precedence:

+ = 1

* = 2

Then push the new +

Converting Infix to Postfix Notation

Idea: keep lower-precedence operators on the stack.

Input: a + b * c + d

Output: a b c * + d

+

Order of Precedence:

+ = 1

* = 2

Then push the new +

Converting Infix to Postfix Notation

Idea: keep lower-precedence operators on the stack.

Input: a + b * c + d

Output: a b c * + d +

Order of Precedence:
+ = 1
* = 2

Pop remaining stack elements.

Converting Infix to Postfix Algorithm Sketch

```
• for c in input
    • if c is an operand: print c
    • if c is "+", "*":
        • while stack is not empty and
          priority(stack.top()) ≥ priority(c):
            • print stack.pop()
        • push c

• while stack is not empty:
      print  stack.pop()
```

Converting Infix to Postfix
Dealing with ()

Idea: Put “(“ on stack. When “)” is seen, reduce stack until matching “(“.

Input: a * (b + c) * d + e

Output: a

Order of Precedence:
+ = 1
* = 2

Converting Infix to Postfix
Dealing with ()

Idea: Put “(“ on stack. When “)” is seen, reduce stack until matching “(“.

Input: a * (b + c) * d + e

Output: a

*

Order of Precedence:

+ = 1

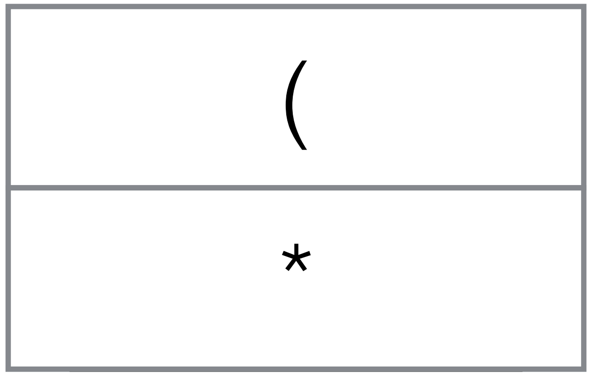
* = 2

Converting Infix to Postfix
Dealing with ()

Idea: Put "(" on stack. When ")" is seen, reduce stack until matching "(".

Input: a * (b + c) * d + e

Output: a


(
*

Order of Precedence:
+ = 1
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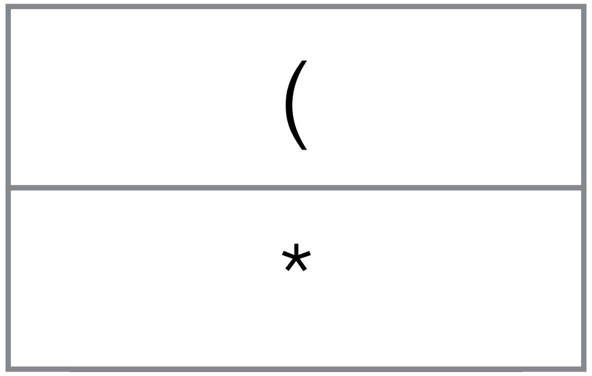
Converting Infix to Postfix
Dealing with ()

Idea: Put “(“ on stack. When “)” is seen, reduce stack until matching “(“.

Input: a * (b + c) * d + e

Output: a b



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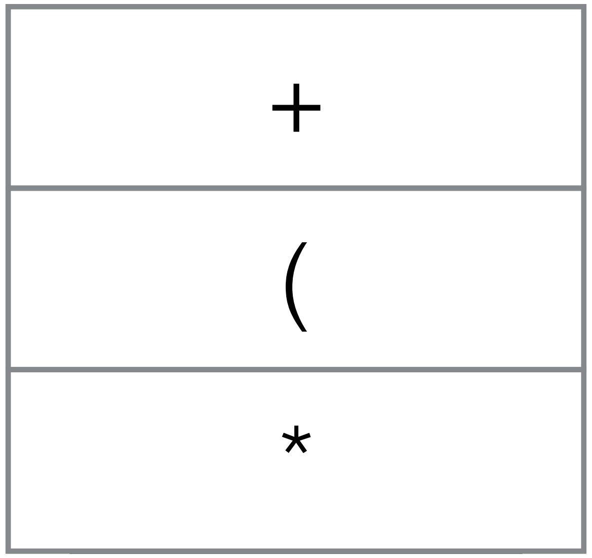

+

(

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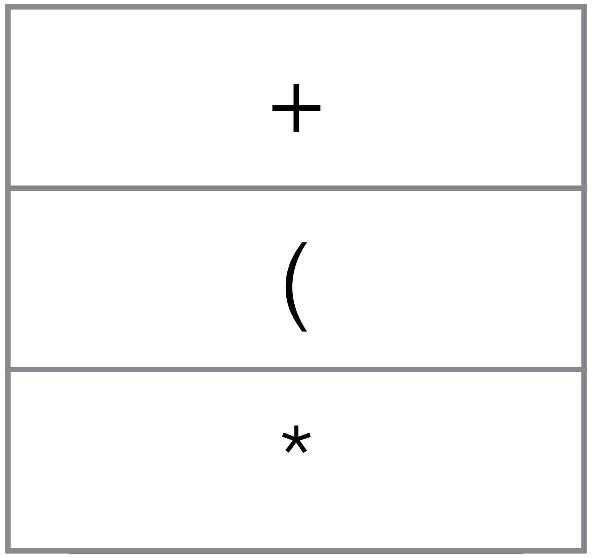
Converting Infix to Postfix
Dealing with ()

Idea: Put “(“ on stack. When “)” is seen, reduce stack until matching “(“.

Input: a * (b + c) * d + e

Output: a b c



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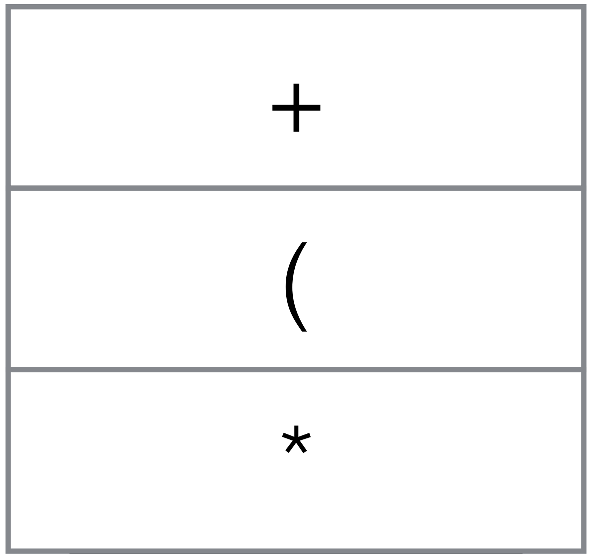
Output: a b c



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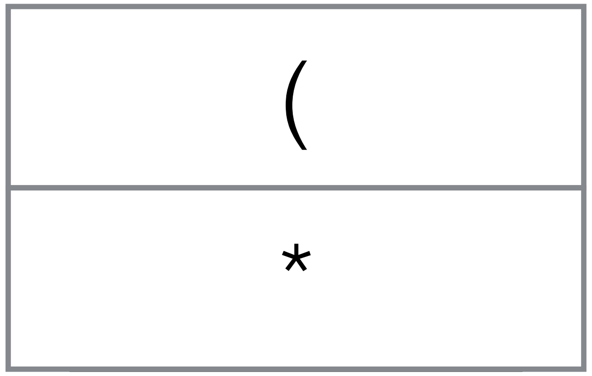
Converting Infix to Postfix
Dealing with ()

Idea: Put “(“ on stack. When “)” is seen, reduce stack until matching “(“.

Input: a * (b + c) * d + e

Output: a b c +



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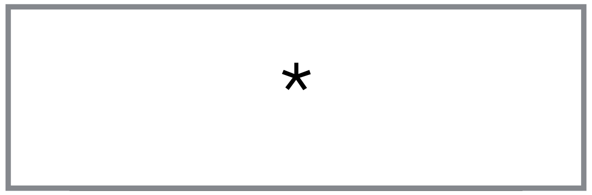

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Converting Infix to Postfix Dealing with ()

Idea: Put “(“ on stack. When “)” is seen, reduce stack until matching “(“.

Input: a * (b + c) * d + e

Output: a b c +

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Order of Precedence:
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Converting Infix to Postfix
Dealing with ()

Idea: Put “(“ on stack. When “)” is seen, reduce stack until matching “(“.

Input: a * (b + c) * d + e

Output: a b c + *

Order of Precedence:
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Idea: Put “(“ on stack. When “)” is seen, reduce stack until matching “(“.

Input: a * (b + c) * d + e

Output: a b c + *

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+ = 1
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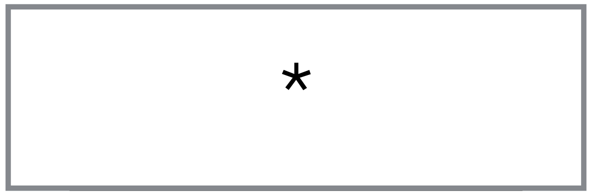
Output: a b c + * d



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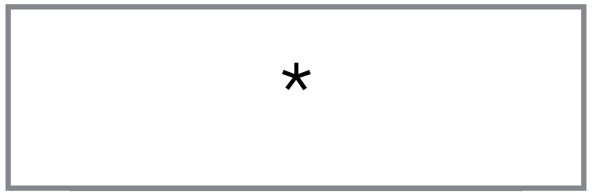
*

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Output: a b c + * d *

+

Order of Precedence:

+ = 1

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Converting Infix to Postfix
Dealing with ()

Idea: Put “(“ on stack. When “)” is seen, reduce stack until matching “(“.

Input: a * (b + c) * d + e

Output: a b c + * d * e

+

Order of Precedence:
+ = 1
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Converting Infix to Postfix

Dealing with ()

Idea: Put “(“ on stack. When “)” is seen, reduce stack until matching “(“.

Input: a * (b + c) * d + e

Output: a b c + * d * e +

Order of Precedence:
+ = 1
* = 2

Stacks in Hardware

- Stack as a memory abstraction:
  - CPU implement a hardware stack (use register to point to “top” location in main memory).
  - CPU operations push, pop will write/get value and increase or decrease register with a single byte code instruction.

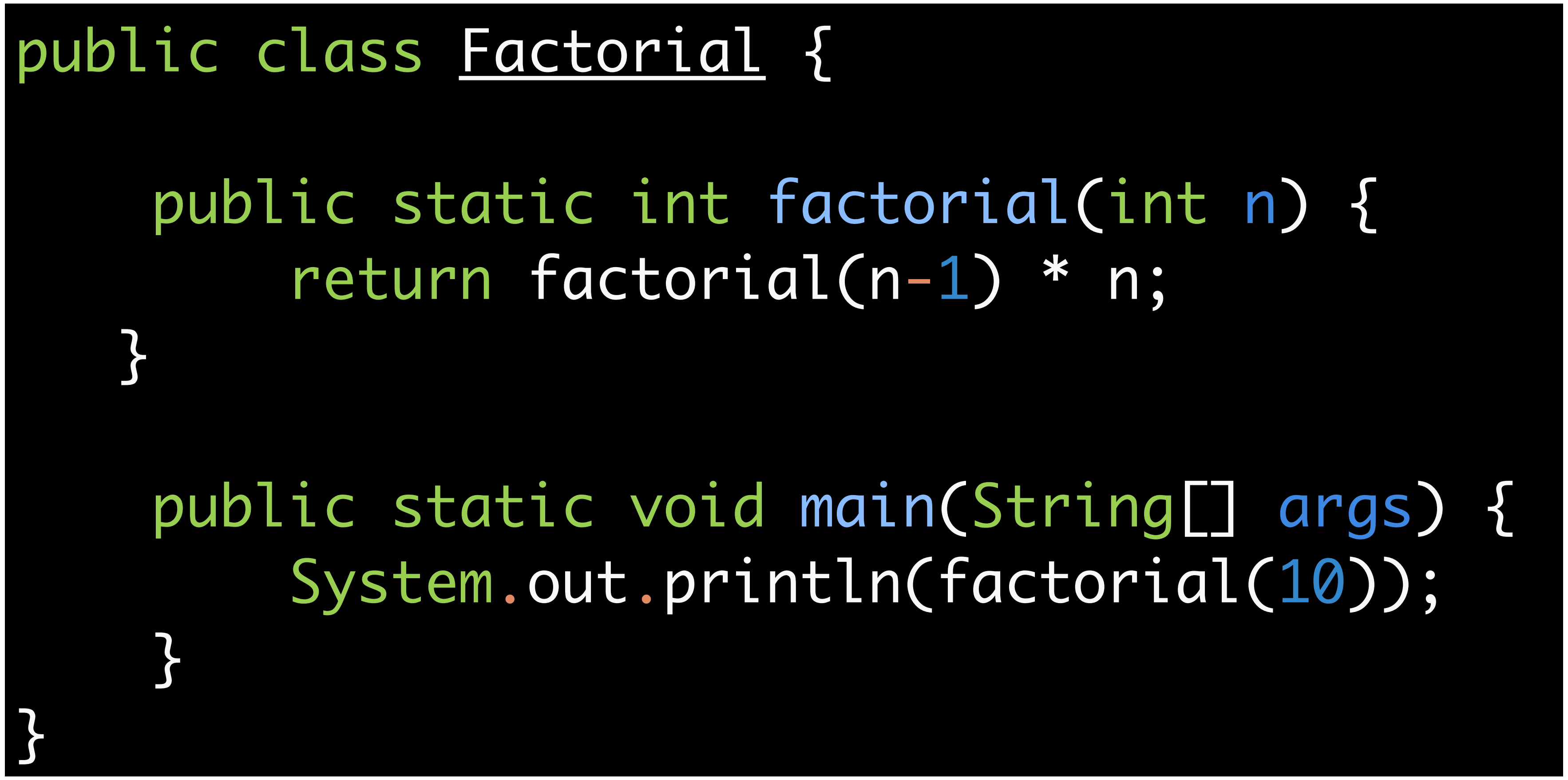
Stack Machines

- Most modern computers are register machines.
  To compute 2+3:
  - `mov eax,2`
  - `move ebx,3`
  - `add eax,abx` which stores the result in `eax`

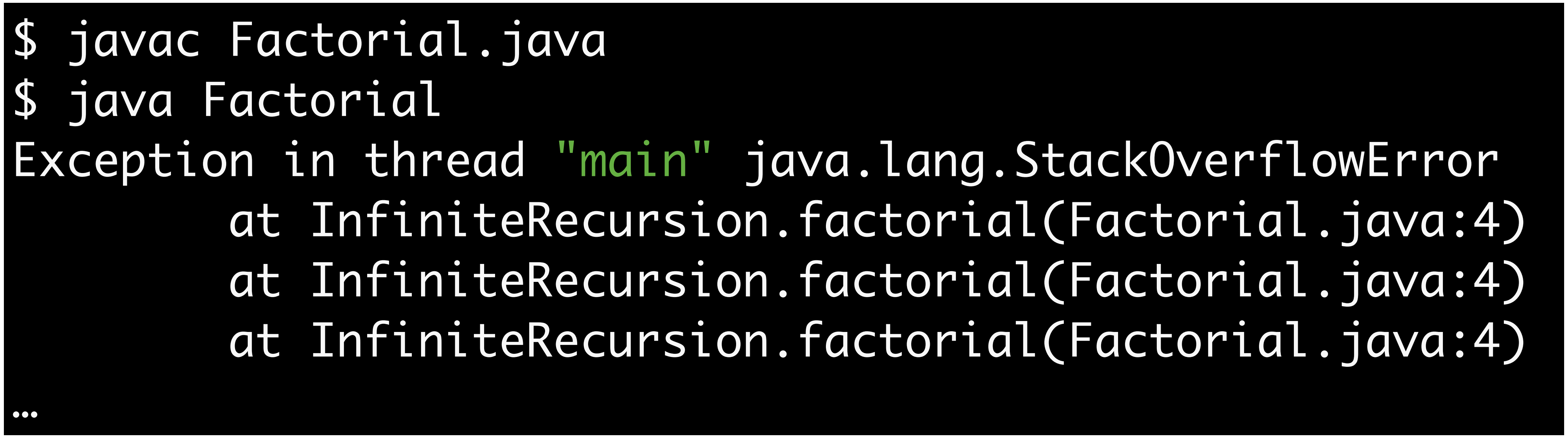
- In a Stack Machine:
  - `push 2`
  - `push 3`
  - `add` which stores the result back on the stack.

- Hardware stack machines are rare, but most virtual machines (including JVM) are stack machines.

What’s wrong with this program?



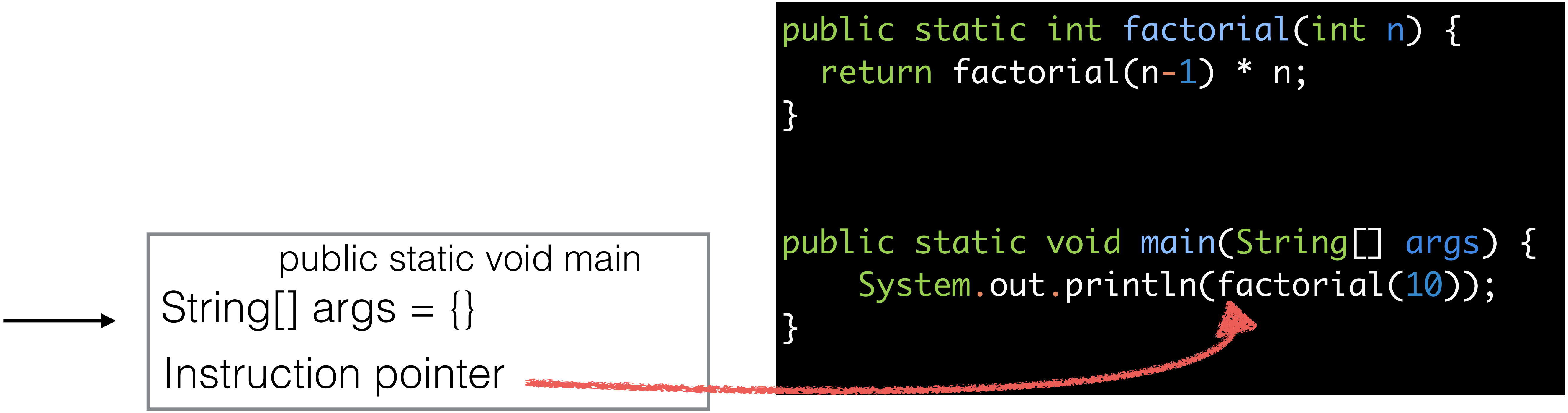
```java
public class Factorial {

    public static int factorial(int n) {
        return factorial(n-1) * n;
   }

    public static void main(String[] args) {
        System.out.println(factorial(10));
    }
}
```



```
$ javac Factorial.java
$ java Factorial
Exception in thread "main" java.lang.StackOverflowError
        at InfiniteRecursion.factorial(Factorial.java:4)
        at InfiniteRecursion.factorial(Factorial.java:4)
        at InfiniteRecursion.factorial(Factorial.java:4)
…
```

Method Call Stacks

- Every function keeps an *activation record* on the method call stack.
  - Represent current state of execution of this function.
  - Includes instruction pointer, value of variables, parameters, intermediate results.



public static void main

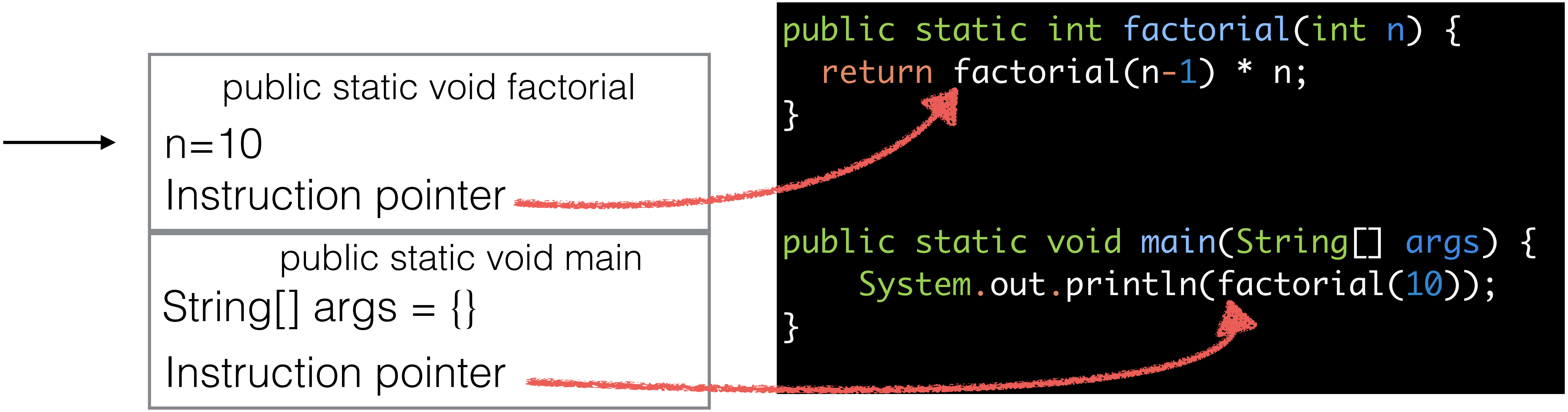
String[] args = {}

Instruction pointer

```
public static int factorial(int n) {
  return factorial(n-1) * n;
}


public static void main(String[] args) {
    System.out.println(factorial(10));
}
```

Method Call Stacks (2)

- When a function is called
  - Execution of the current function is suspended.
  - A new activation record is pushed to the stack.
  - The new function is run.



public static void factorial

n=10

Instruction pointer

public static void main

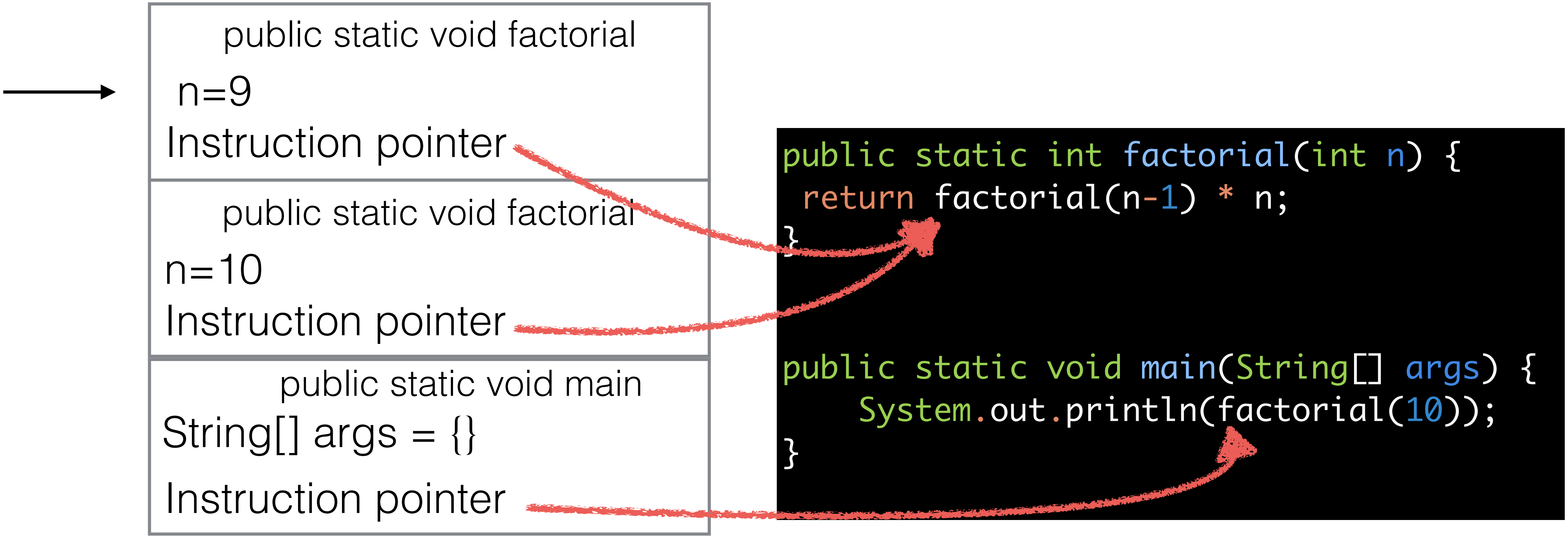
String[] args = {}

Instruction pointer

```
public static int factorial(int n) {
  return factorial(n-1) * n;
}

public static void main(String[] args) {
    System.out.println(factorial(10));
}
```

Runaway Recursion

- Recursion will quickly grow the method call stack.
- Execution of the current function is suspended.



public static void factorial

n=9

Instruction pointer

public static void factorial

n=10

Instruction pointer

public static void main

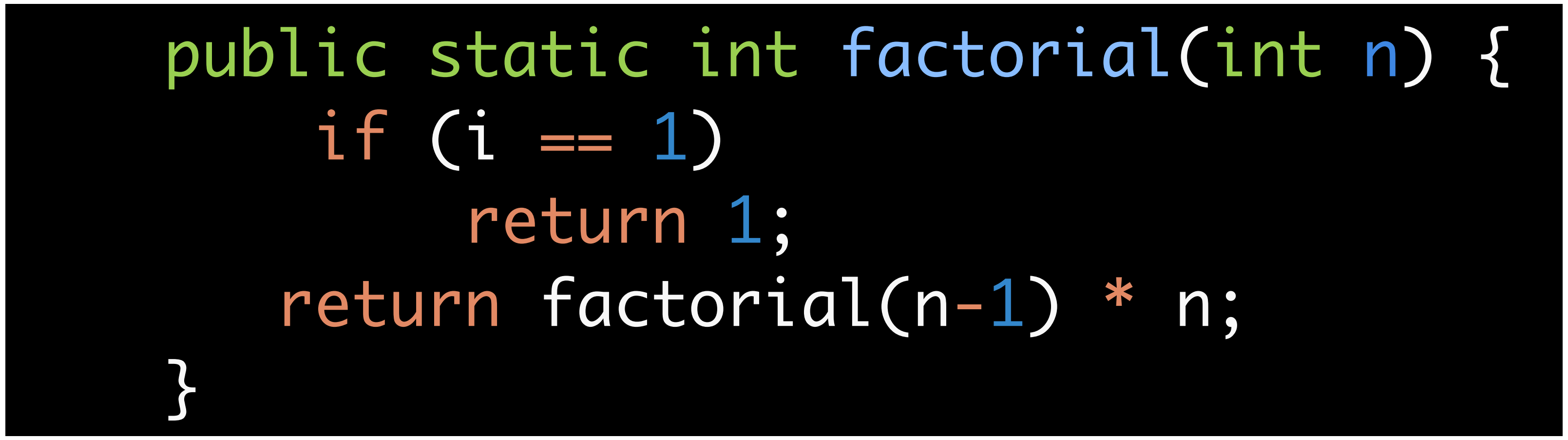
String[] args = {}

Instruction pointer

```
public static int factorial(int n) {
 return factorial(n-1) * n;
}


public static void main(String[] args) {
    System.out.println(factorial(10));
}
```

Fixing Runaway Recursion

- We forgot to add the base case:

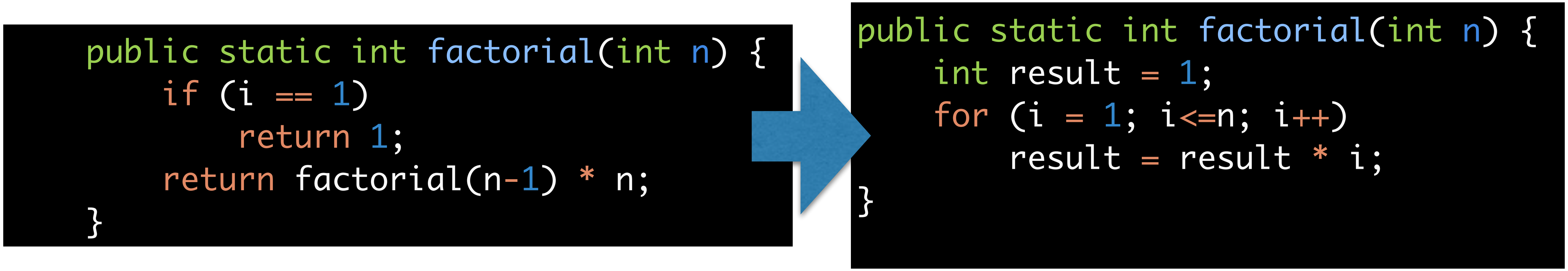


```
public static int factorial(int n) {
    if (i == 1)
        return 1;
   return factorial(n-1) * n;
}
```

- Still can get stack overflows for large *n*.

Rewriting Recursion

- This is a stupid use for recursion.



```
public static int factorial(int n) {
    if (i == 1)
        return 1;
    return factorial(n-1) * n;
}
```

```
public static int factorial(int n) {
    int result = 1;
    for (i = 1; i<=n; i++)
        result = result * i;
}
```

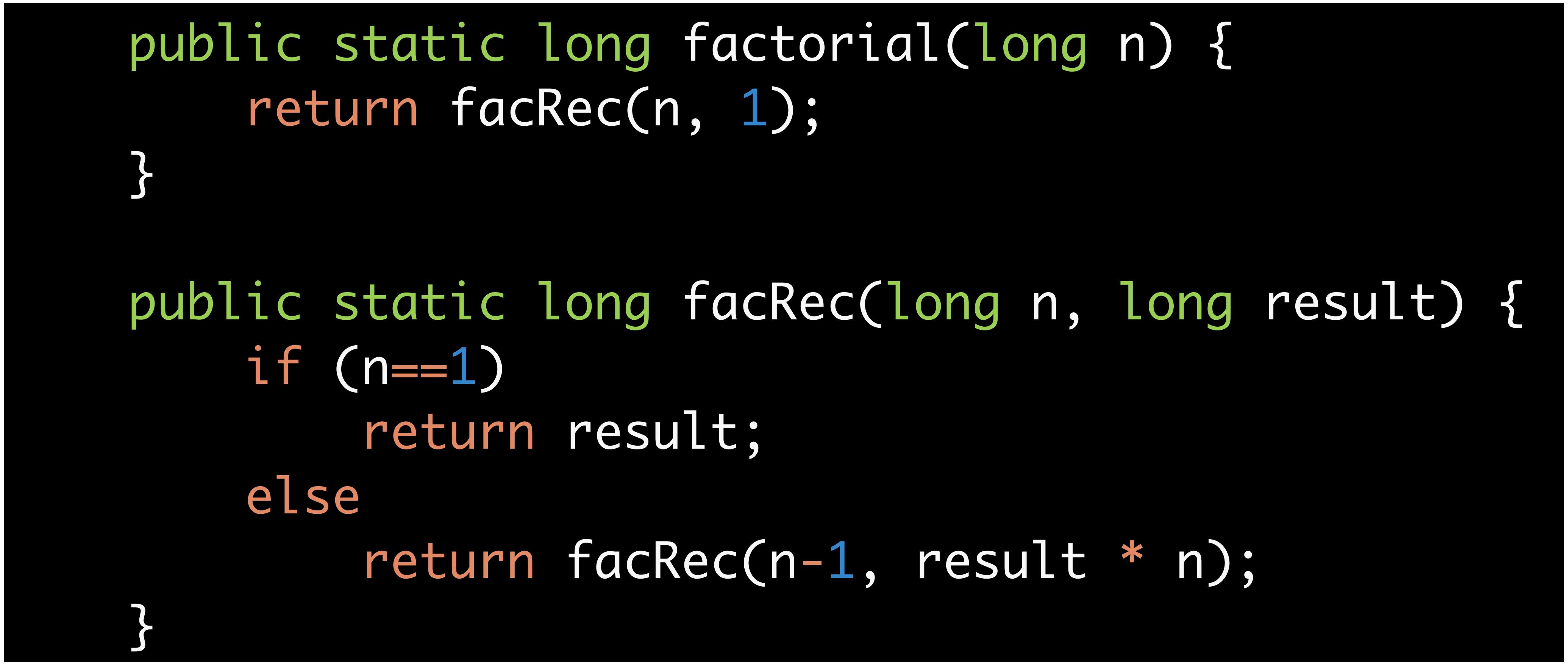
- In general, any recursion can be removed, but this will often lead to unreadable code.

- But recursion is often more readable.

Tail Recursion

- Compilers can detect and remove some types of recursion.

- A method is *tail recursive* if the last thing it does is call itself. Compilers can turn this into a loop.



```
public static long factorial(long n) {
    return facRec(n, 1);
}

public static long facRec(long n, long result) {
    if (n==1)
        return result;
    else
        return facRec(n-1, result * n);
}
```