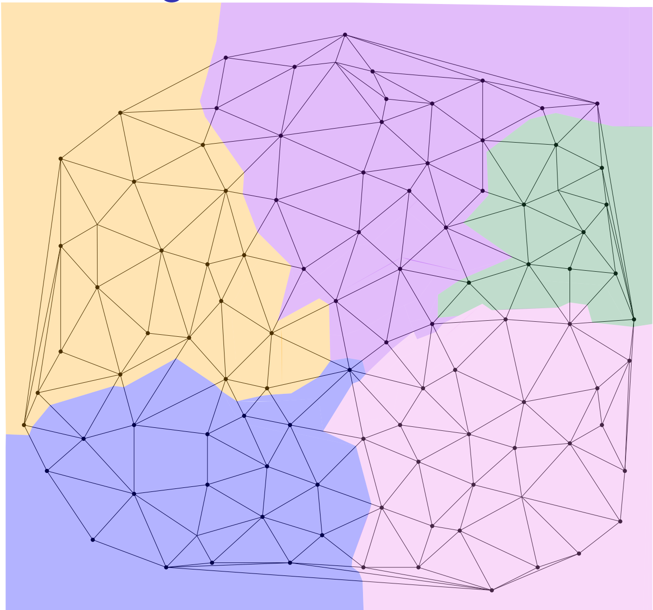
On Strong Diameter Padded Decompositions

Arnold Filtser

Columbia University

November 05, 2019

Clustering Problems: Stochastic Decompositions



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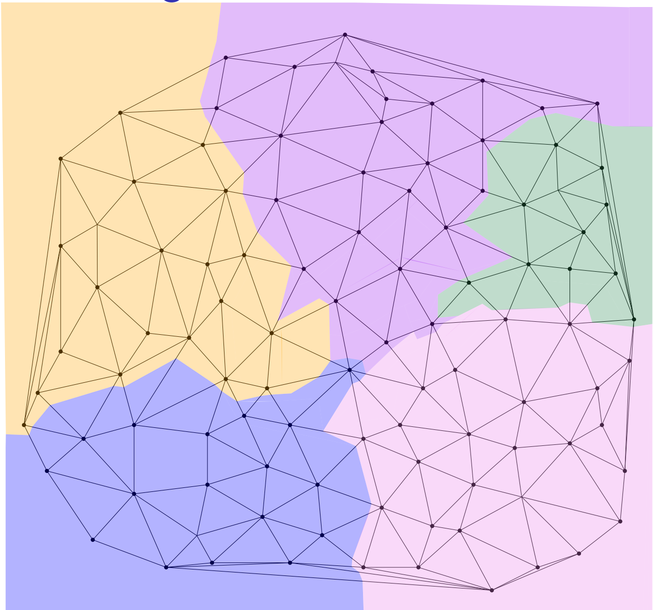

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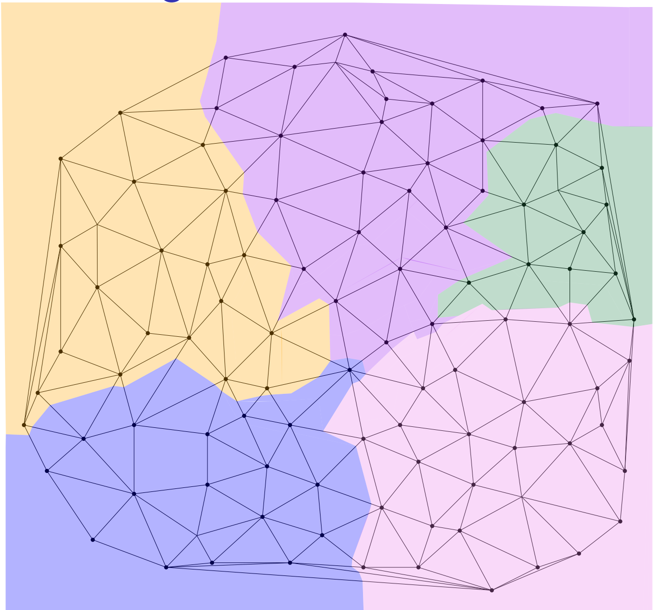

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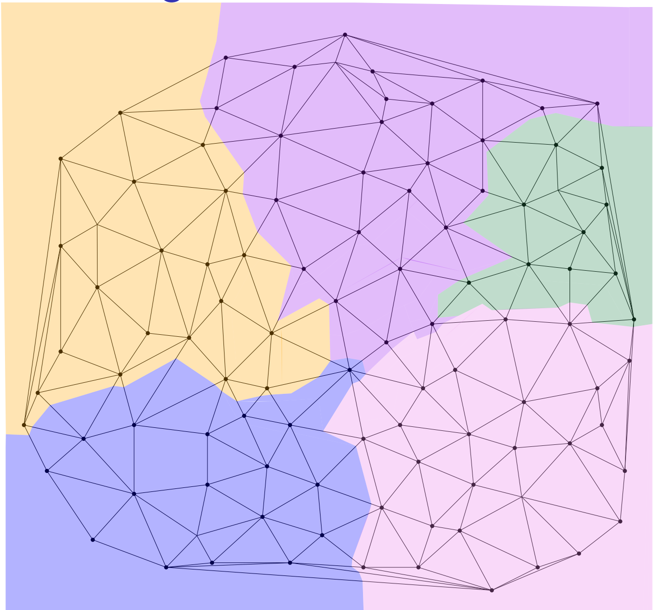

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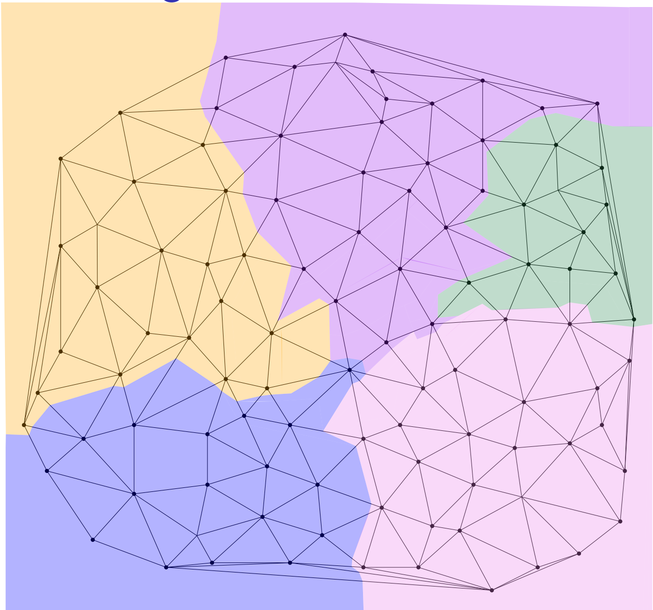

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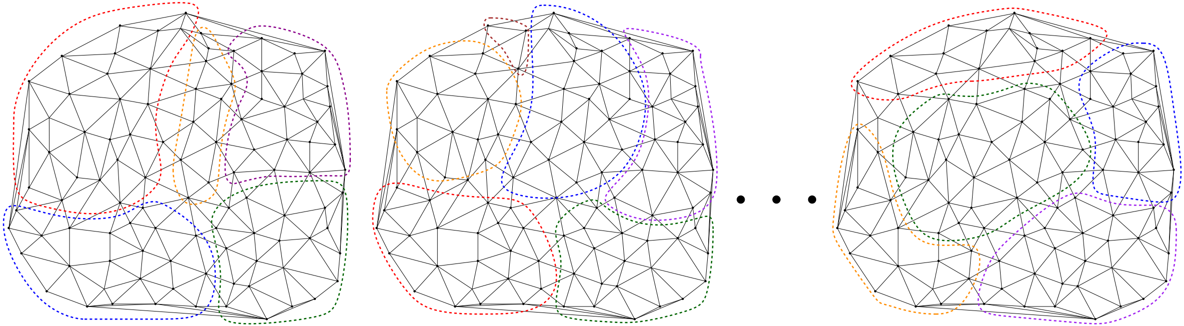
- Small diameter.
- Connectivity.
- Nearby vertices are **likely** to be clustered together.

Definition (Padded Decomposition)

Weighted graph $G = (V, E, w)$.
Distribution $\mathcal{D}$ over partitions of G is (β, Δ)-padded decomposition if:



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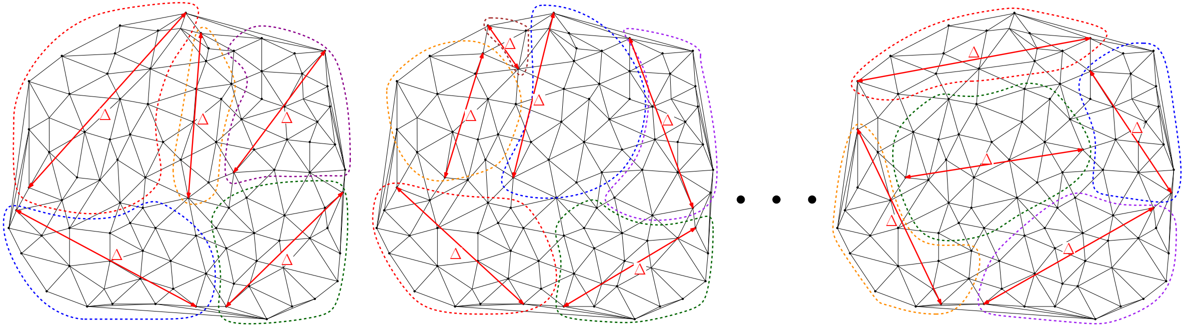

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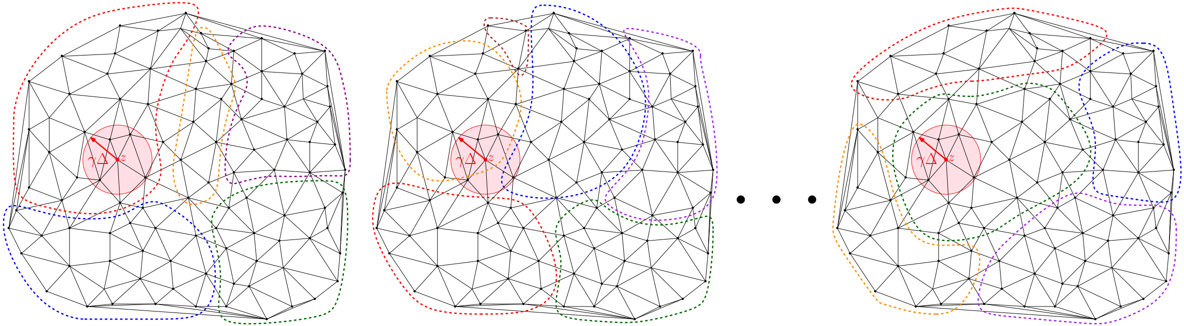

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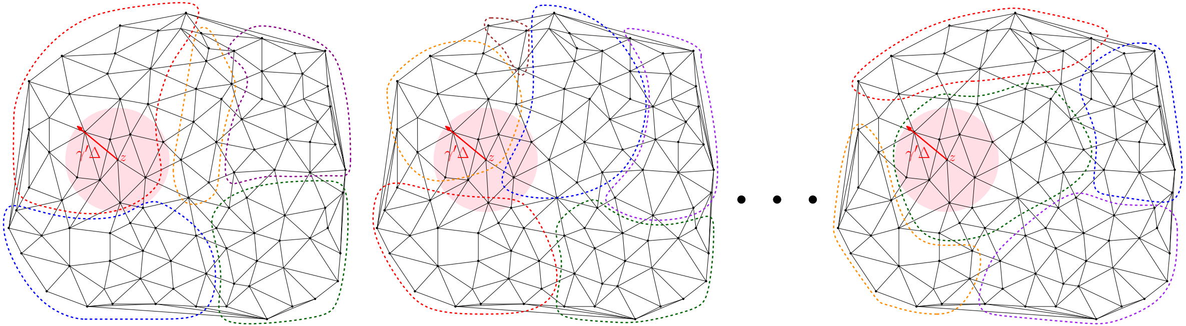

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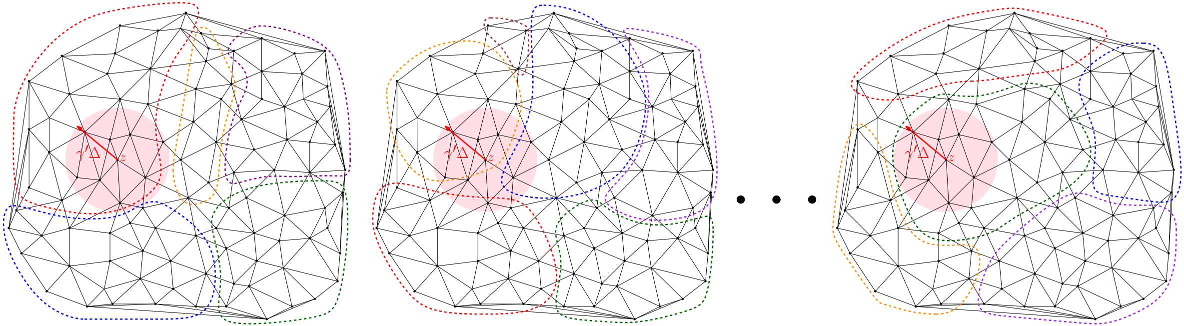

G admits a β-padded decomposition **scheme**:

$\forall \Delta > 0$, G admits (β, Δ)-padded decomposition.

Strong Vs. Weak Diameter

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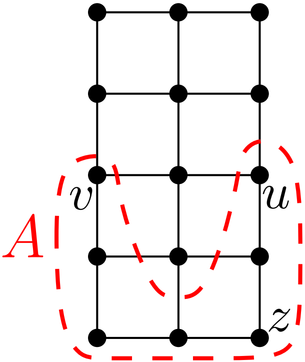

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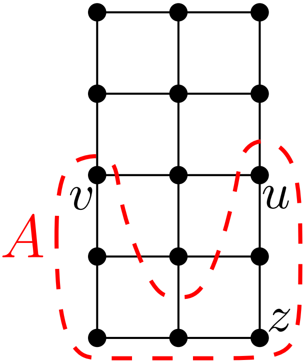

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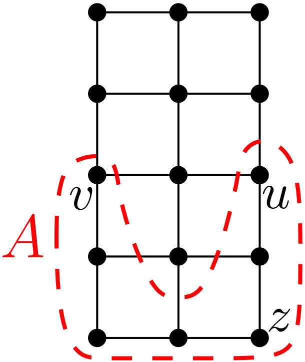

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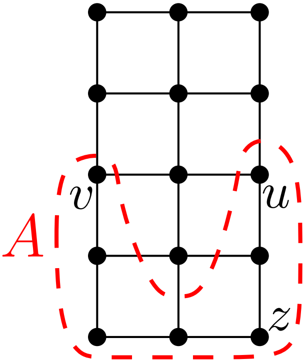

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Weak diameter of $A = 4$.
Strong diameter of $A = 6$.

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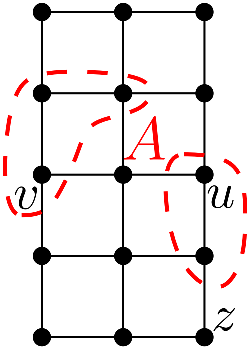

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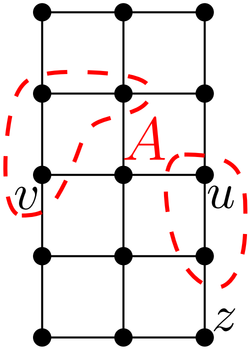

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History and Results

Family	**Diam**	**Padding**	**Ref**
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General n vertex graphs	Strong	$O(\log n)$	[Bartal 96]

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Cops, Robbers
and Threatening Skeletons
Padded Decompositions
for Graphs Excluding a Fixed Minor

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Path reporting Distance Oracles.

Lemma (Sparse Net $\Rightarrow$ Padded Decomposition)

$G = (V, E, w)$, $\Delta > 0, \tau = \Omega(1)$.

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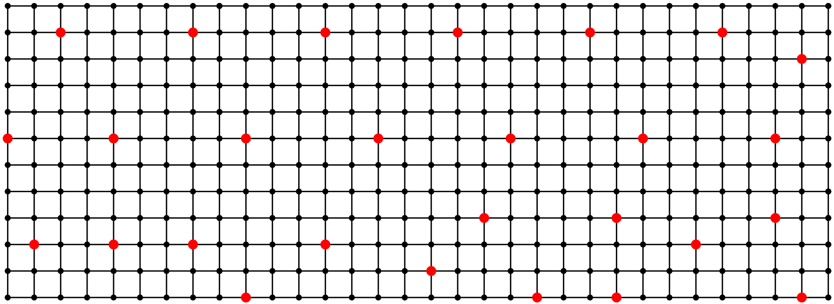

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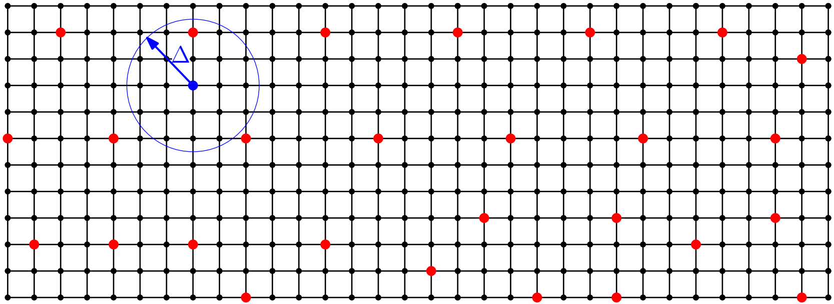

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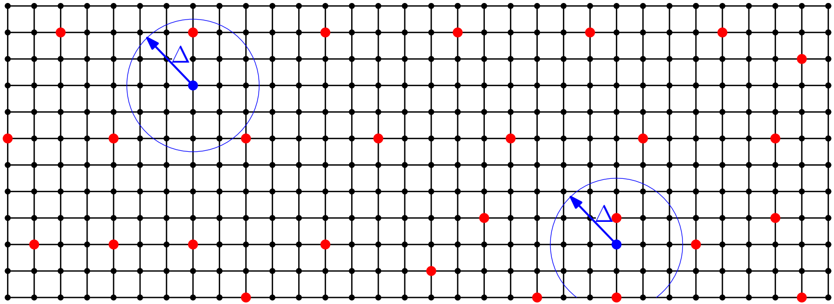

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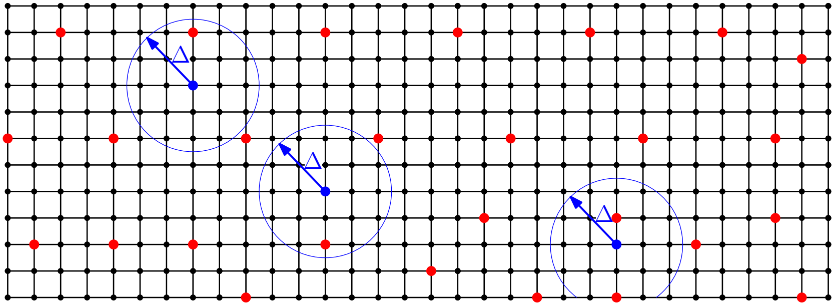

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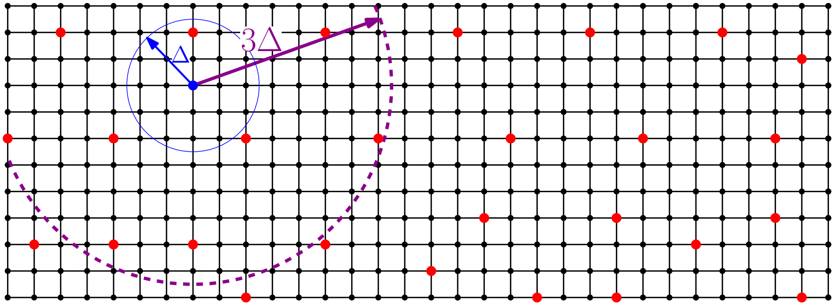

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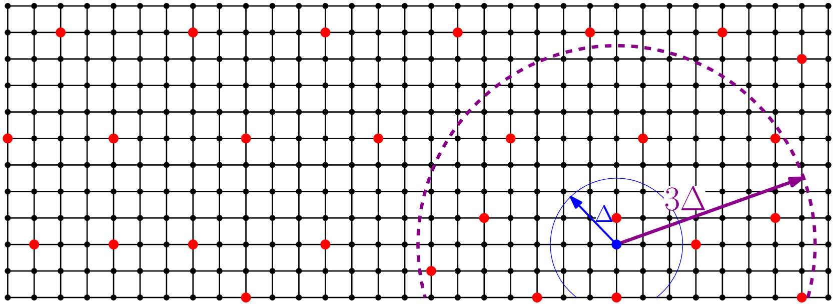

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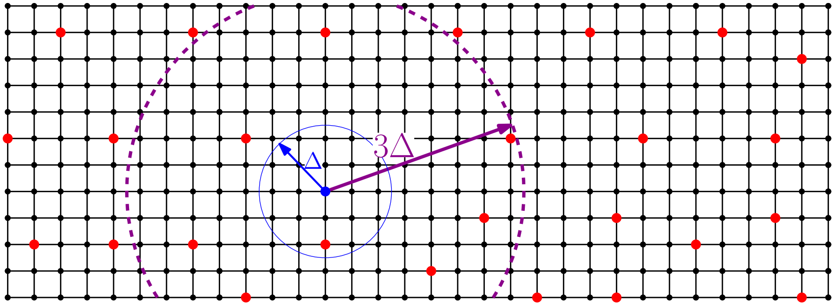

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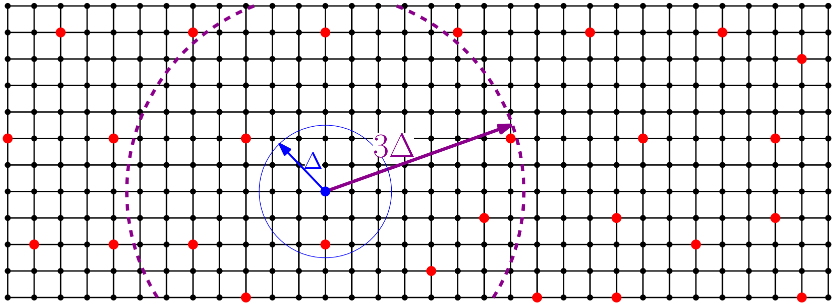
Then G *admits a* **strongly** $(O(\ln \tau), 4\Delta)$*-padded decomposition.*



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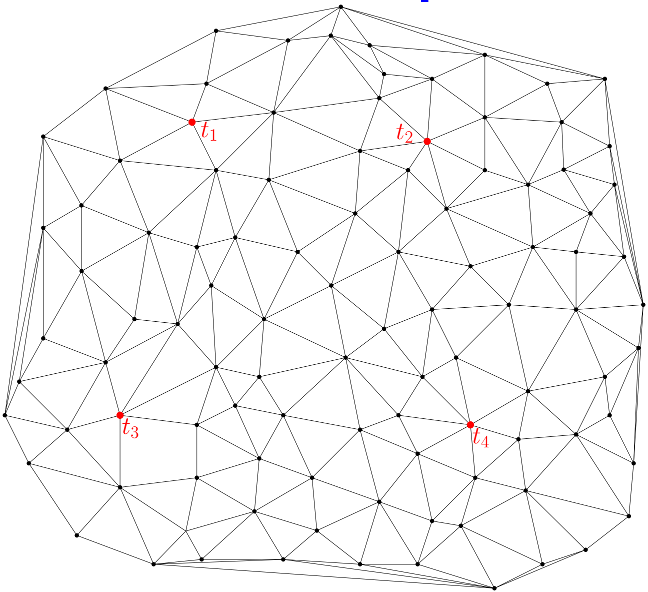

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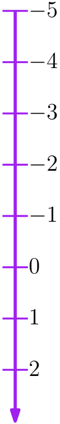

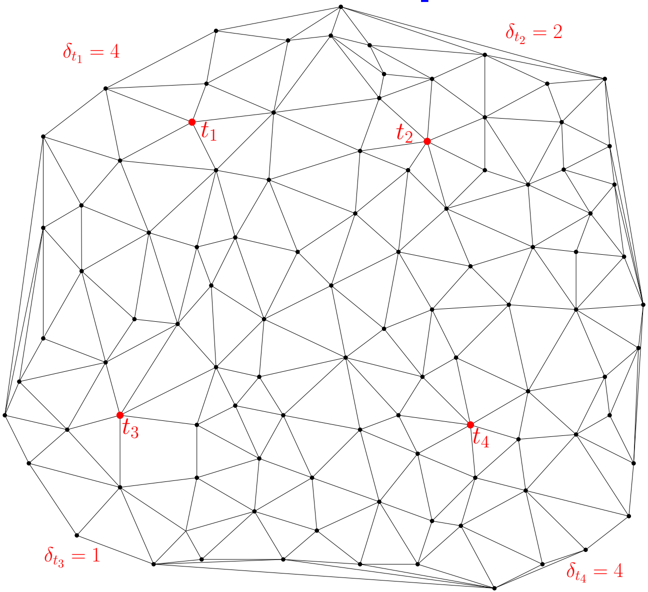

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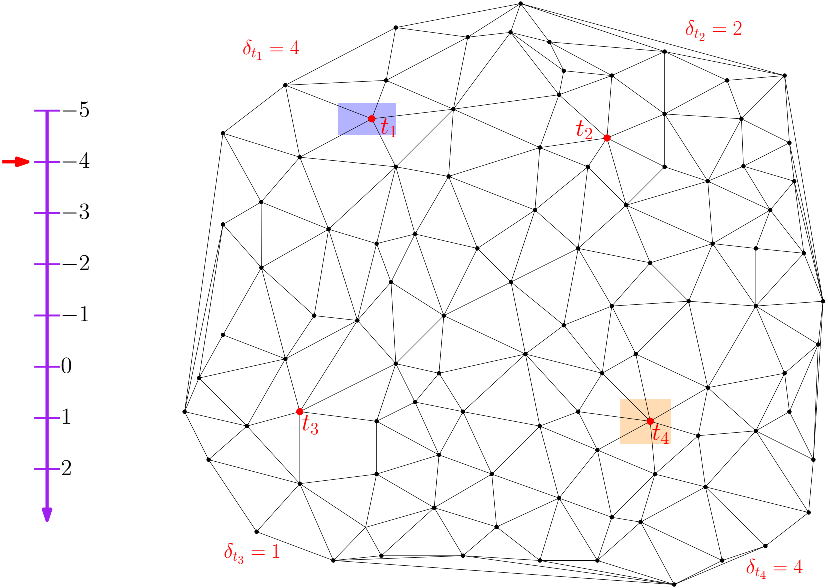

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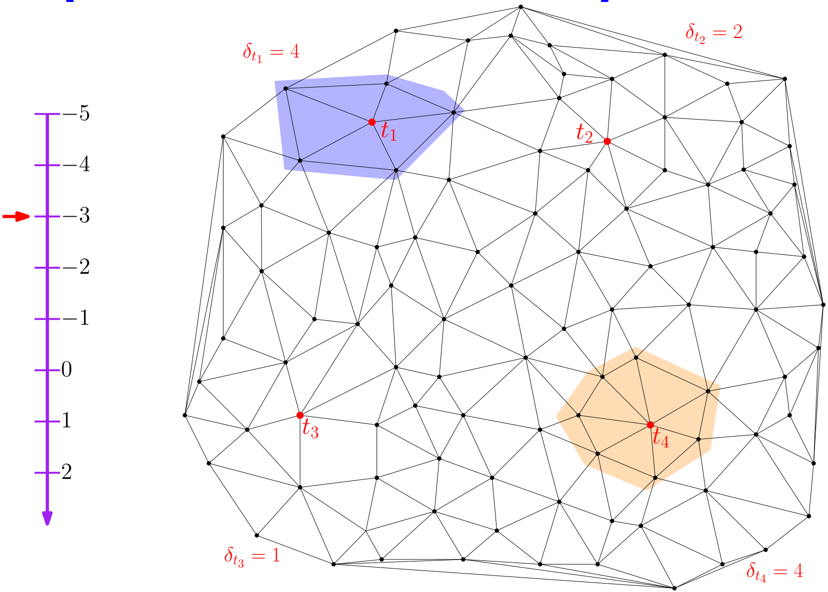

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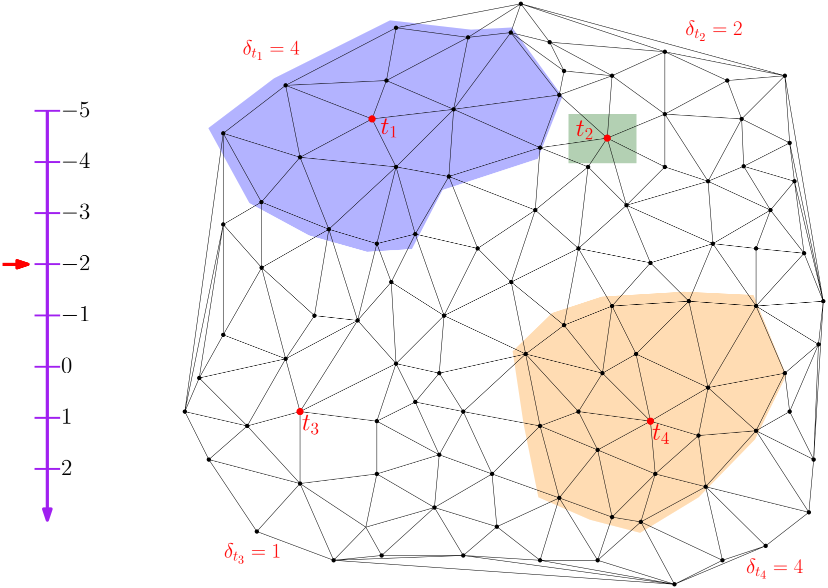

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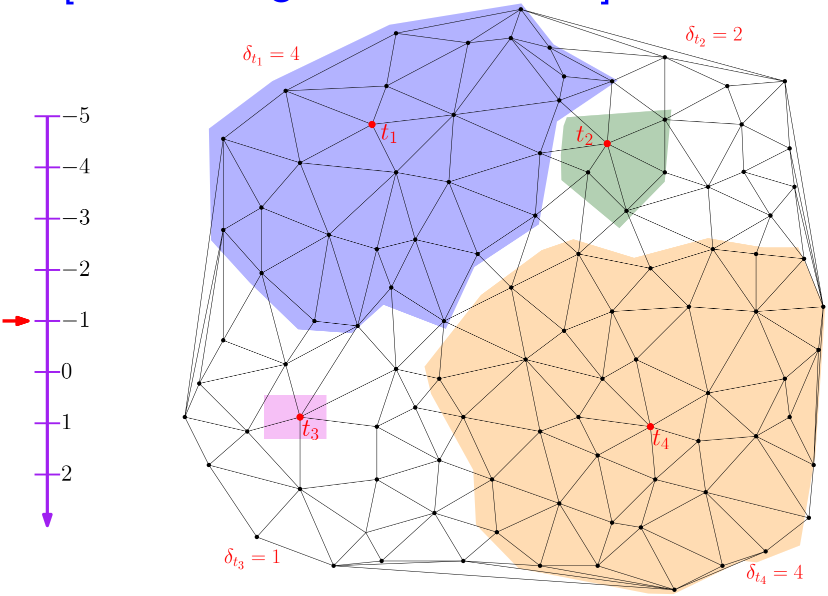

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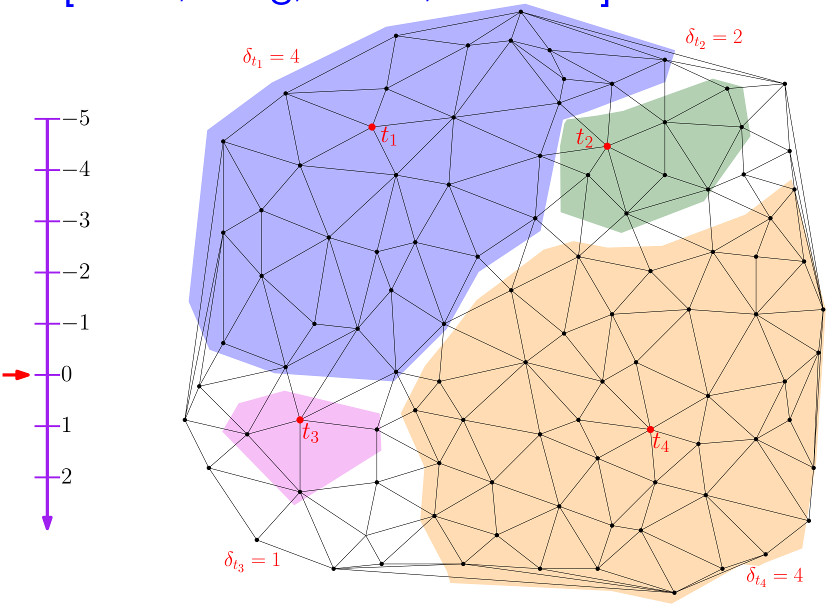

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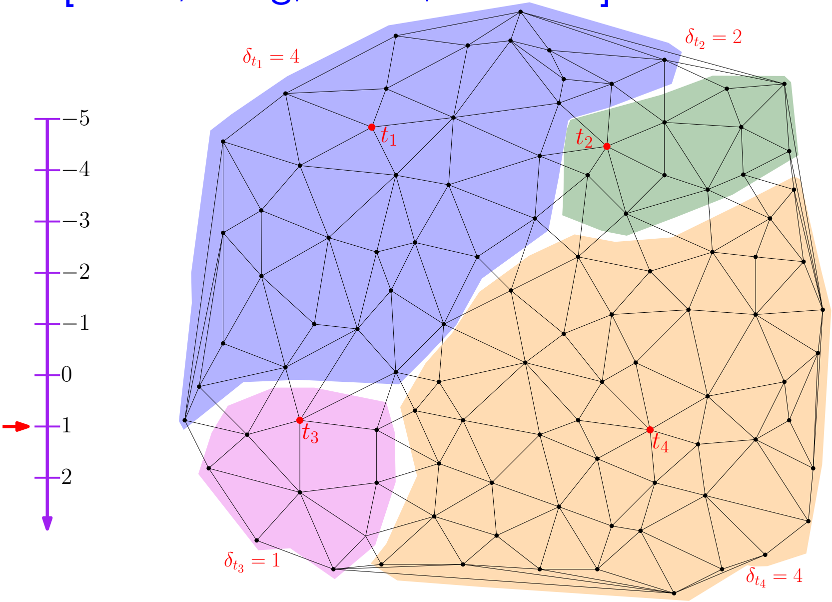

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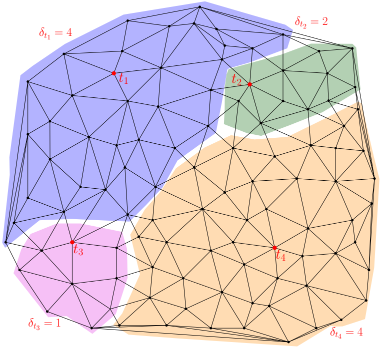

Inherently **connected**!

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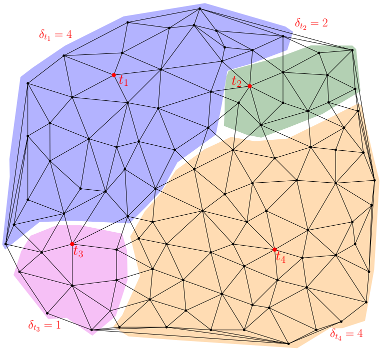

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Formally, for $v \in V$ set $\boldsymbol{f_v}(t) = \delta_t - d_G(v, t)$.

v **joins** the cluster C_t of the center t **maximizing** f_v.

MPVX to Technical Lemma

Lemma (Sparse Net $\Rightarrow$ Padded Decomposition)

$G = (V, E, w)$, $\Delta > 0, \tau = \Omega(1)$.
Suppose $\exists$ *set* $N \subseteq V$ *of* **centers** *s.t.* $\forall v \in V$*:*

- COVERING. $\exists t \in N$ *s.t.* $d_G(v, t) \leq \Delta$.
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Then G *admits a* **strongly** $(O(\ln \tau), 4\Delta)$*-padded decomposition.*

Partition Algorithm: For each center $t \in N$ **sample** $\delta_t \in [0, 2\Delta]$ according to **truncated exponential** distribution with parameter $\boldsymbol{\lambda} = \frac{\Delta}{\ln \tau}$.
Run [MPVX15] (v goes to $\arg\max f_v(t) = \delta_t - d_G(v, t)$).

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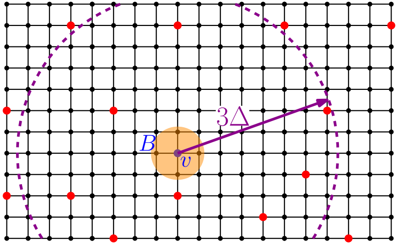

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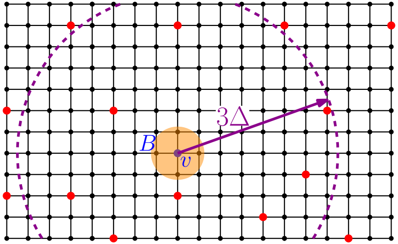

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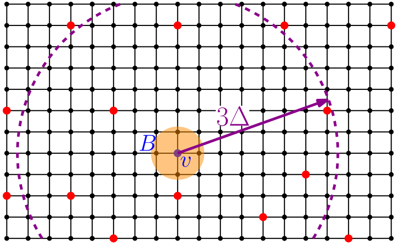

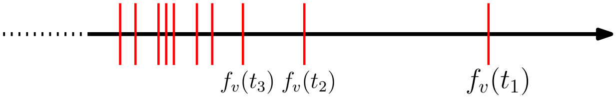

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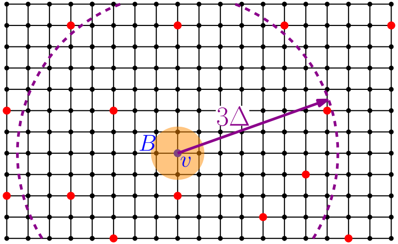

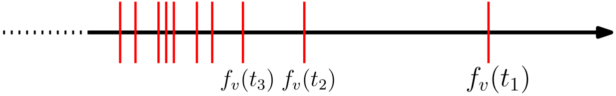

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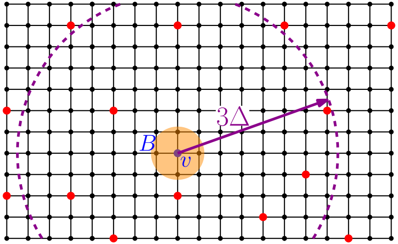

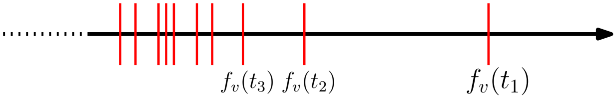

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"memoryless"

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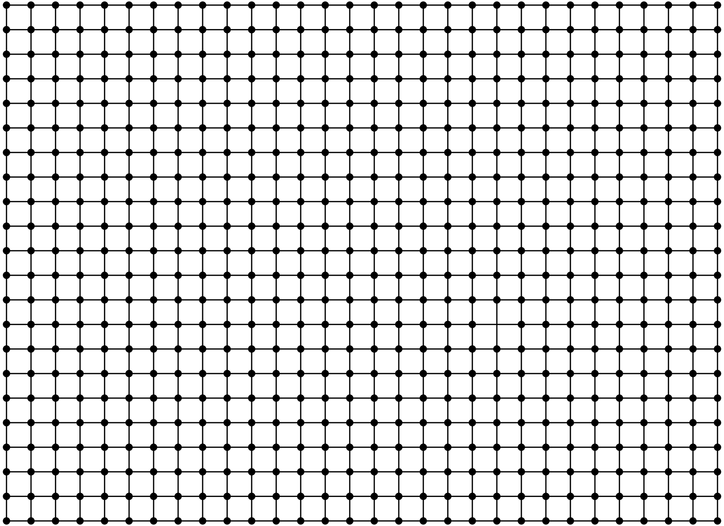

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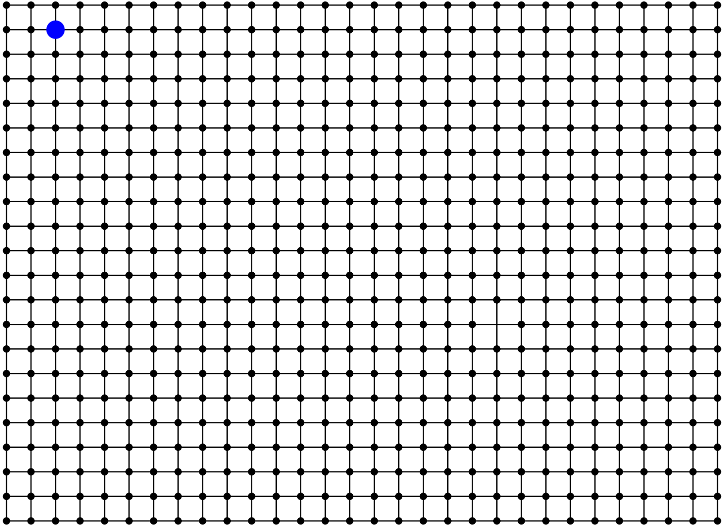

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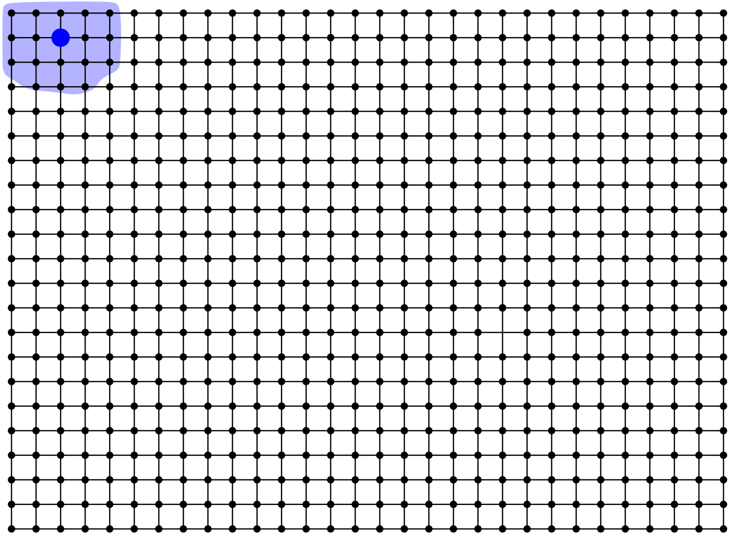

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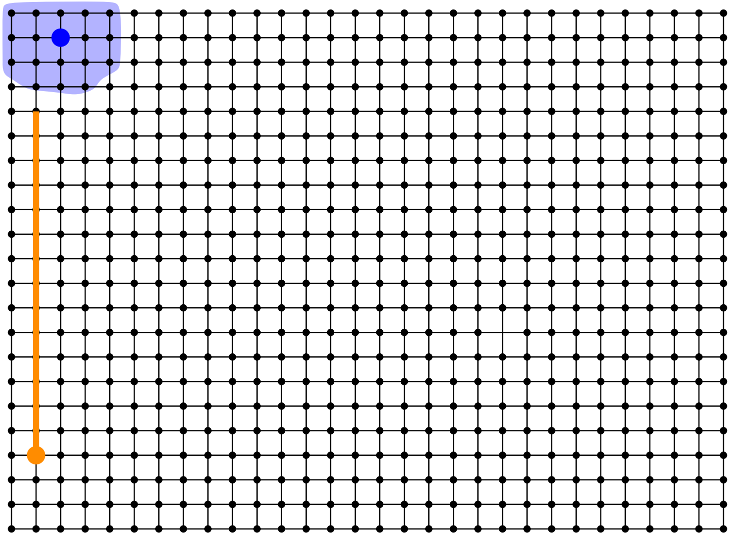

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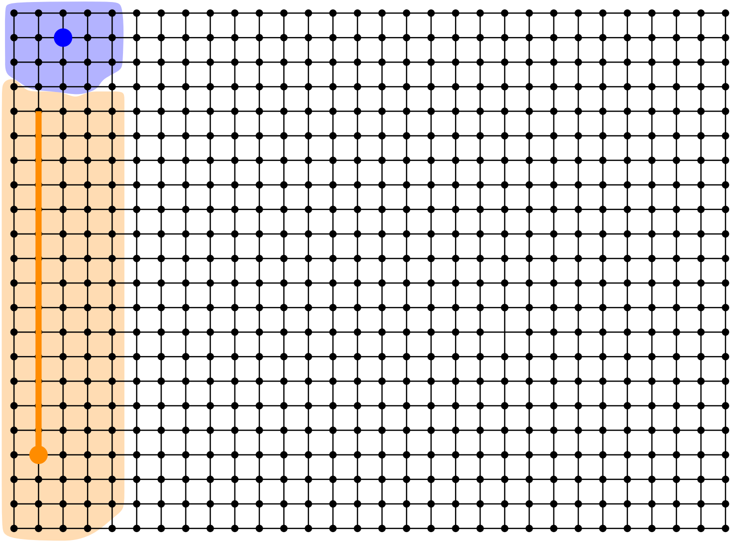

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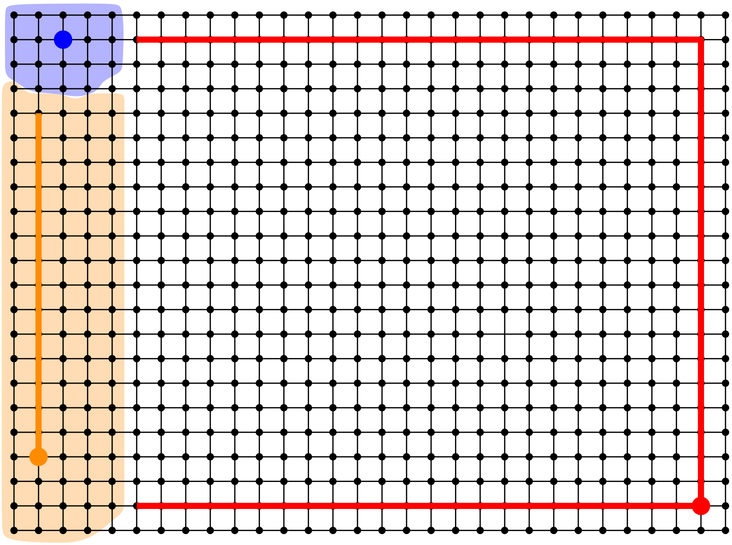

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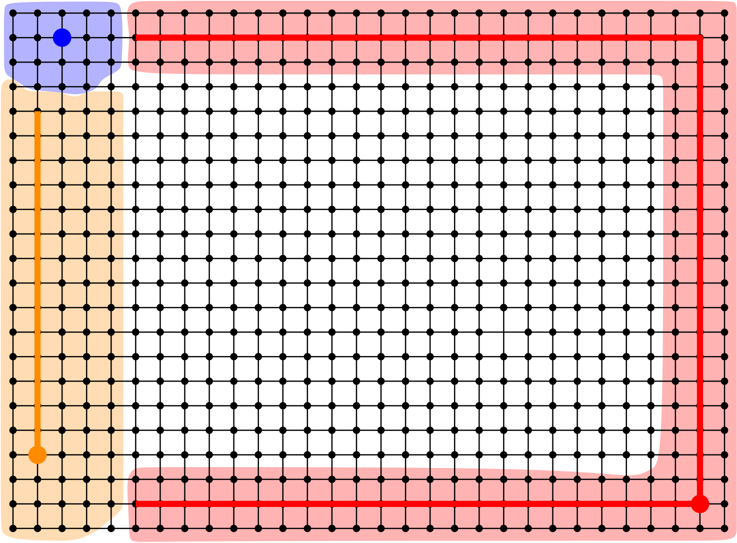

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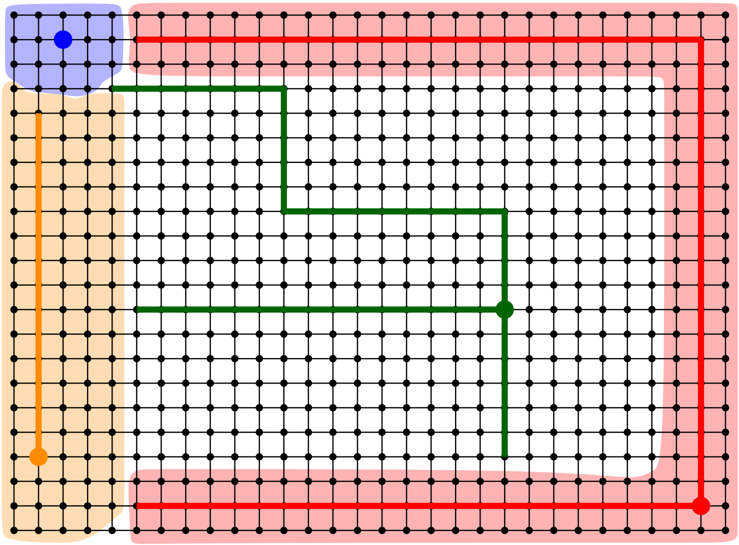

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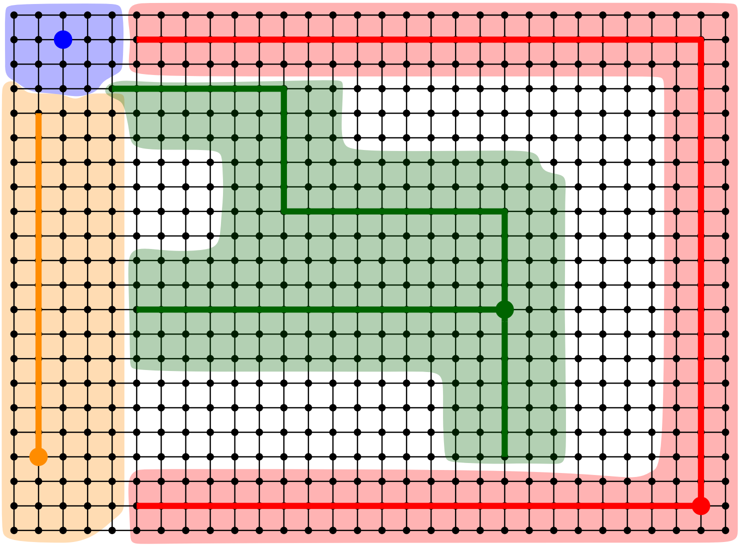

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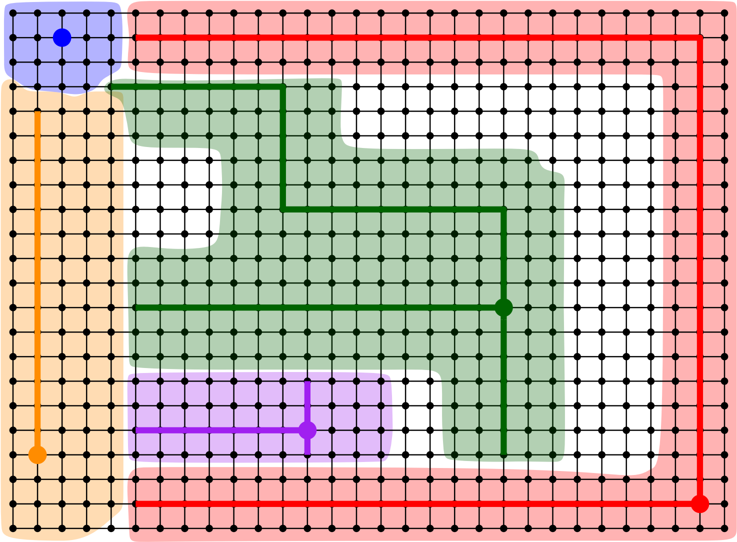

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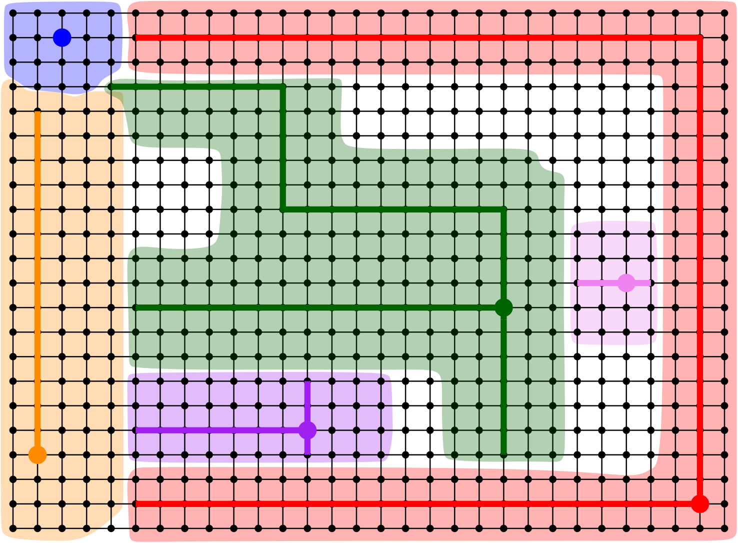

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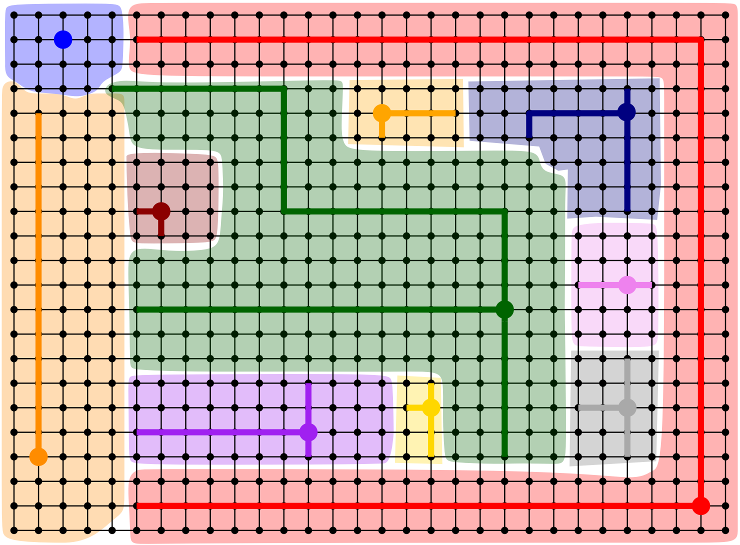

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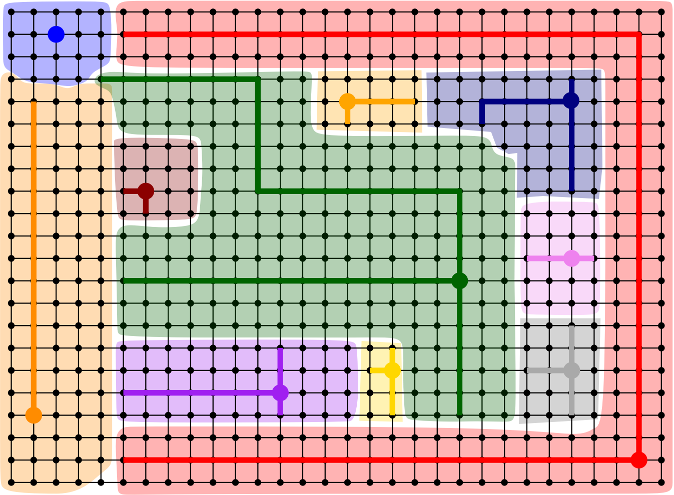

Lemma

If G is K_r **minor free**, then each core tree has at most r **leafs**.

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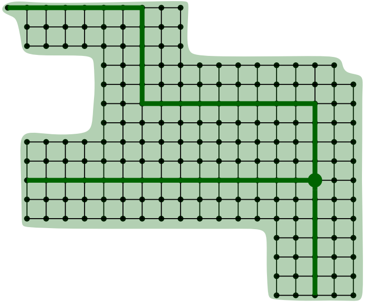

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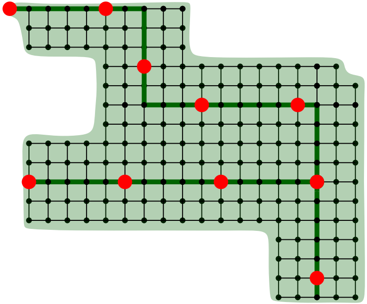

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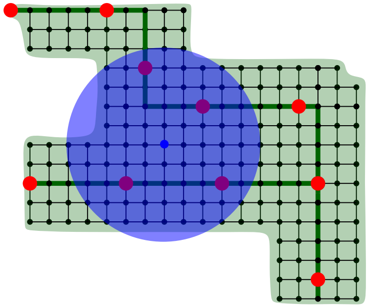

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Theorem (Padded Decomposition for Minor Free Graphs)

*The family of K_r-**minor free** graphs admits strong $O(r)$-padded decomposition scheme.*

Open Question

Conjecture ([AGGNT14])

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Thank You!