

Informed search

Use domain knowledge!

- Are we getting close to the goal?
- Use a heuristic function that estimates how close a state is to the goal
- A heuristic does NOT have to be perfect!

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- Are we getting close to the goal?
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- A heuristic does NOT have to be perfect!
- Example of strategies:
 1. Greedy best-first search
 2. A* search
 3. IDA*

Greedy search

- Evaluation function $h(n)$ (*heuristic*)
- $h(n)$ estimates the cost from n to the closest goal
- Example: $h_{\text{SLD}}(n)$ = straight-line distance from n to Sault Ste Marie
- Greedy search expands the node that **appears** to be closest to goal

Greedy search

```
function GREEDY-BEST-FIRST-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE : /* Cost  $f(n) = h(n)$  */

    frontier = Heap.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.deleteMin()
        explored.add(state)

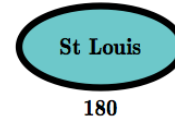
        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier  $\cup$  explored:
                frontier.insert(neighbor)
            else if neighbor in frontier:
                frontier.decreaseKey(neighbor)

    return FAILURE
```

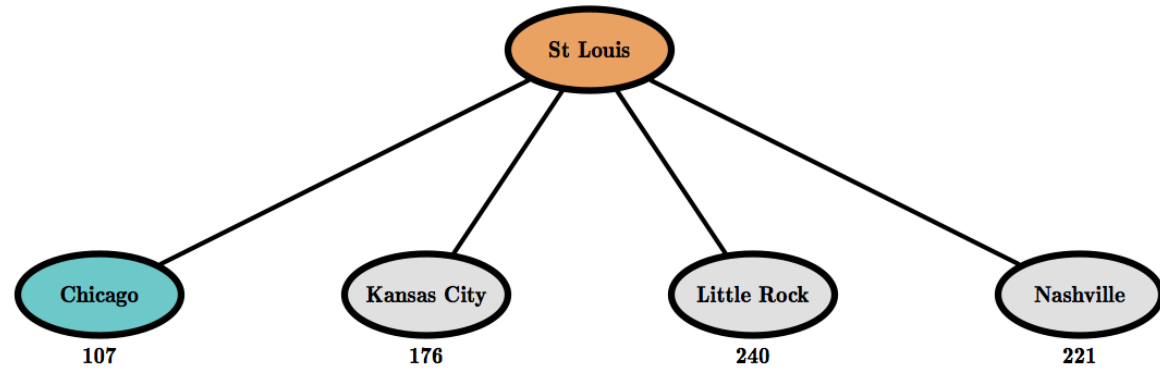
Greedy search example

The initial state:



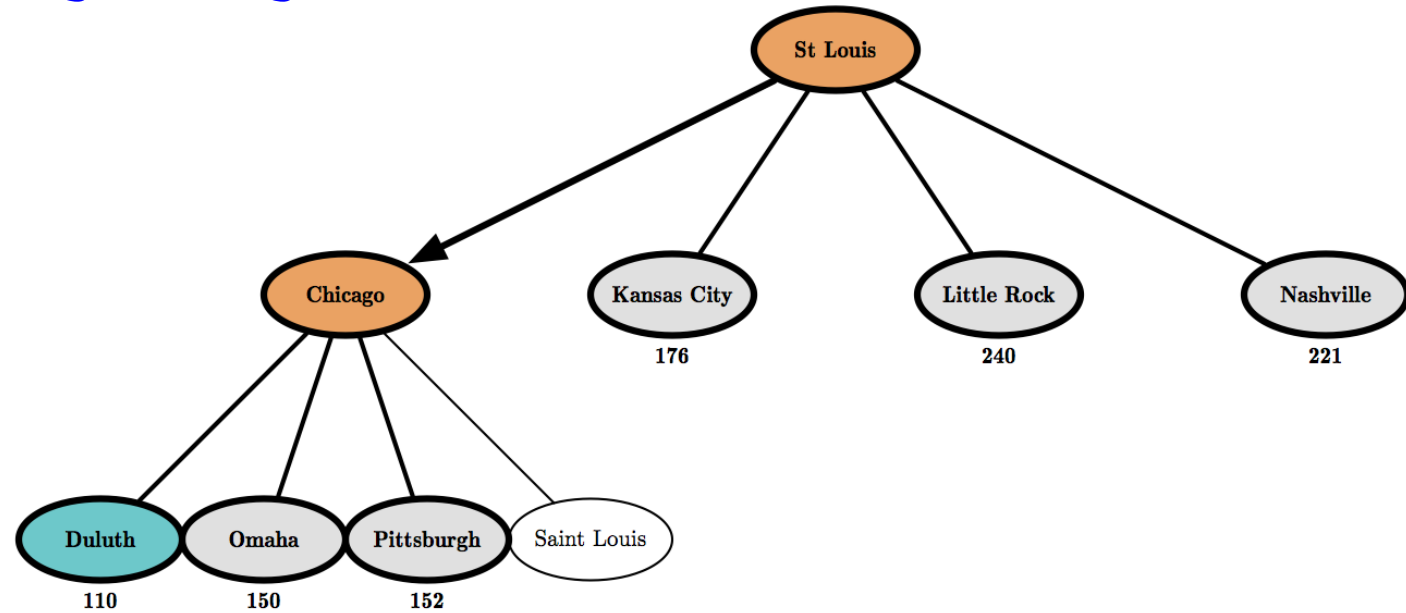
Greedy search example

After expanding St Louis:



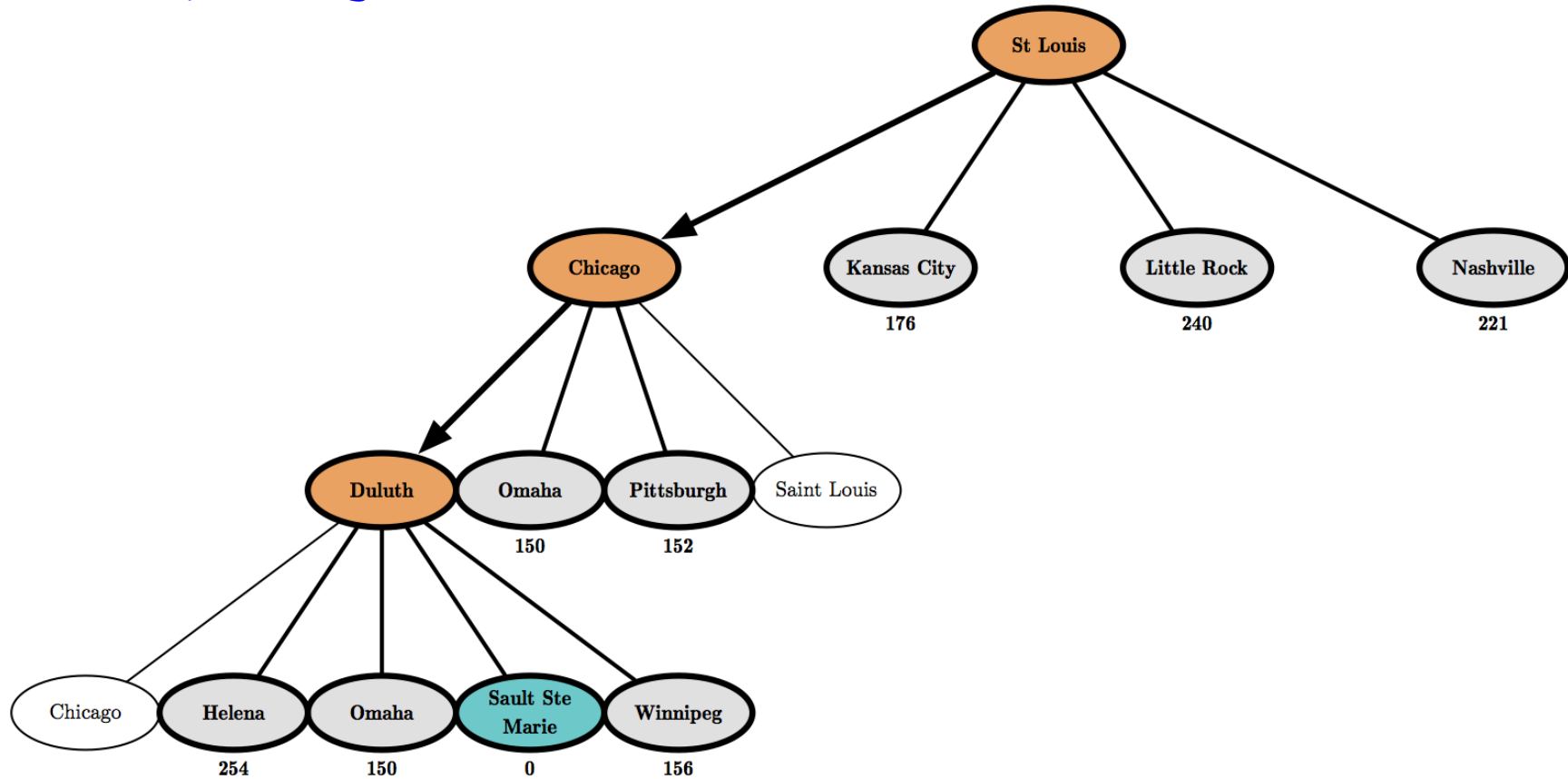
Greedy search example

After expanding Chicago:



Greedy search example

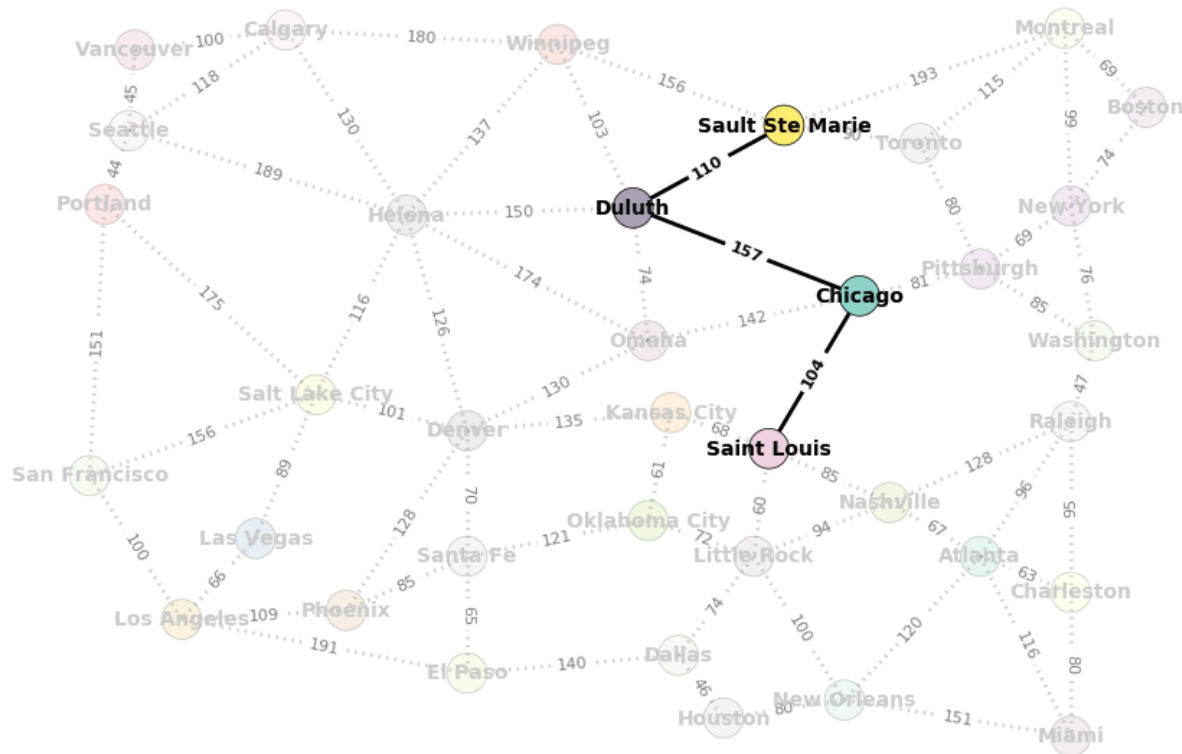
After expanding Duluth:



Examples using the map

Start: Saint Louis

Goal: Sault Ste Marie



Greedy search

A* search

- Minimize the total estimated solution cost
- Combines:
 - $g(n)$: cost to reach node n
 - $h(n)$: cost to get from n to the goal
 - $f(n) = g(n) + h(n)$

$f(n)$ is the estimated cost of the cheapest solution through n

A* search

```
function A-STAR-SEARCH(initialState, goalTest)
    returns SUCCESS or FAILURE : /* Cost  $f(n) = g(n) + h(n)$  */

    frontier = Heap.new(initialState)
    explored = Set.new()

    while not frontier.isEmpty():
        state = frontier.deleteMin()
        explored.add(state)

        if goalTest(state):
            return SUCCESS(state)

        for neighbor in state.neighbors():
            if neighbor not in frontier  $\cup$  explored:
                frontier.insert(neighbor)
            else if neighbor in frontier:
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    return FAILURE
```

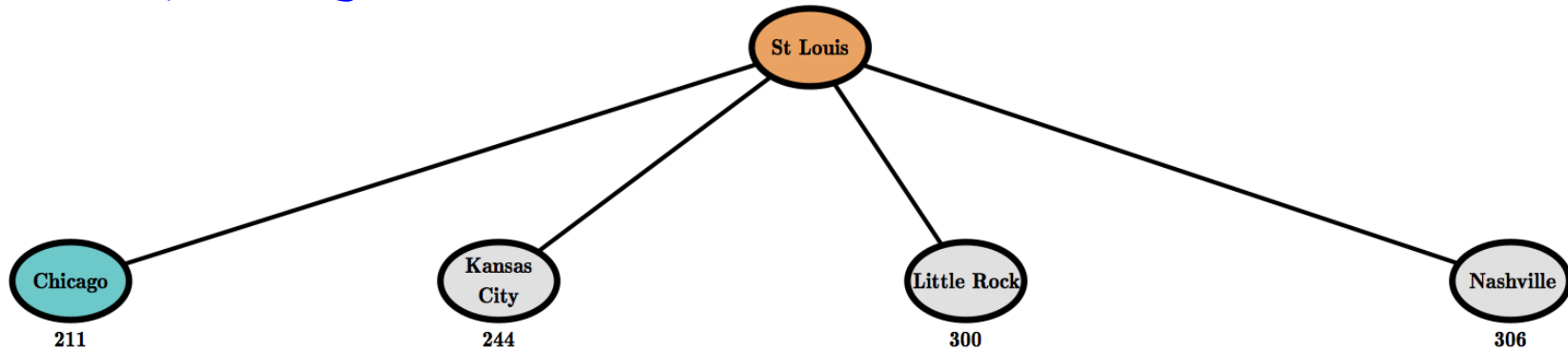
A* search example

The initial state:



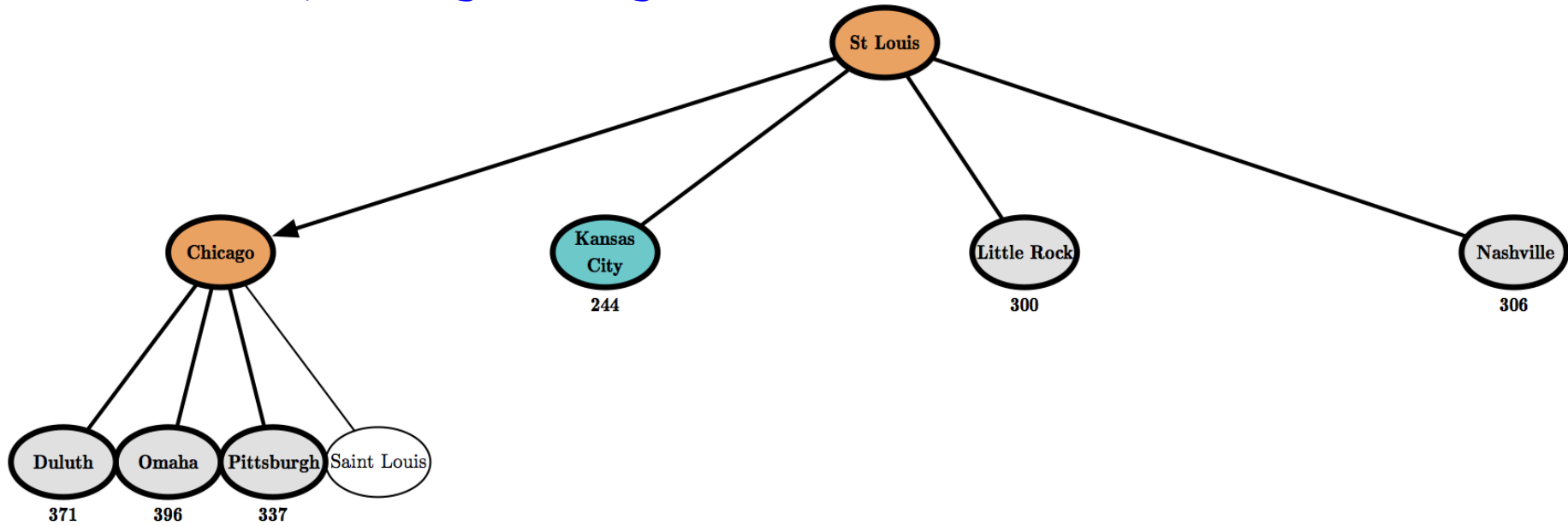
A* search example

After expanding St Louis:



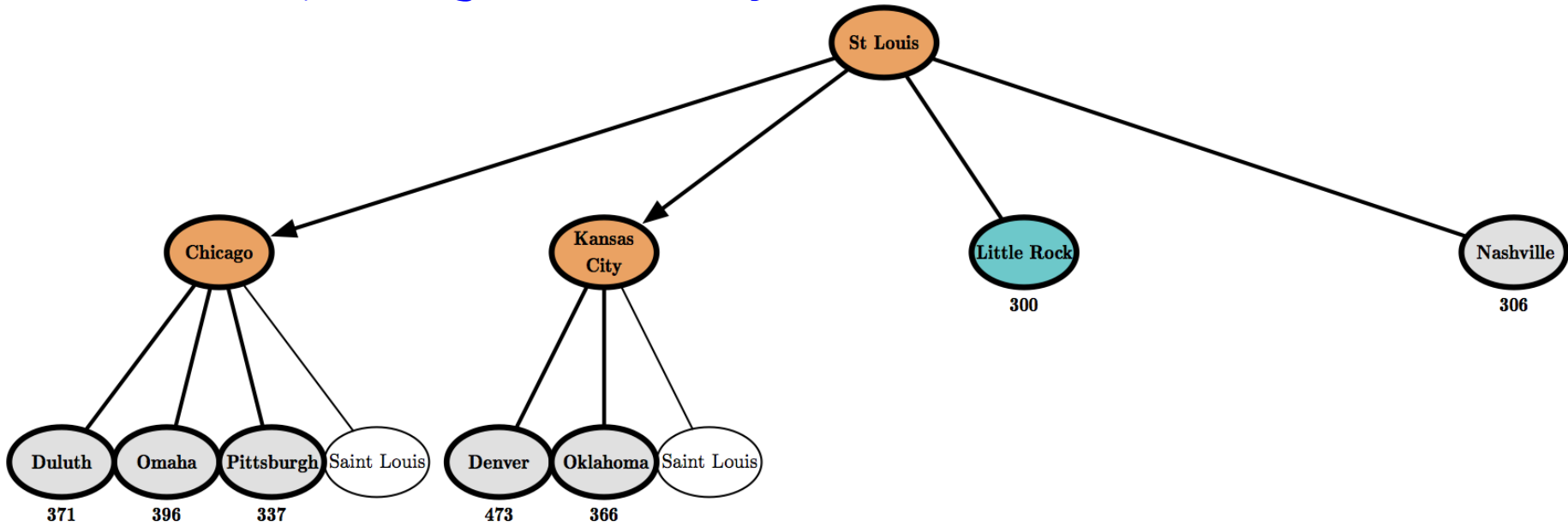
A* search example

After expanding Chicago:



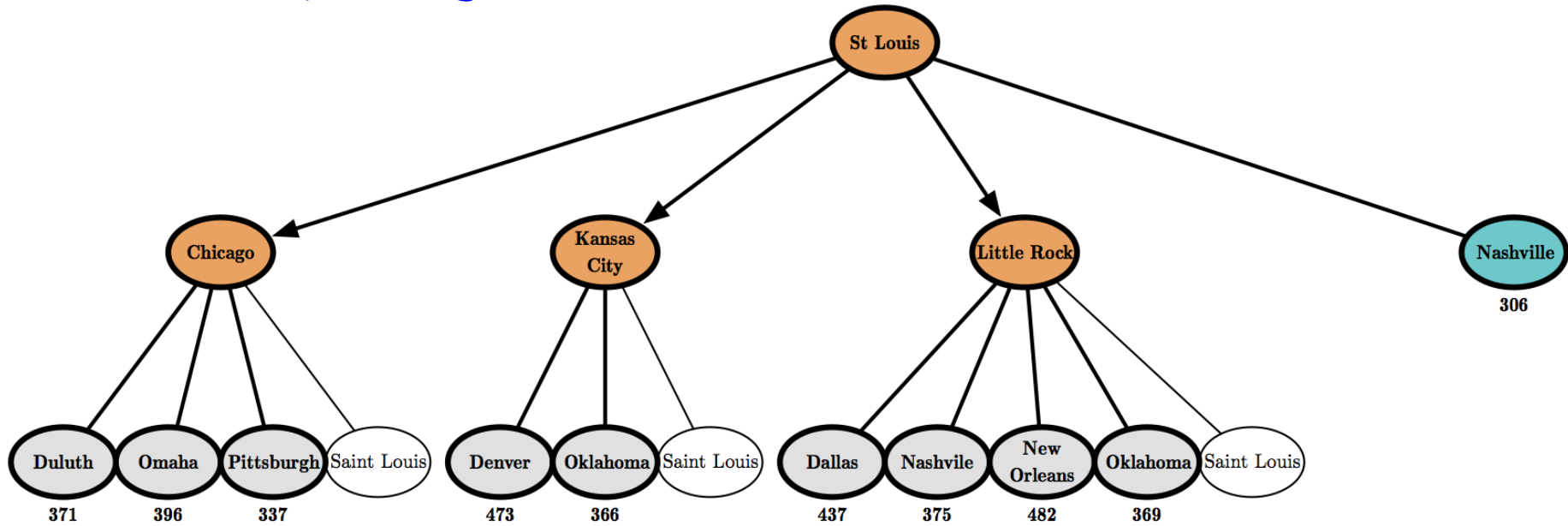
A* search example

After expanding Kansas City:



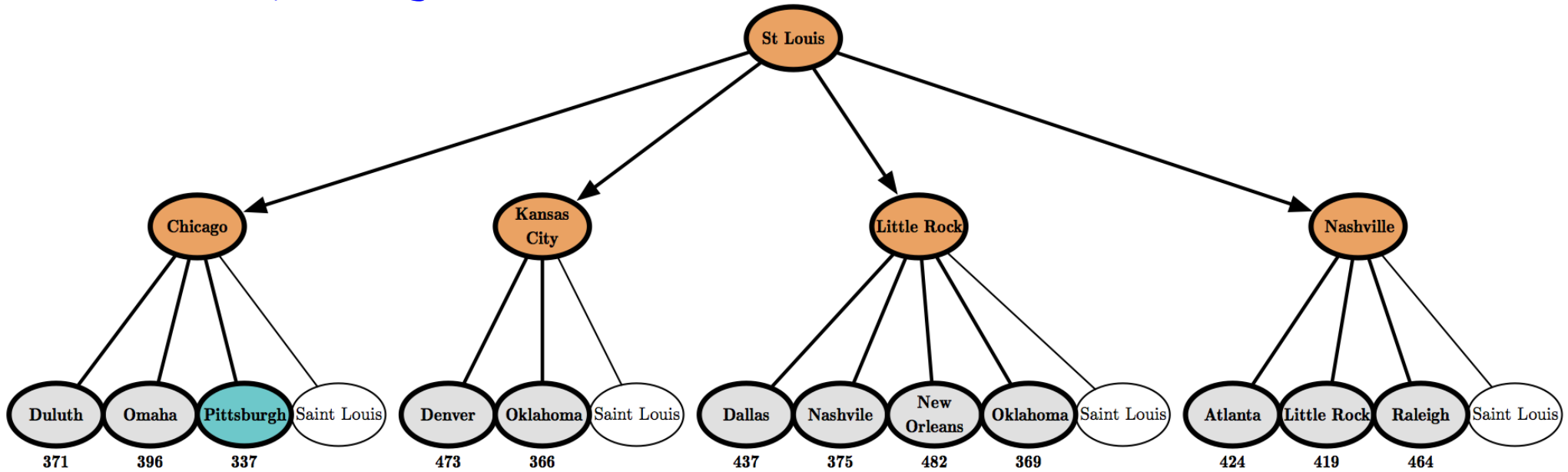
A* search example

After expanding Little Rock:



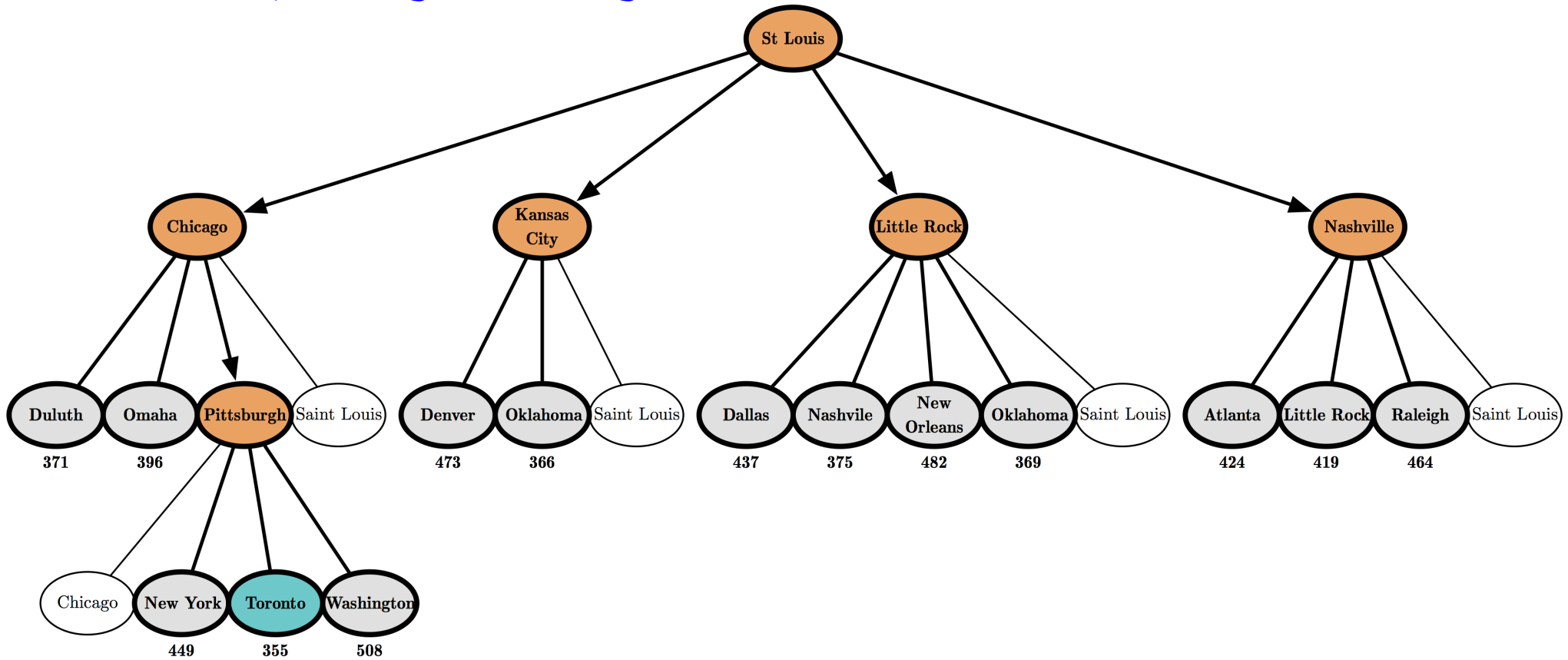
A* search example

After expanding Nashville:



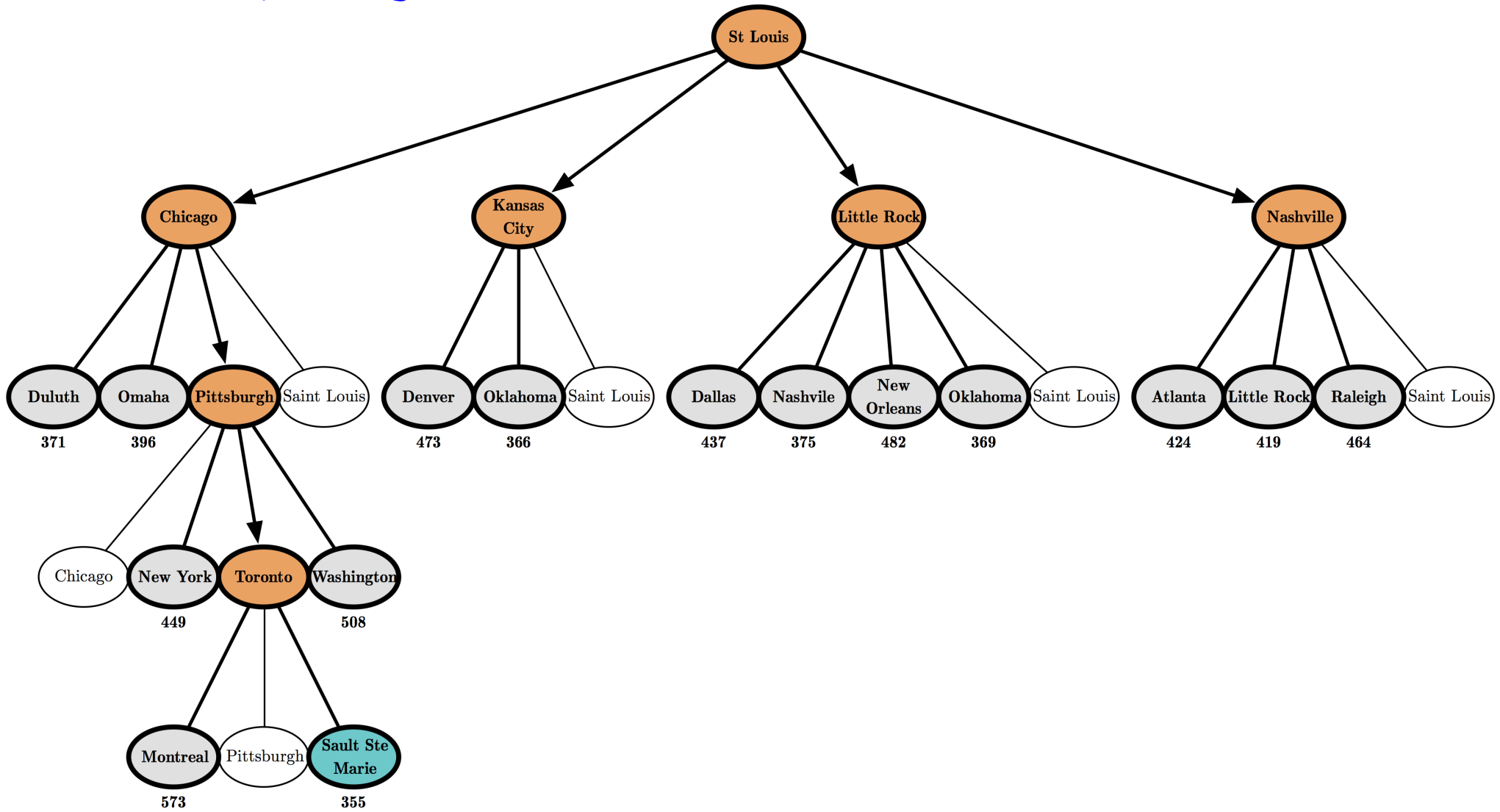
A* search example

After expanding Pittsburgh:



A* search example

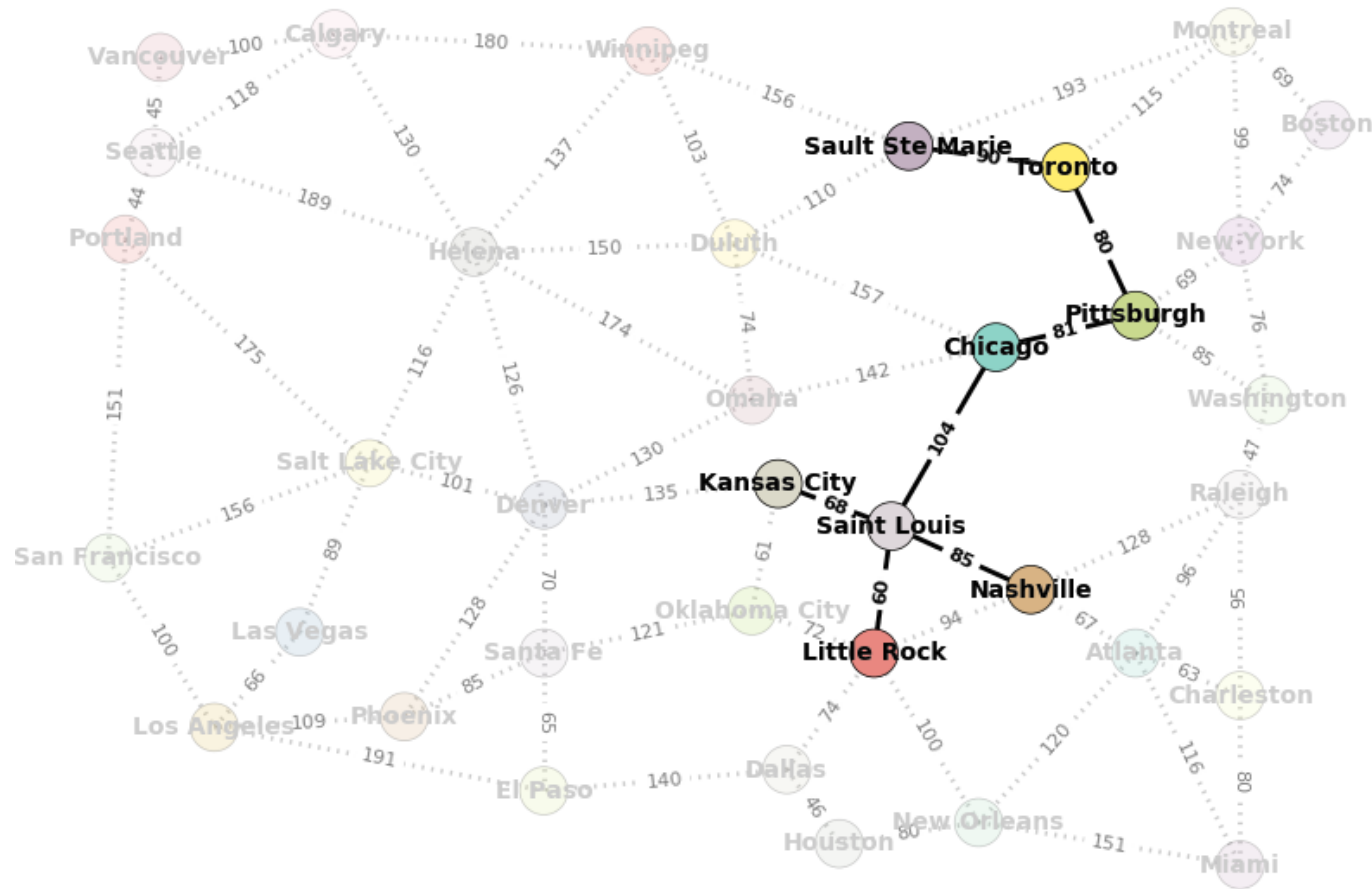
After expanding Toronto:



Examples using the map

Start: Saint Louis

Goal: Sault Ste Marie

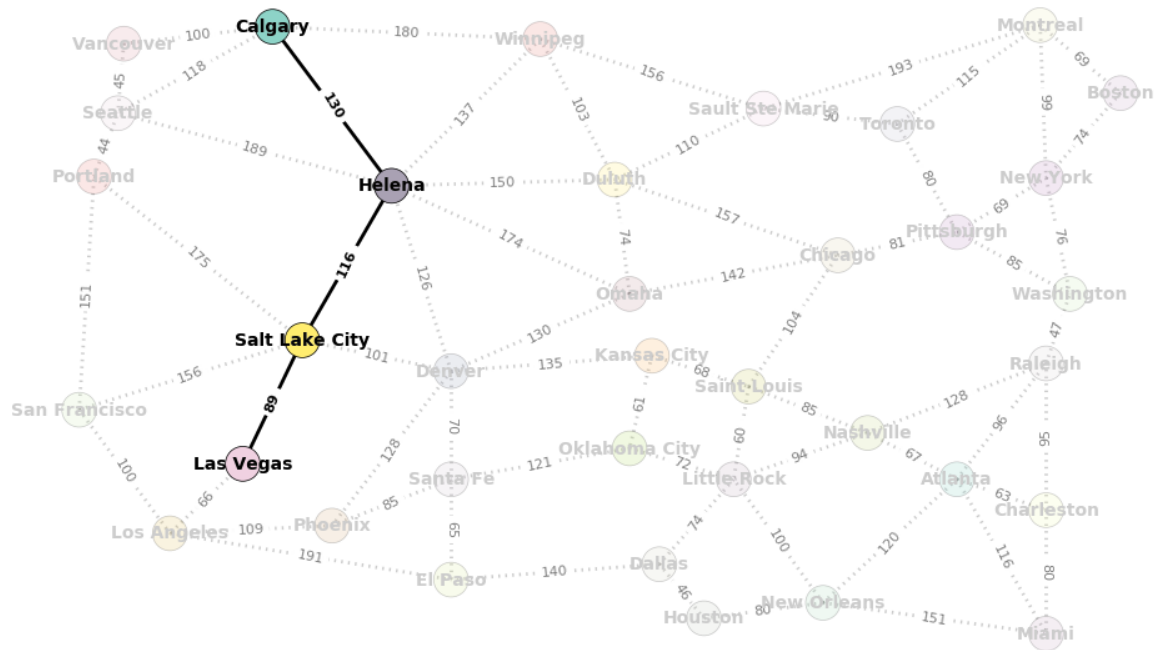


A*

Examples using the map

Start: Las Vegas

Goal: Calgary



A*

Admissible heuristics

A good heuristic can be powerful.

Only if it is of a “good quality”

Admissible heuristics

A good heuristic can be powerful.

Only if it is of a “good quality”

A good heuristic must be admissible.

Admissible heuristics

- An **admissible** heuristic never overestimates the cost to reach the goal, that is it is **optimistic**

- A heuristic h is admissible if

$$\forall \text{ node } n, h(n) \leq h^*(n)$$

where h^* is true cost to reach the goal from n .

- h_{SLD} (used as a heuristic in the map example) is admissible because it is by definition the shortest distance between two points.

A* Optimality

If $h(n)$ is admissible, A* using tree search is optimal.

A* Optimality

If $h(n)$ is admissible, A* using tree search is optimal.

Rationale:

- Suppose G_o is the optimal goal.
- Suppose G_s is some suboptimal goal.
- Suppose n is an unexpanded node in the fringe such that n is on the shortest path to G_o .
- $f(G_s) = g(G_s)$ since $h(G_s) = 0$
 $f(G_o) = g(G_o)$ since $h(G_o) = 0$
 $f(G_s) > g(G_o)$ since G_s is suboptimal
Then $f(G_s) > f(G_o) \dots (1)$
- $h(n) \leq h^*(n)$ since h is admissible
 $g(n) + h(n) \leq g(n) + h^*(n) = g(G_o) = f(G_o)$
Then, $f(n) \leq f(G_o) \dots (2)$
From (1) and (2) $f(G_s) > f(n)$
so A* will never select G_s during the search and hence A* is optimal.

A* search criteria

- **Complete**: Yes
- **Time**: exponential
- **Space**: keeps every node in memory, the biggest problem
- **Optimal**: Yes!

Heuristics

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- The solution is 26 steps long.
- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance (sum of the horizontal and vertical distances).
- $h_1(n) = 8$
- Tiles 1 to 8 in the start state gives: $h_2 = 3 + 1 + 2 + 2 + 2 + 3 + 3 + 2 = 18$ which does not overestimate the true solution.

Search Algorithms: Recap

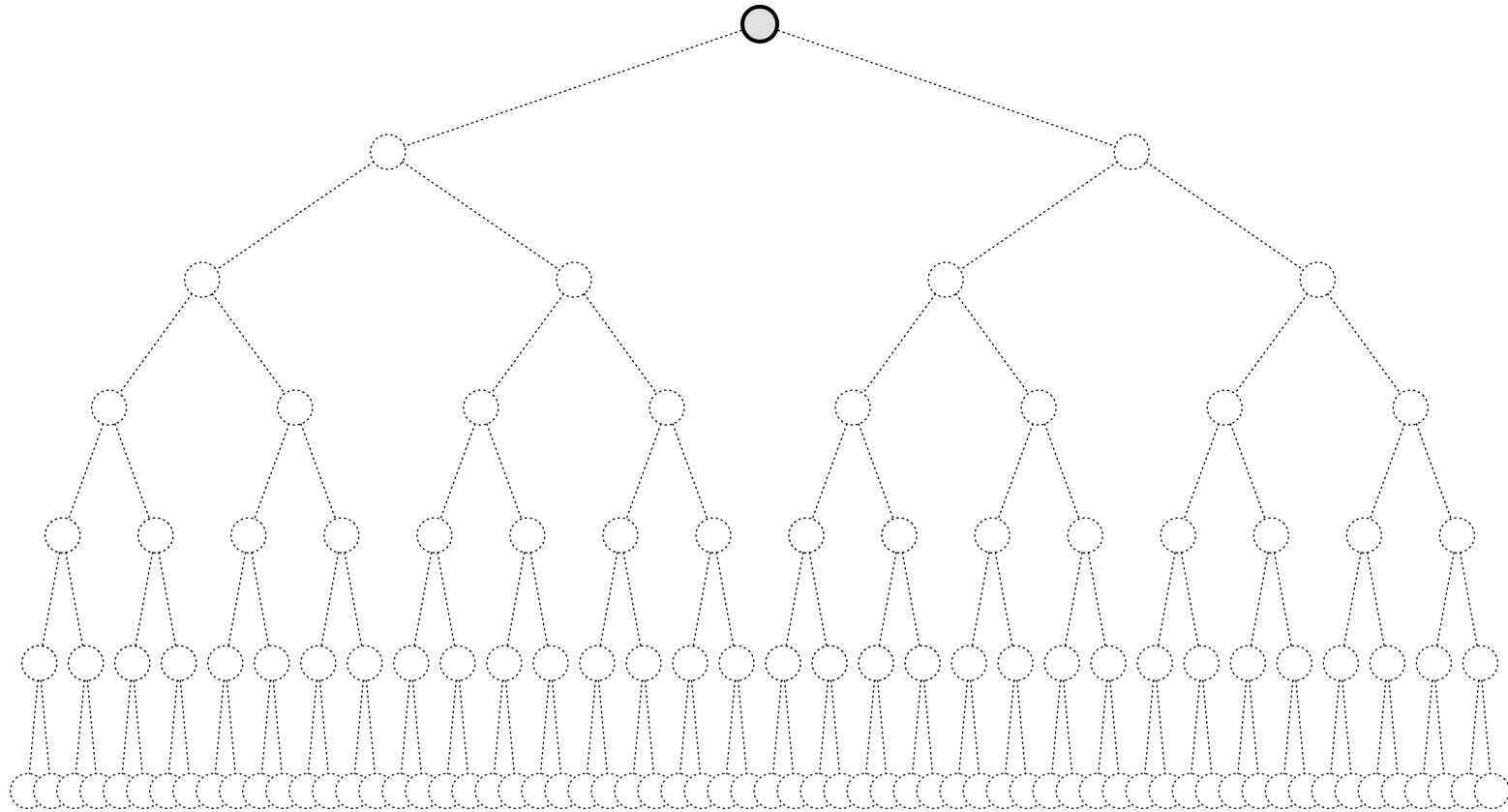
- **Uninformed Search:** Use no domain knowledge.

BFS, DFS, DLS, IDS, UCS.

- **Informed Search:** Use a heuristic function that estimates how close a state is to the goal.

Greedy search, A*, IDA*.

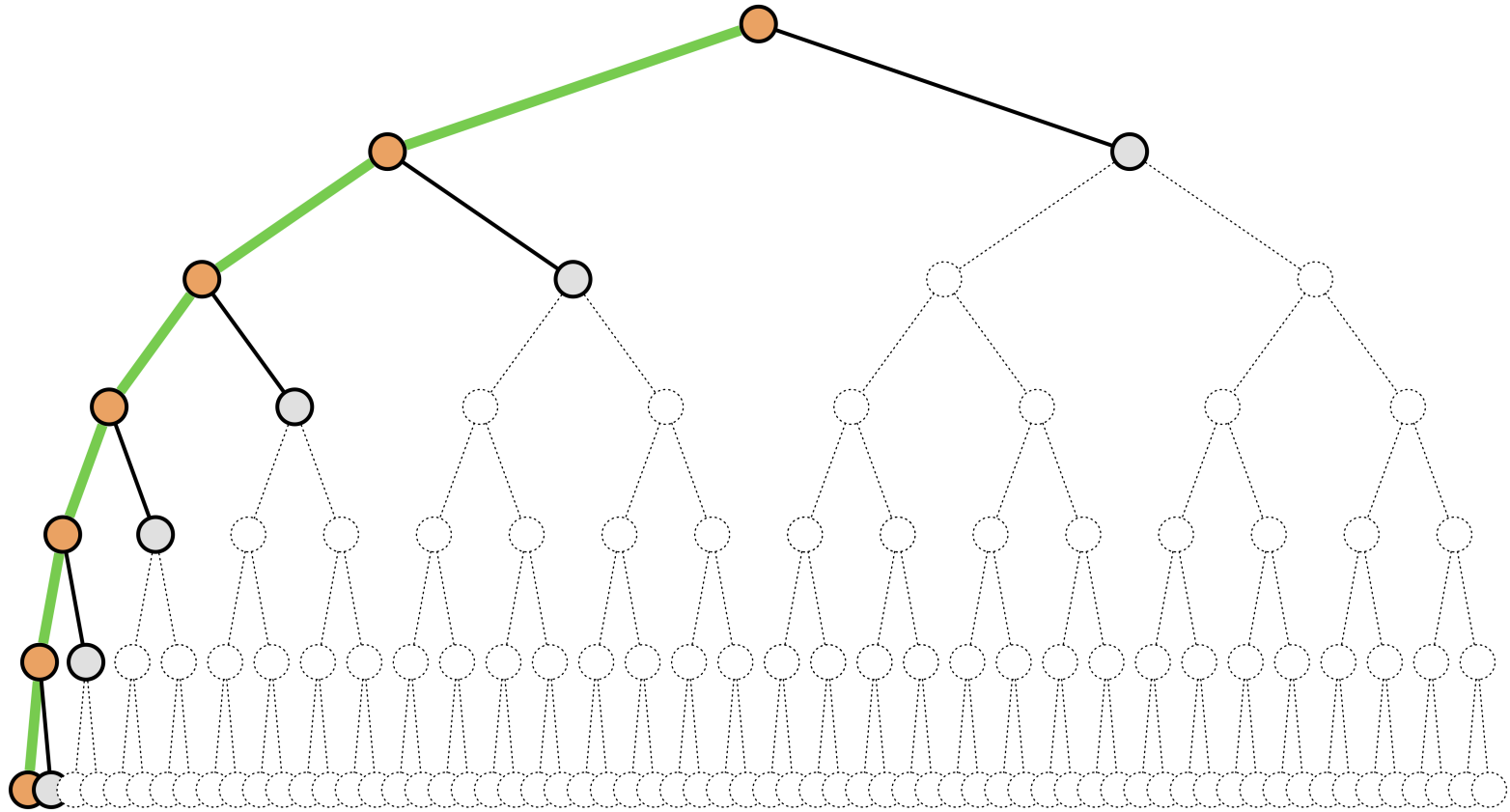
DFS



Searches branch by branch ...

... with each branch pursued to maximum depth.

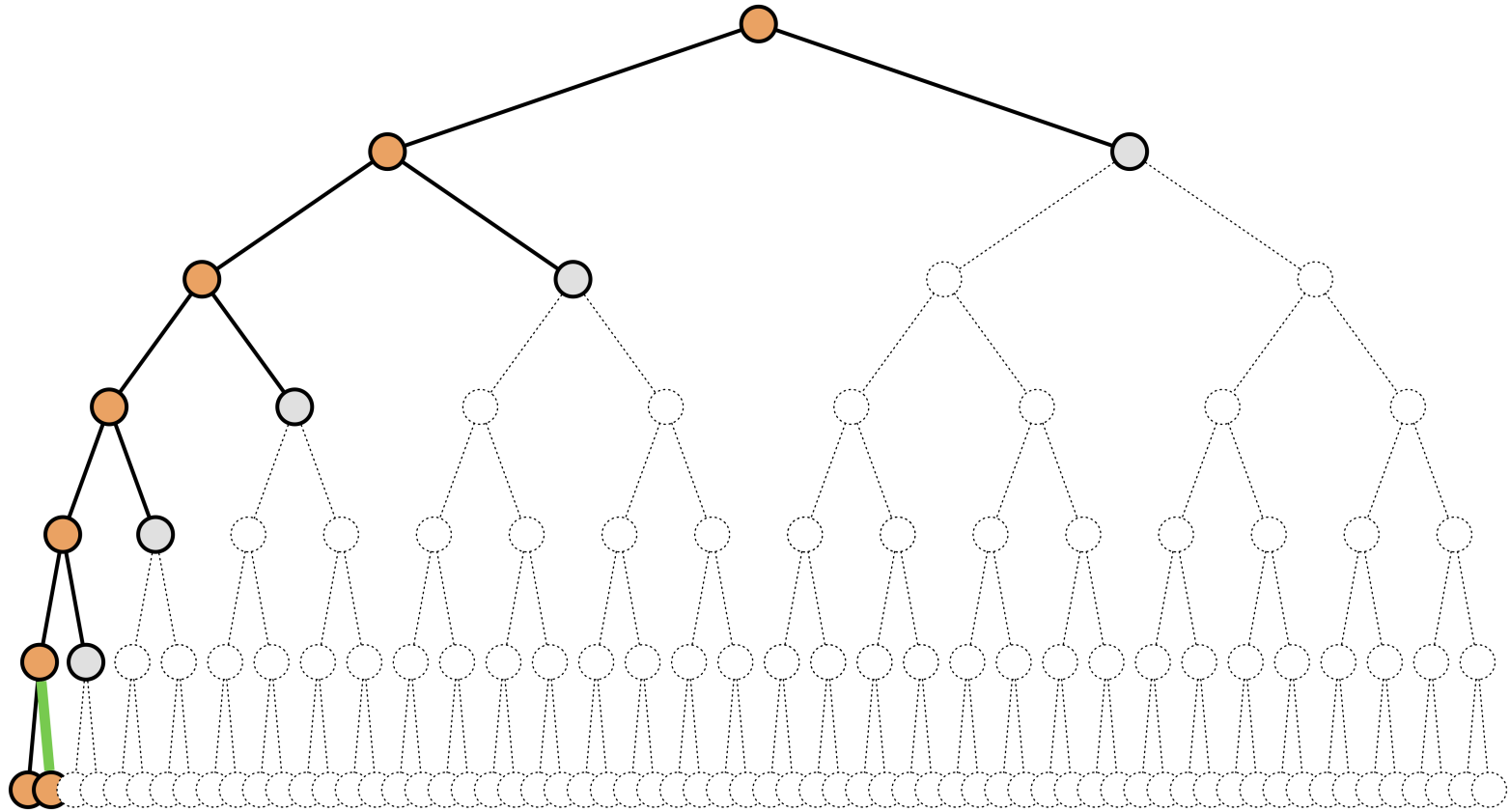
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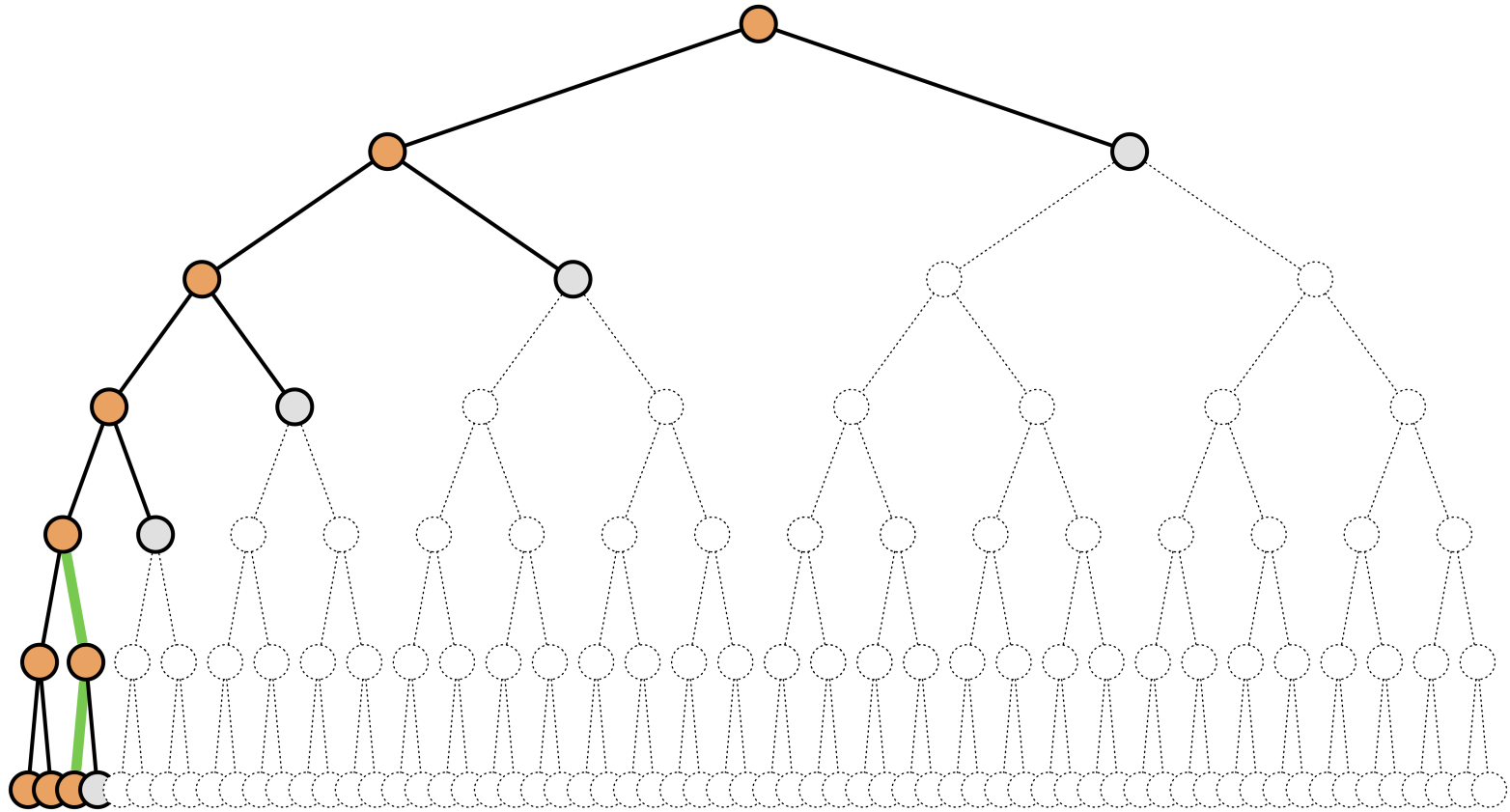
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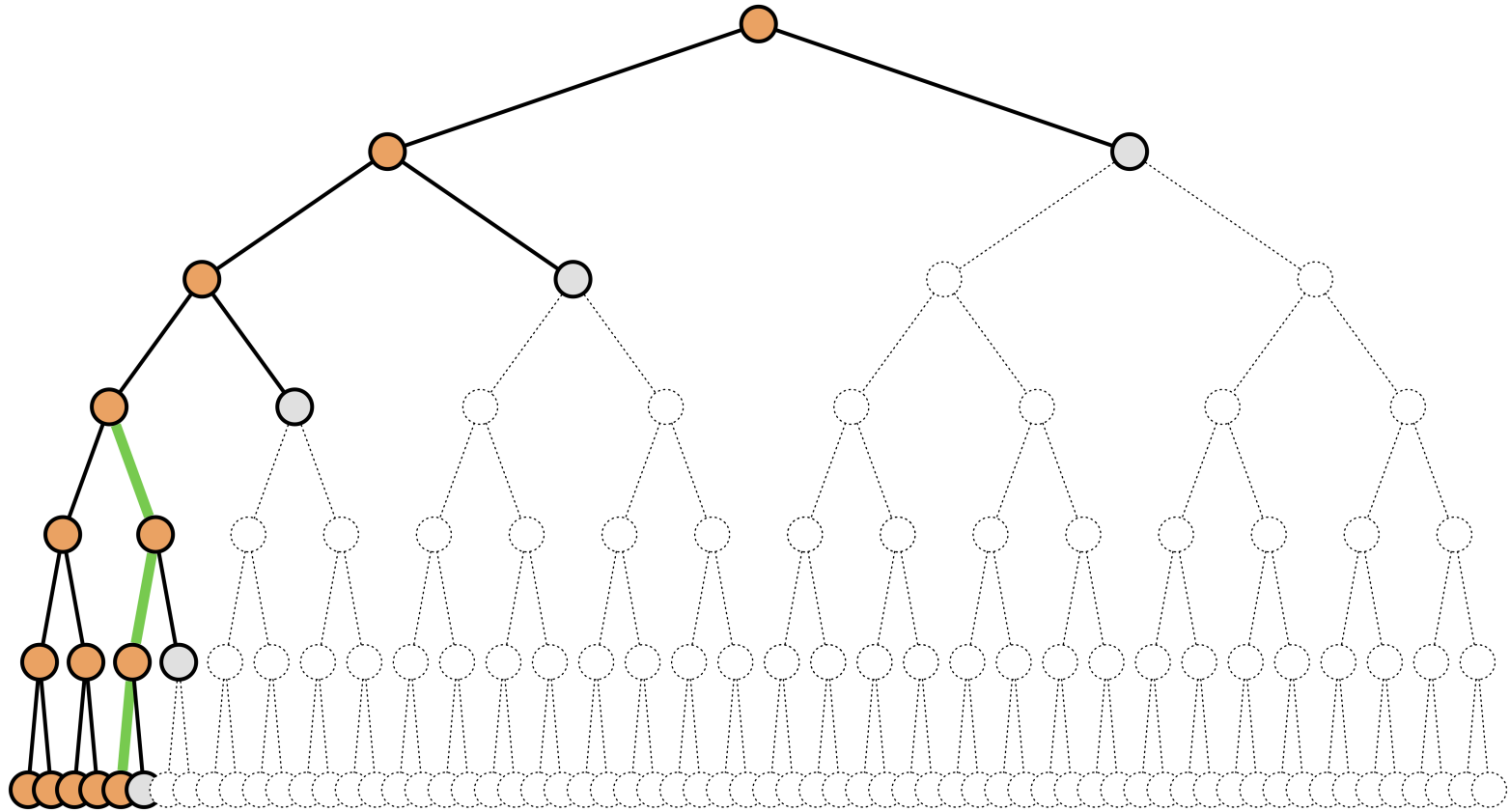
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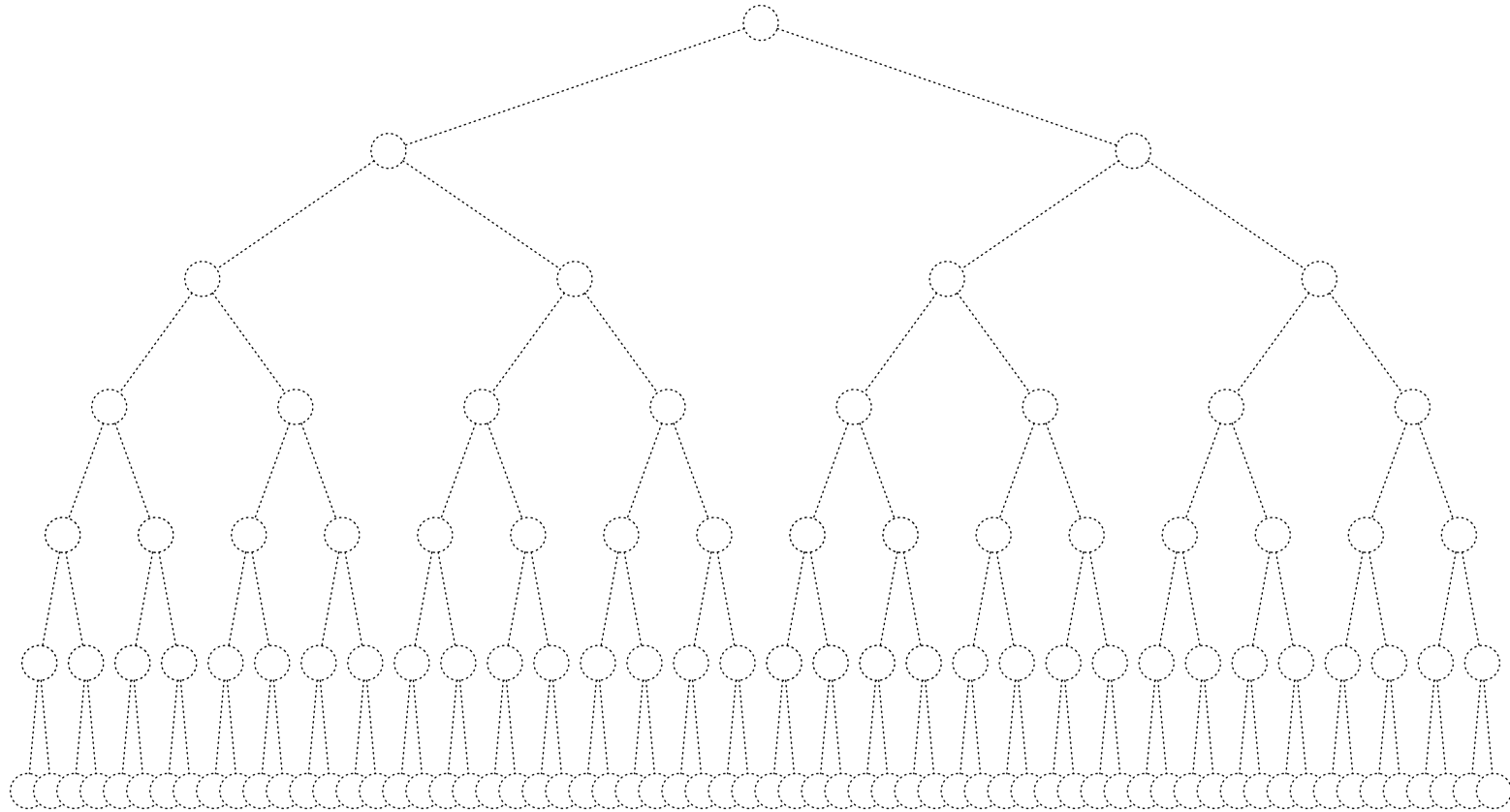
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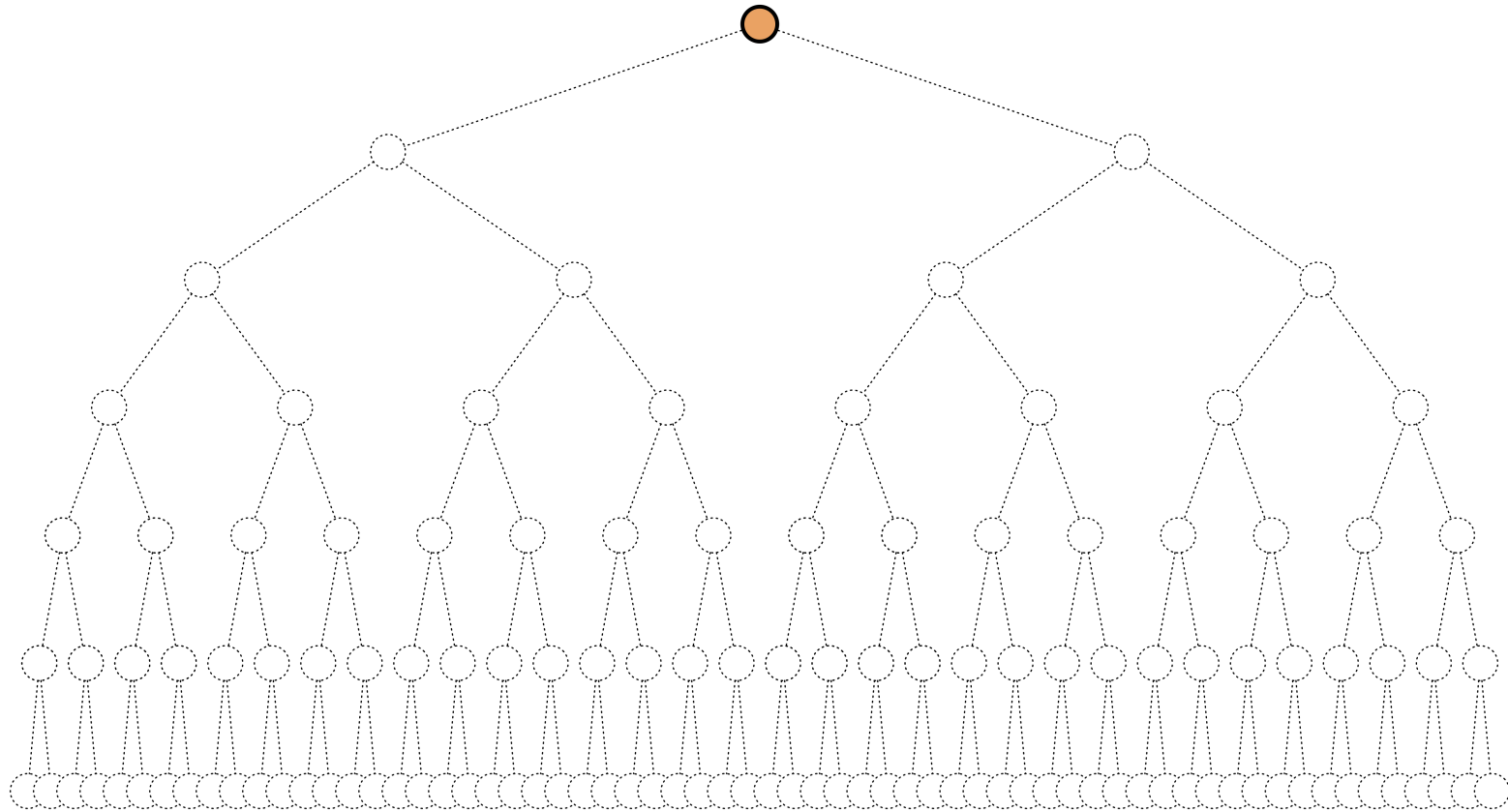
IDS



Searches subtree by subtree ...

... with each subtree increasing by depth limit.

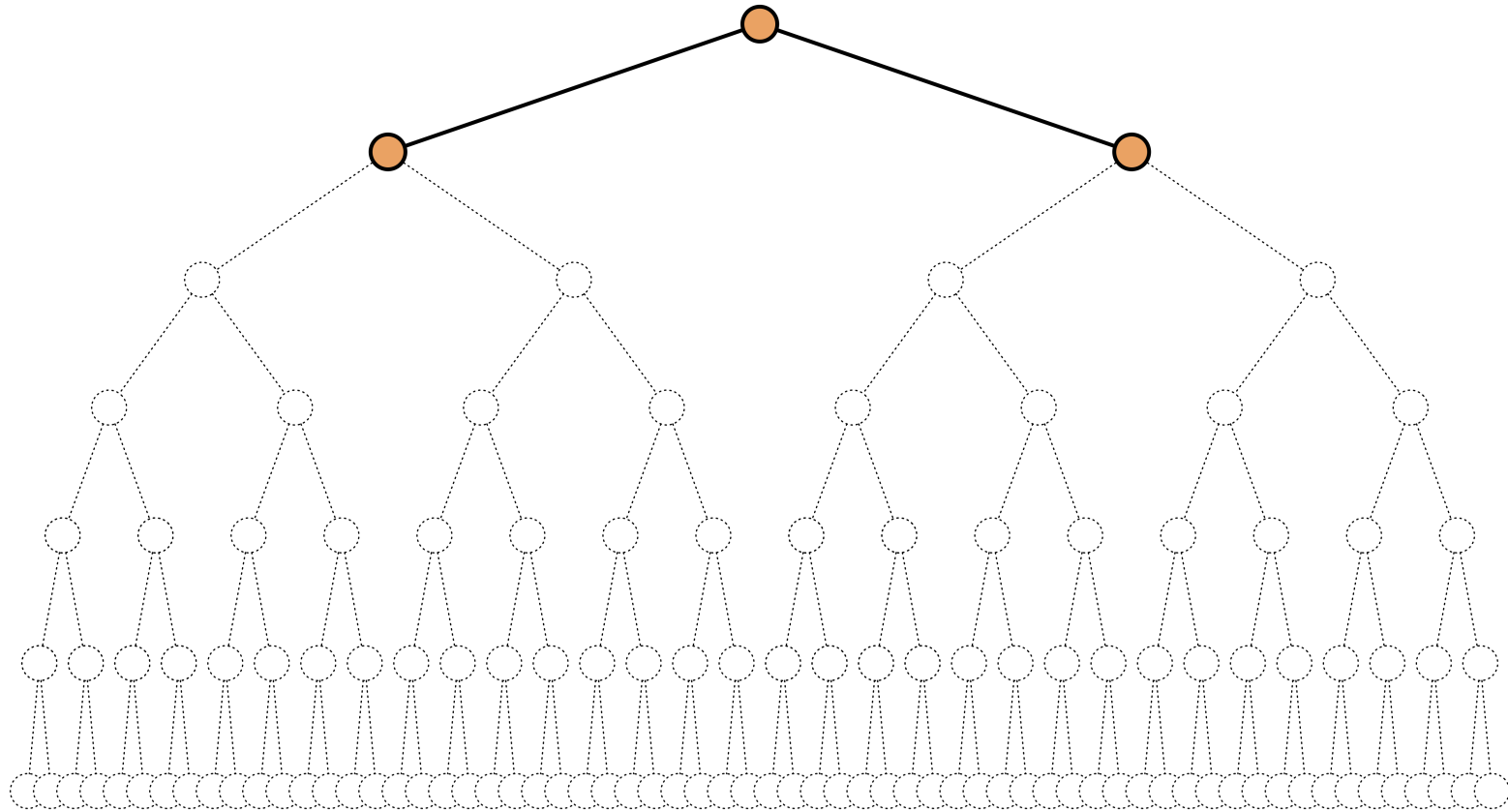
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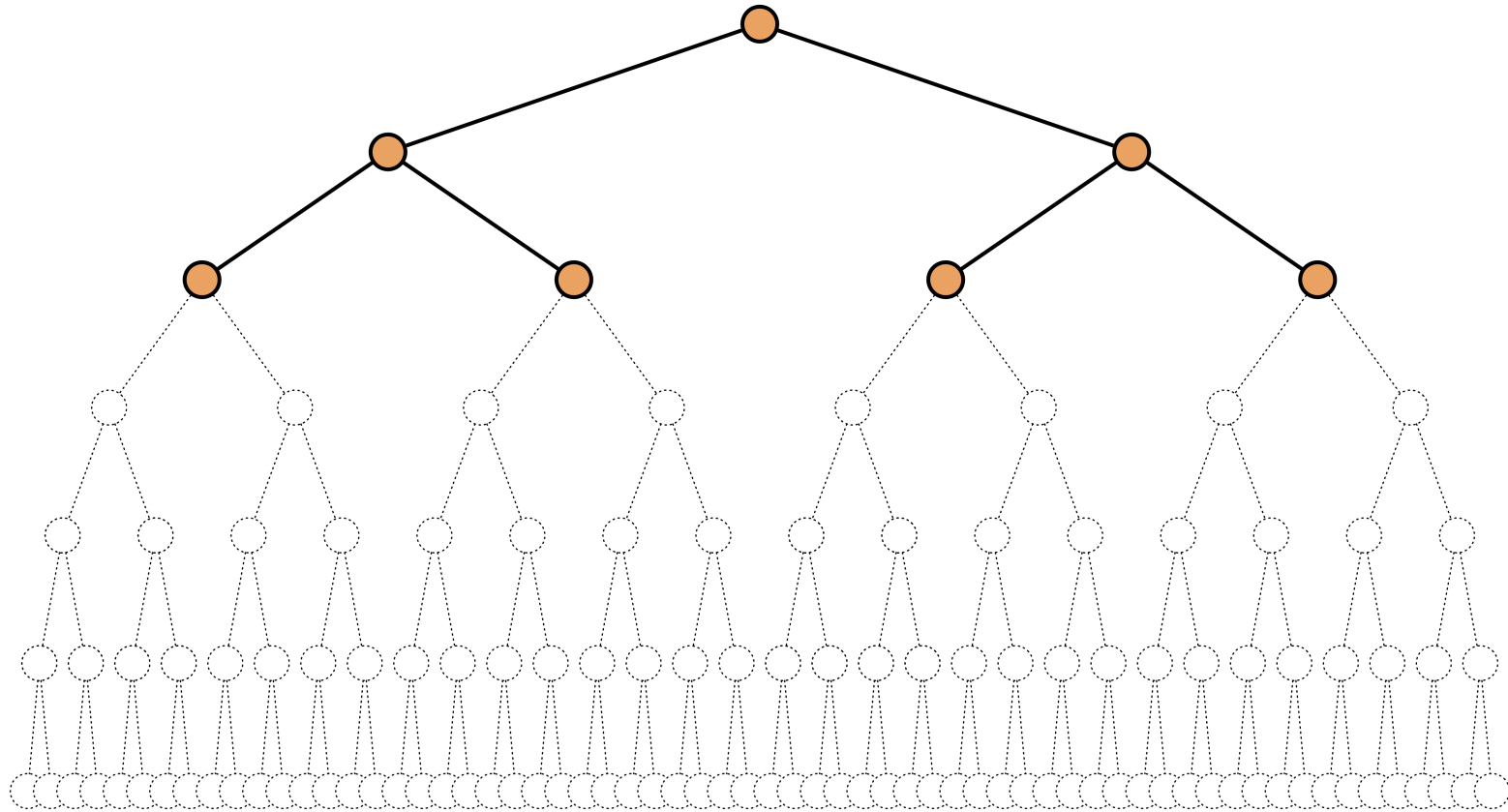
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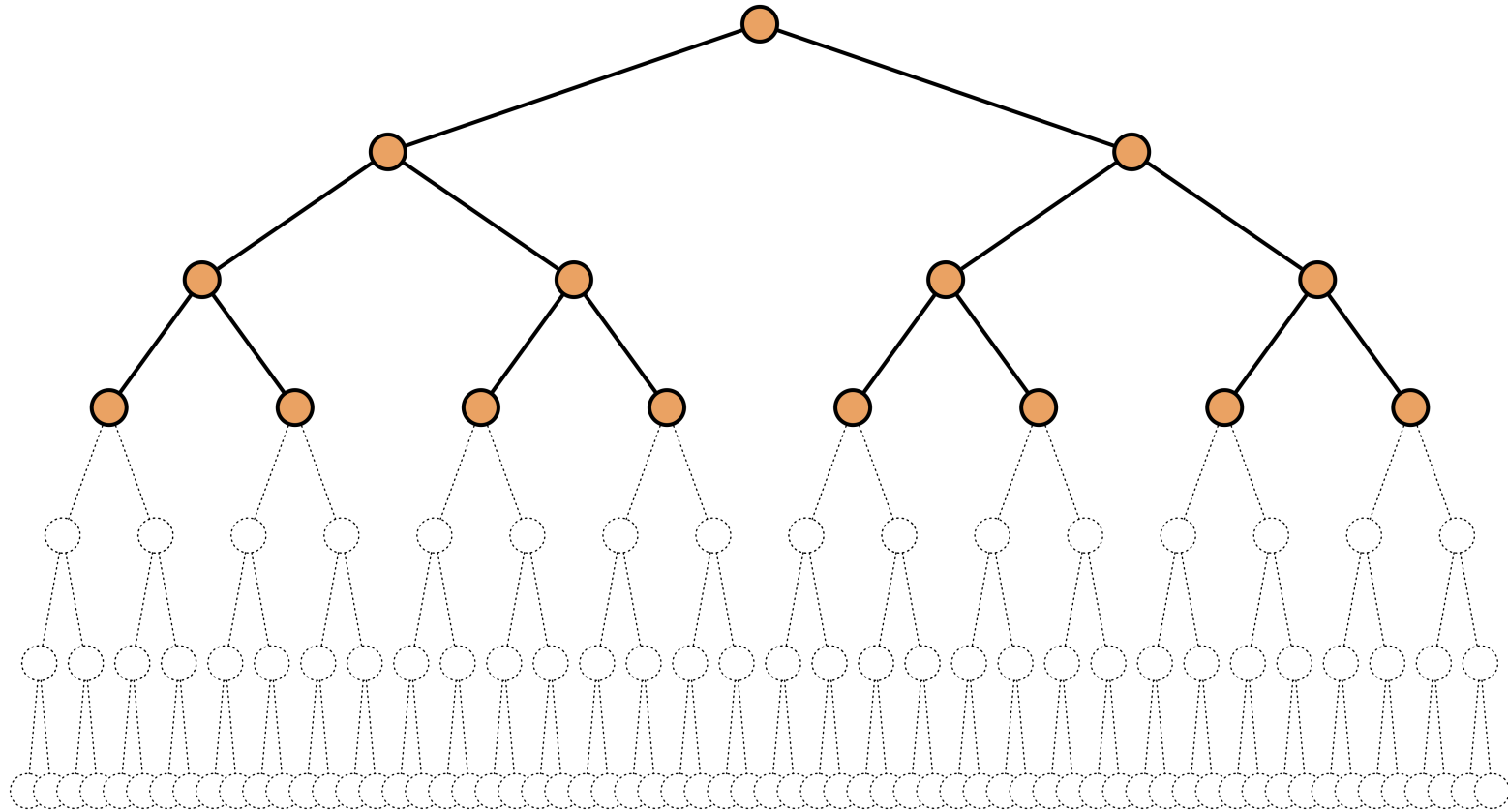
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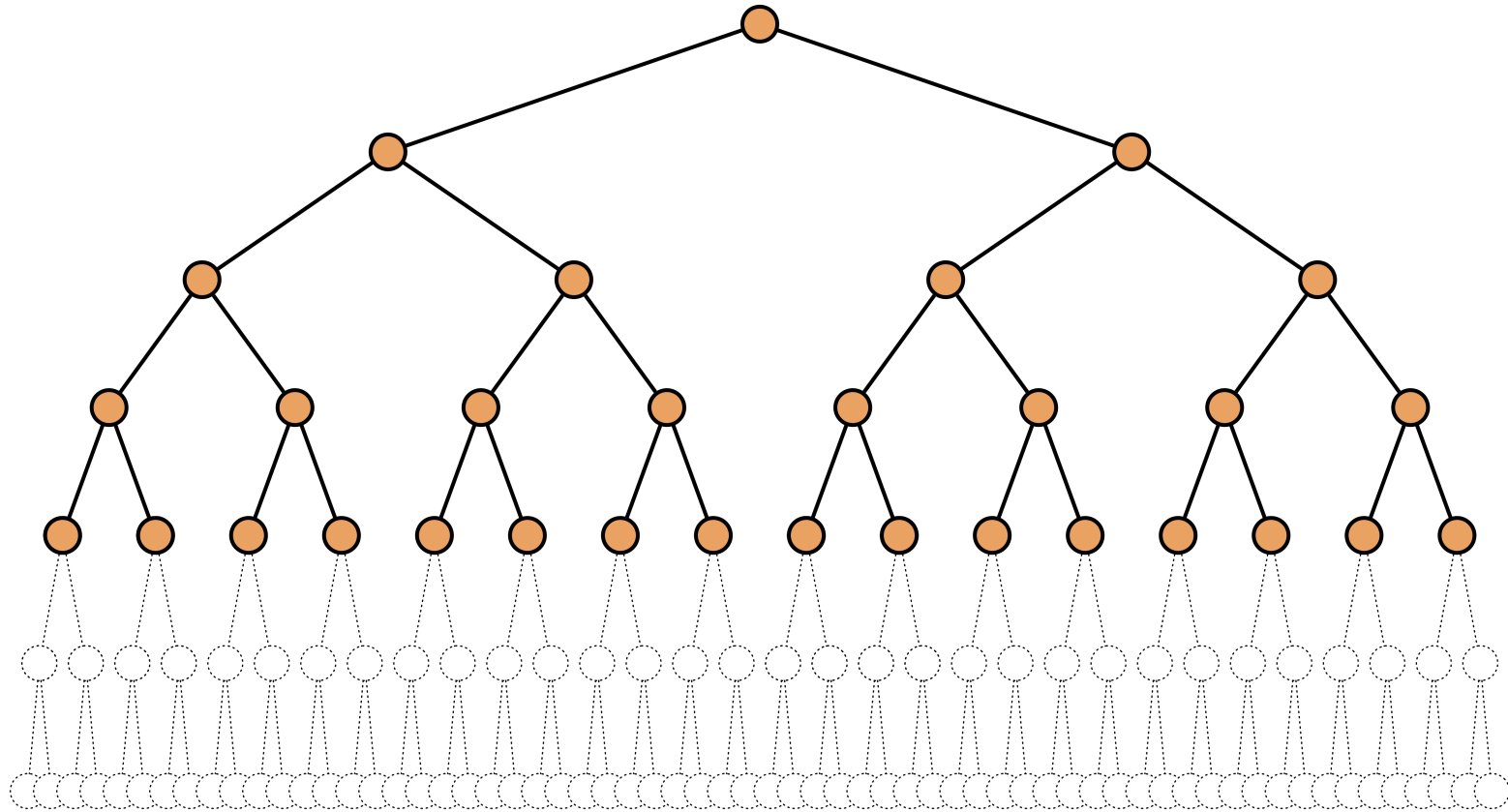
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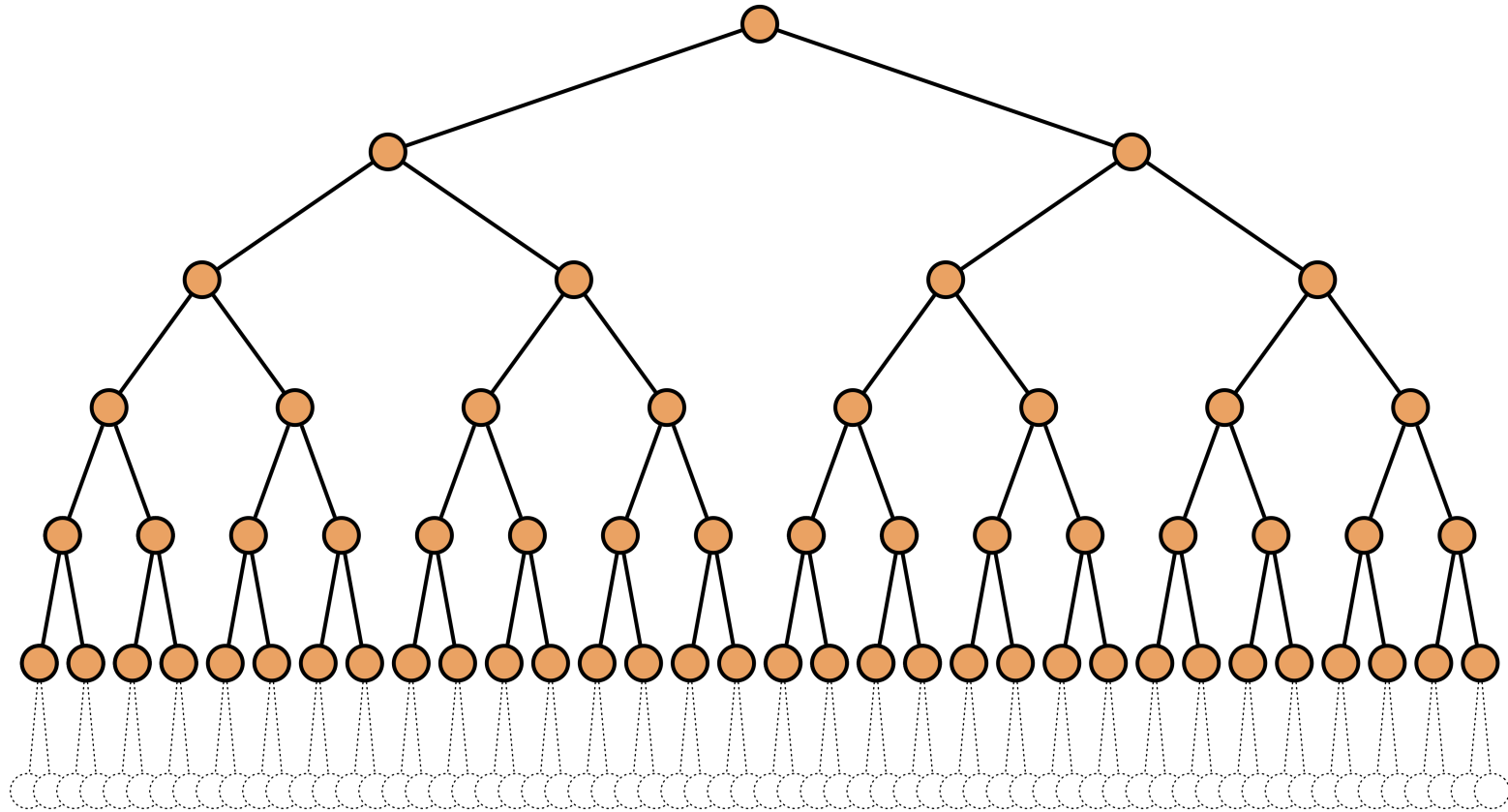
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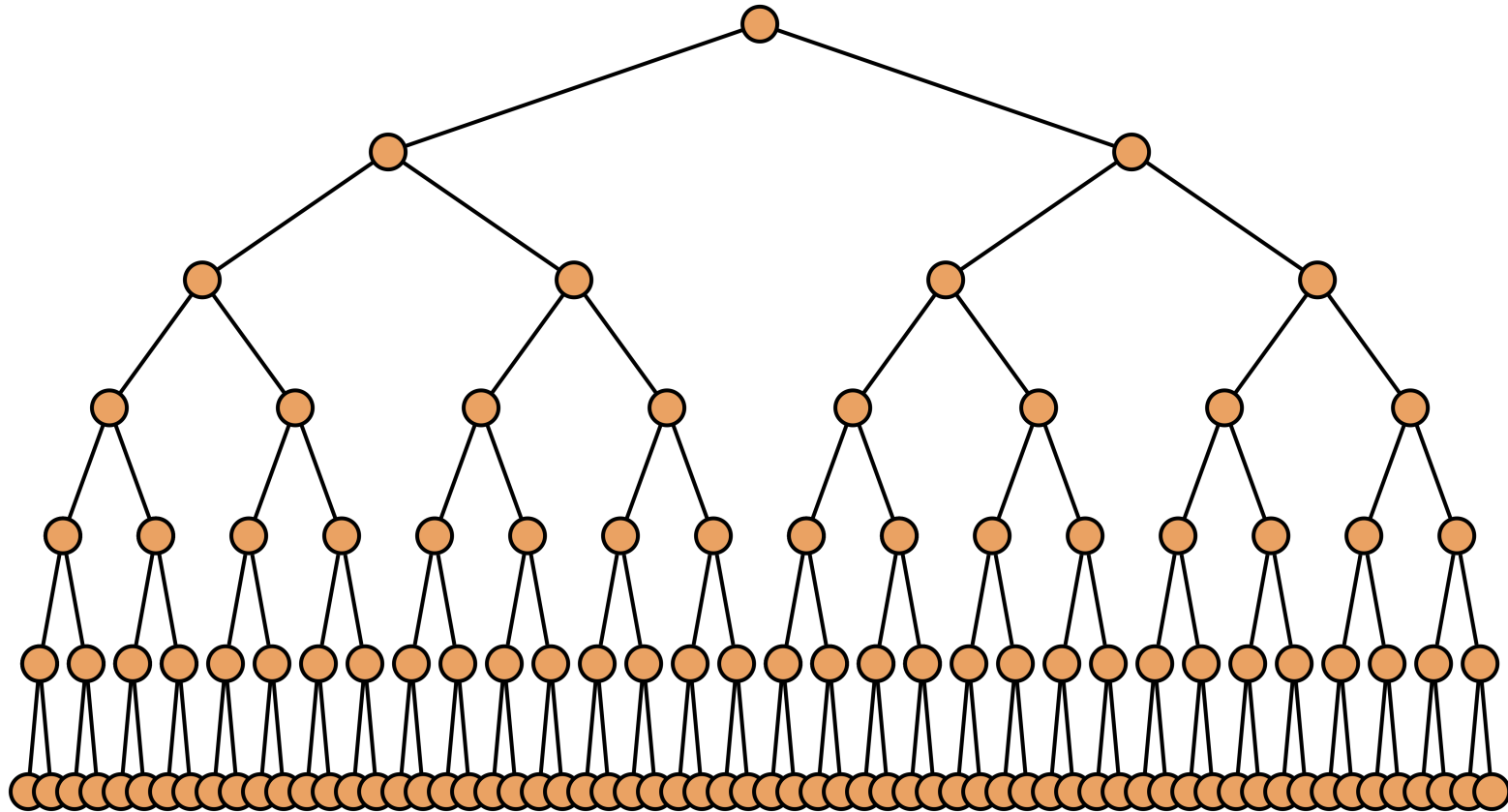
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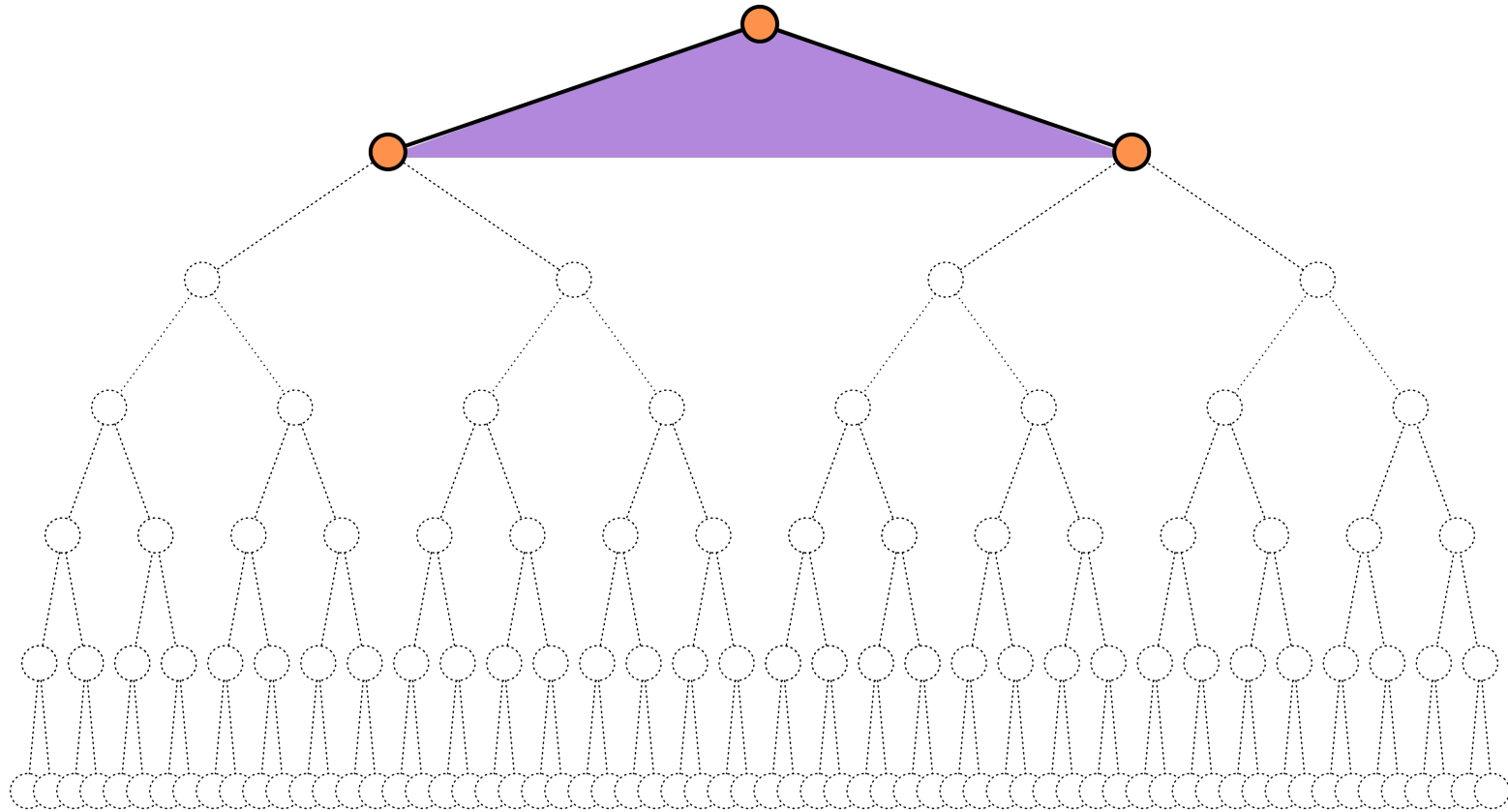
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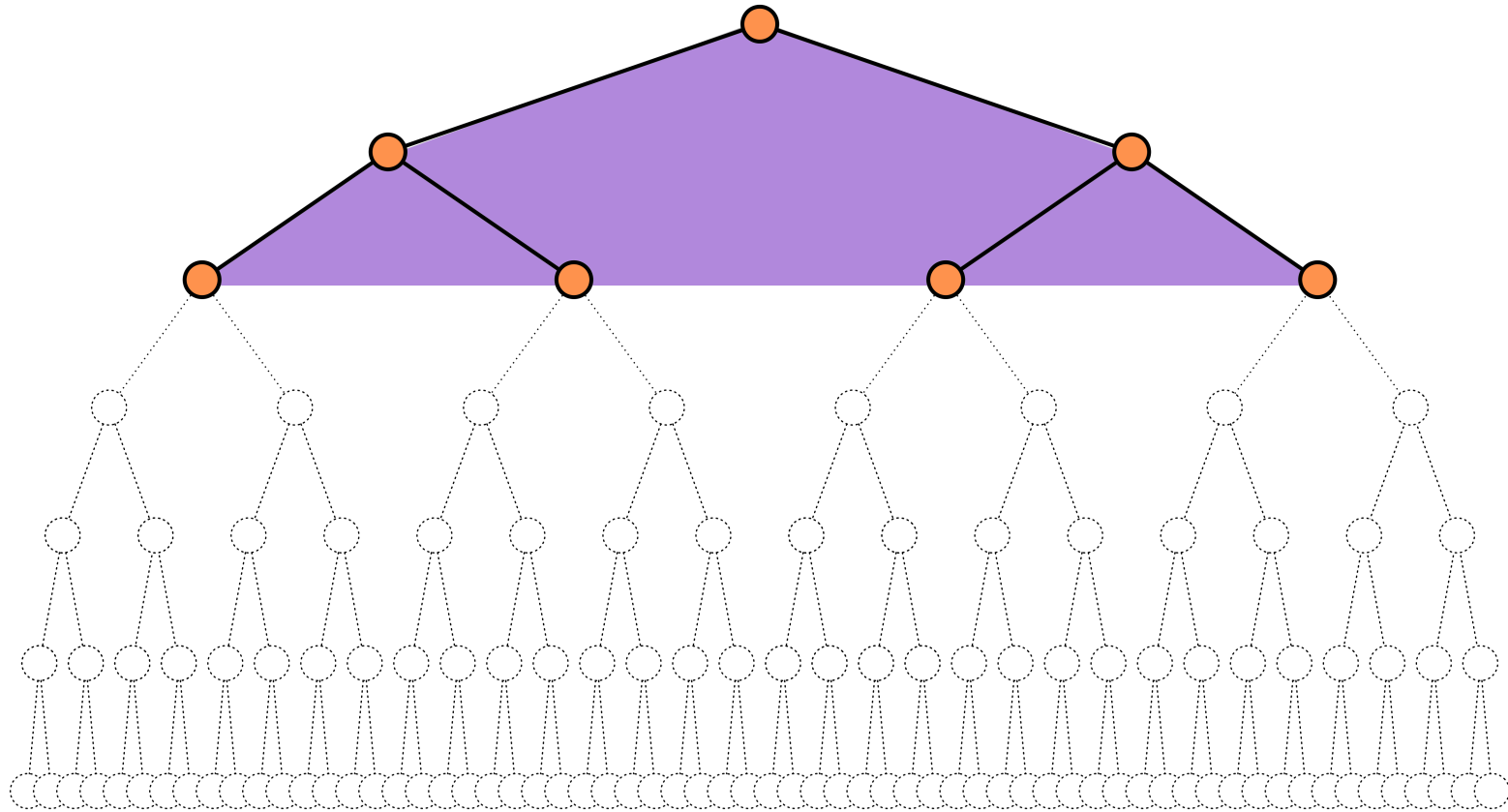
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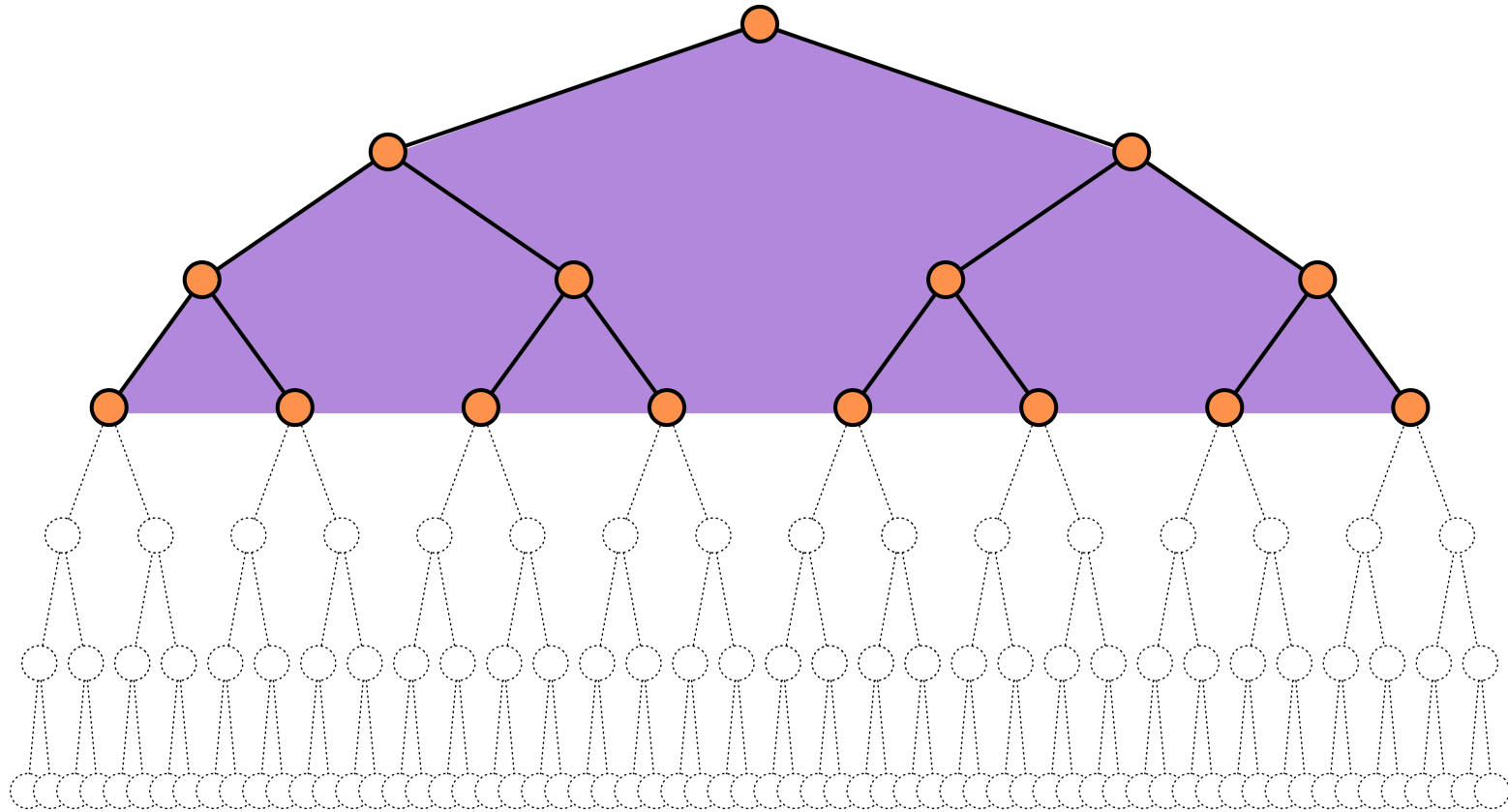
Each iteration is simply an instance of depth-limited search.
Search proceeds by exhausting larger and larger subtrees.

IDS



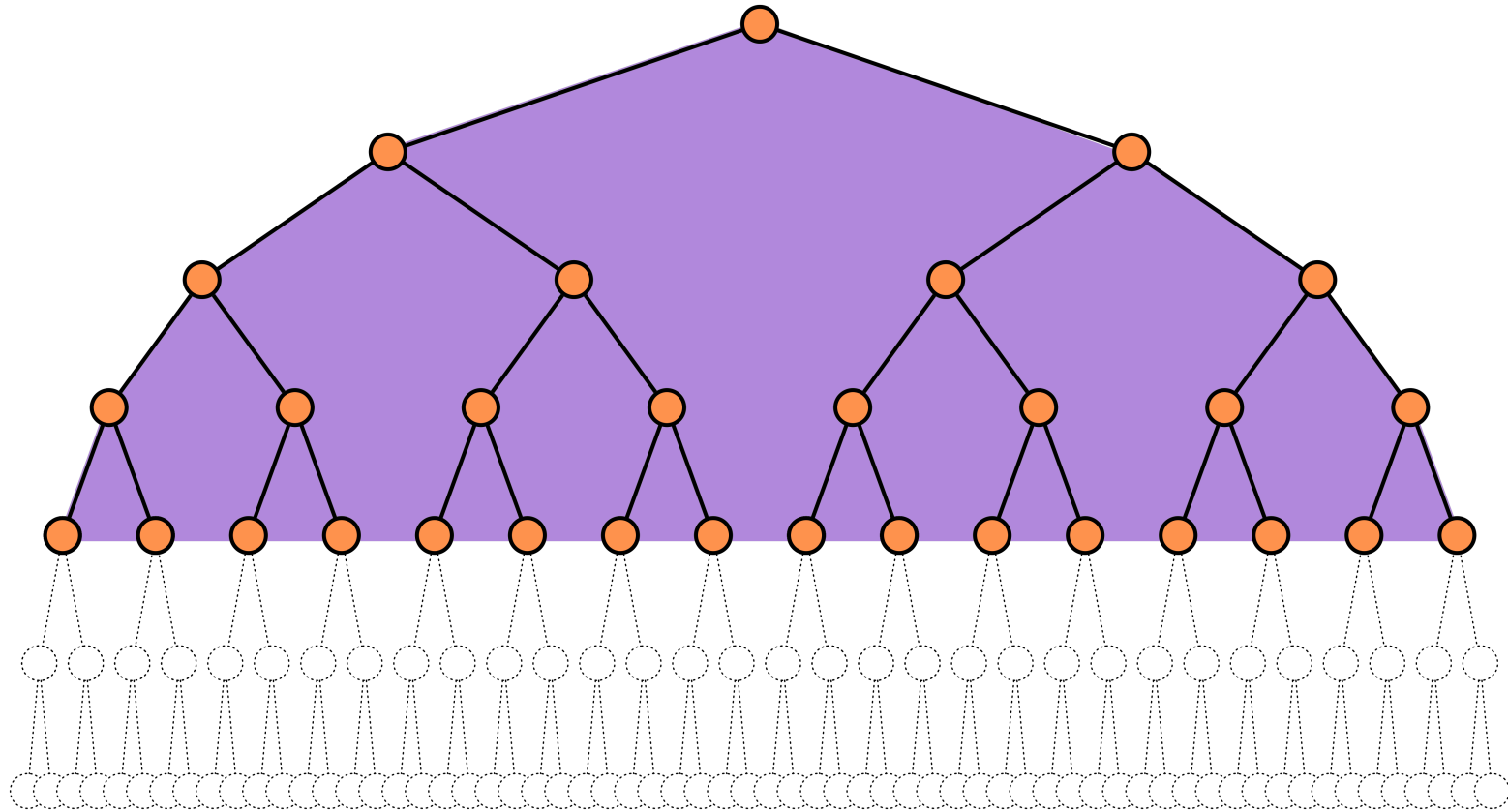
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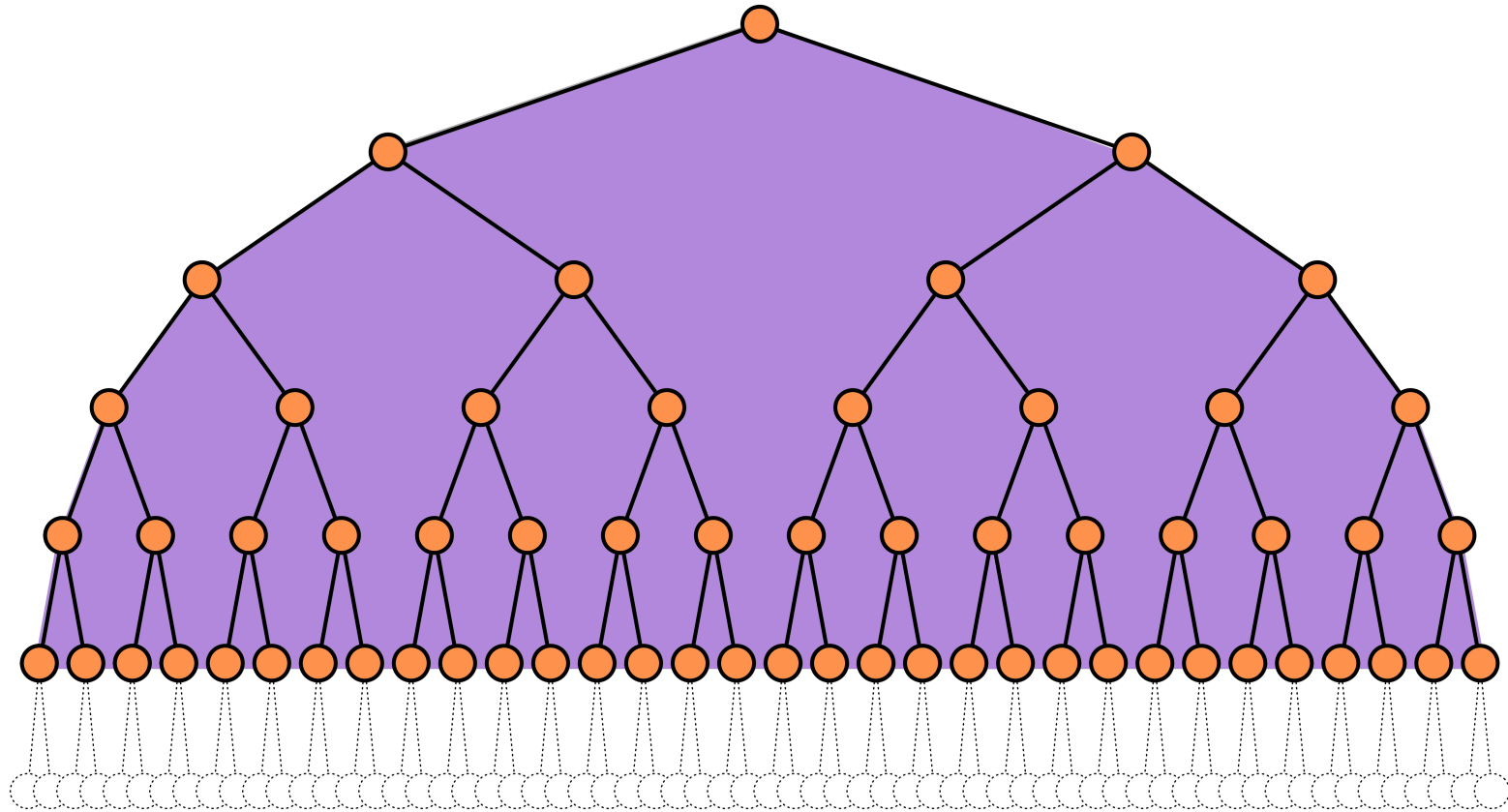
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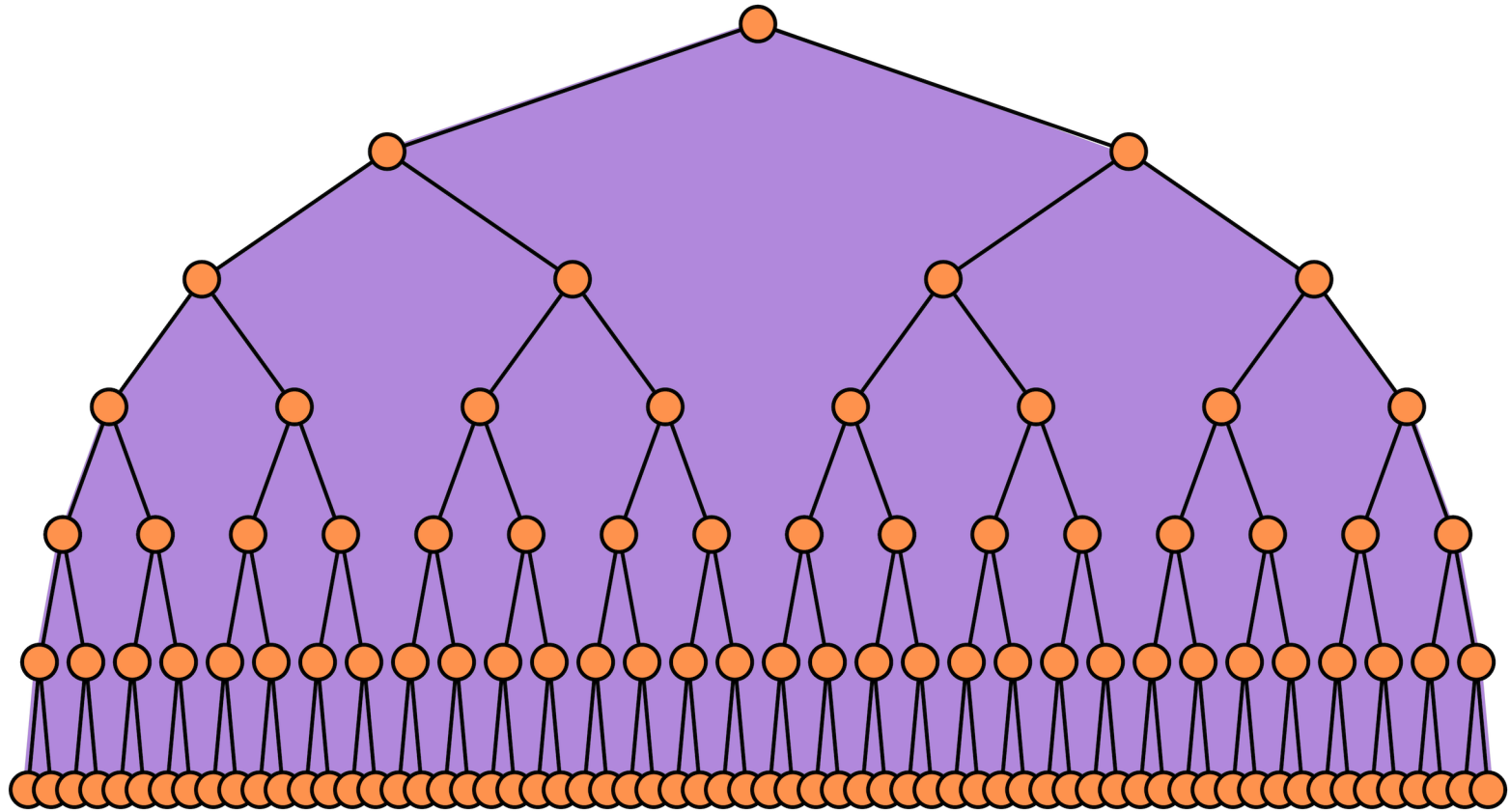
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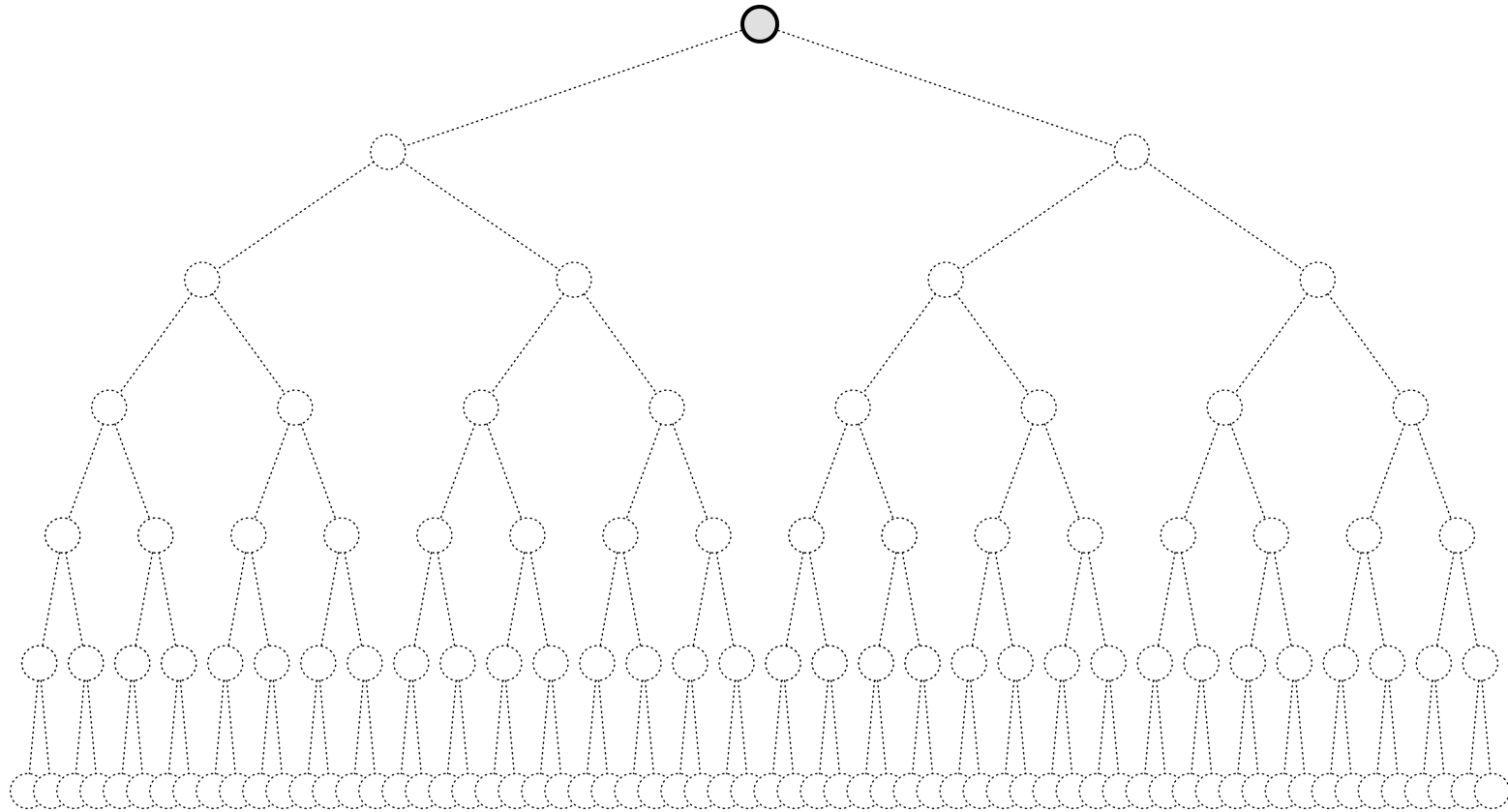
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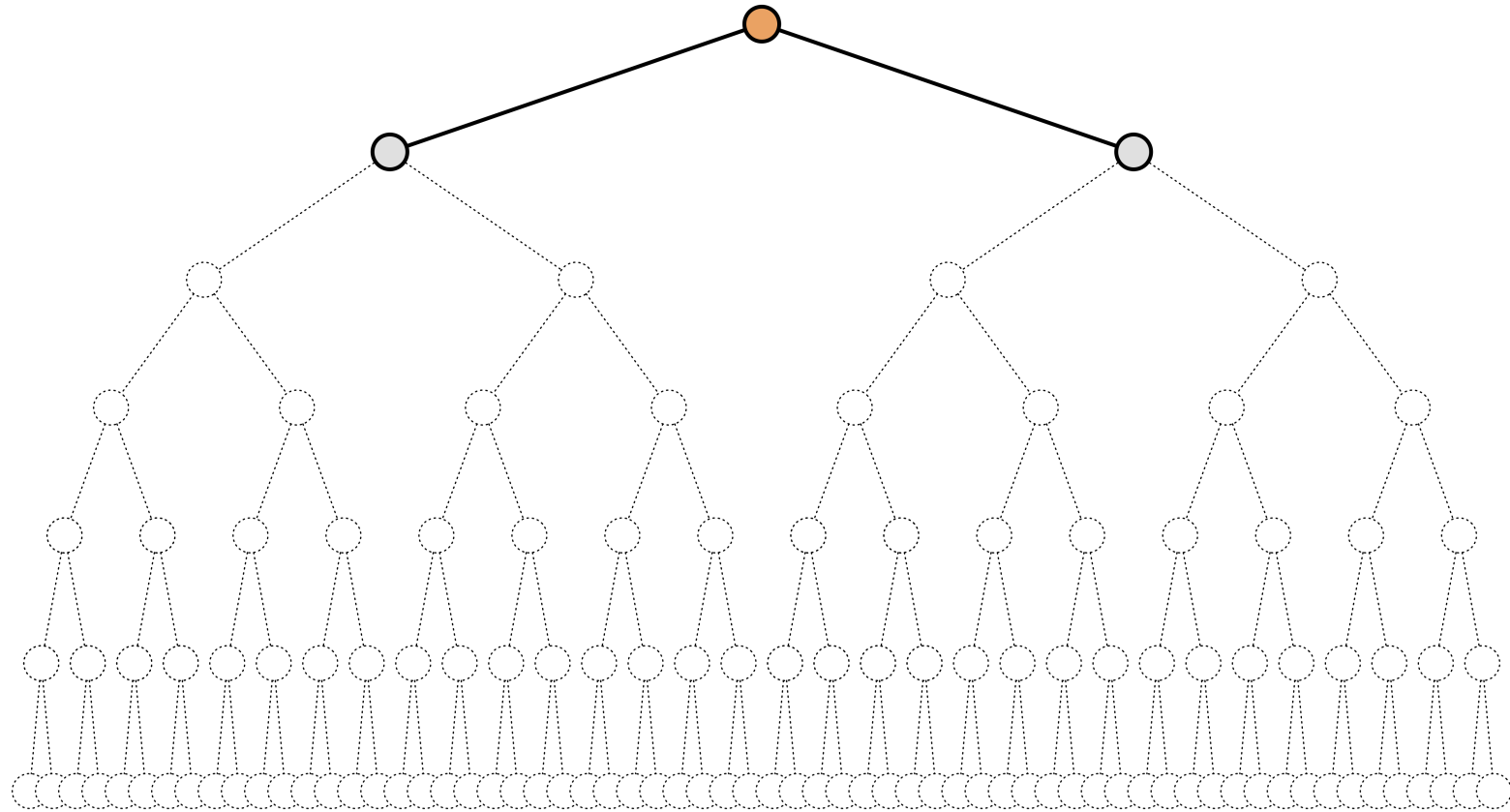
BFS



Searches layer by layer ...

... with each layer organized by node depth.

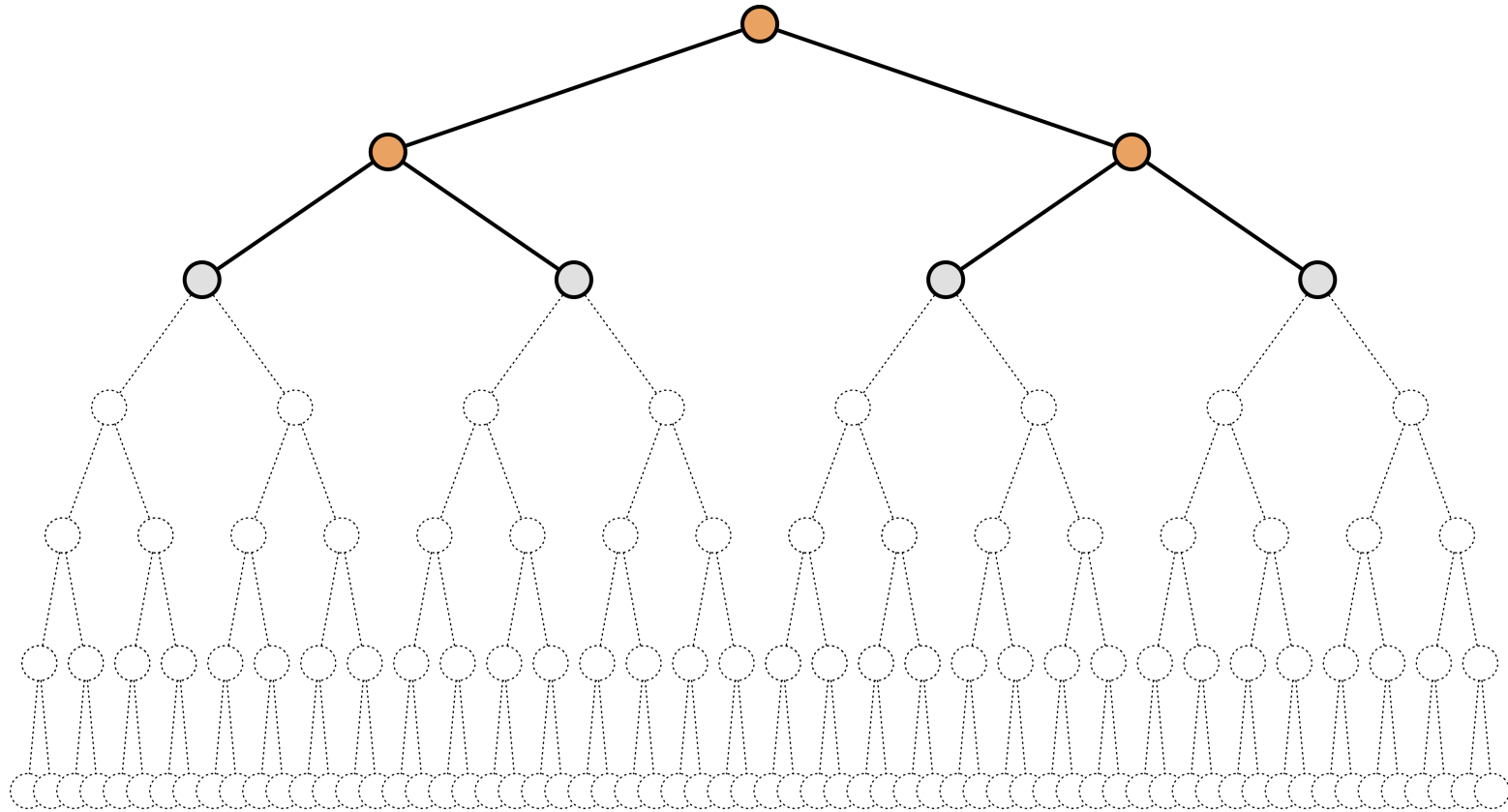
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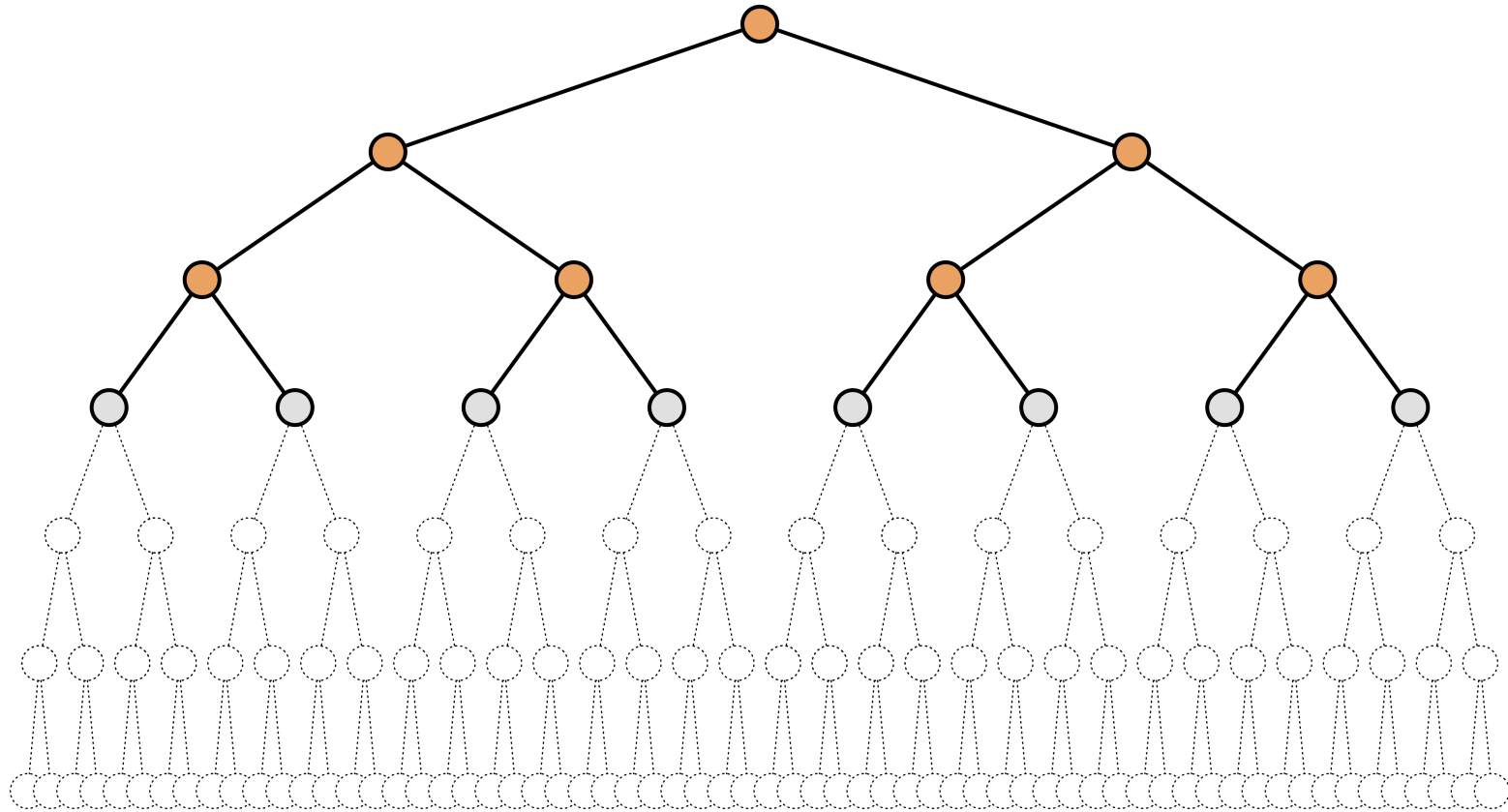
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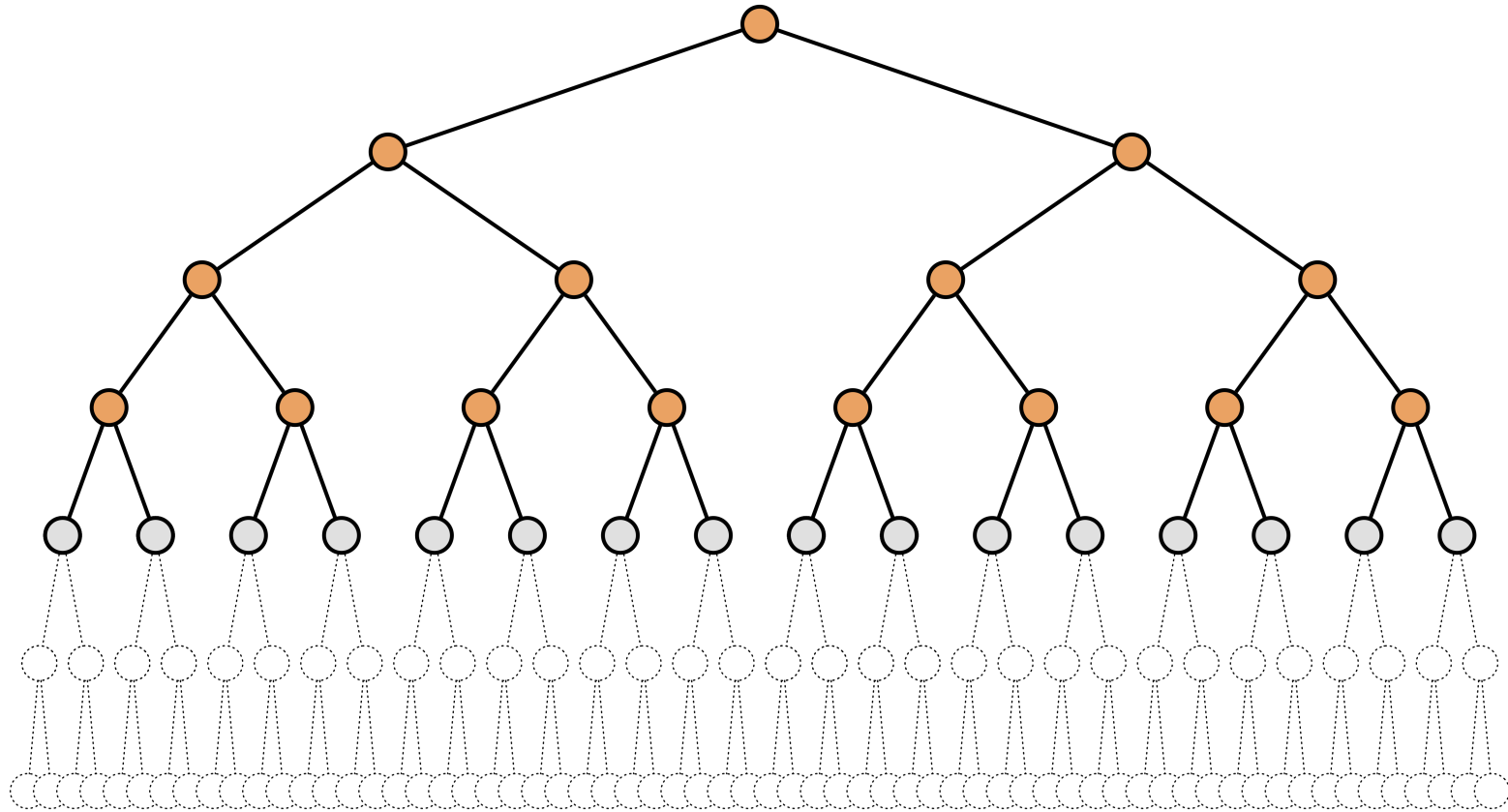
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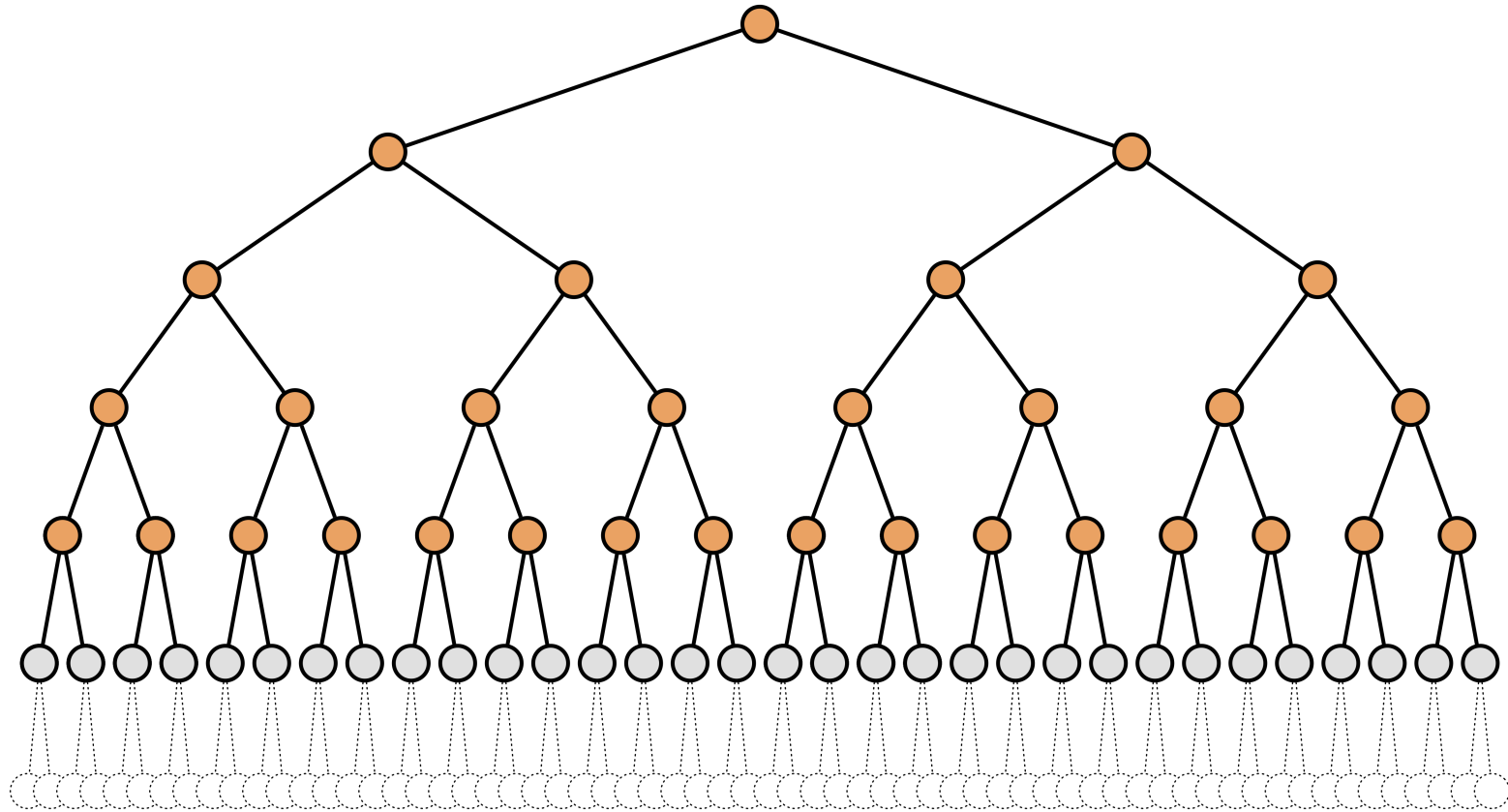
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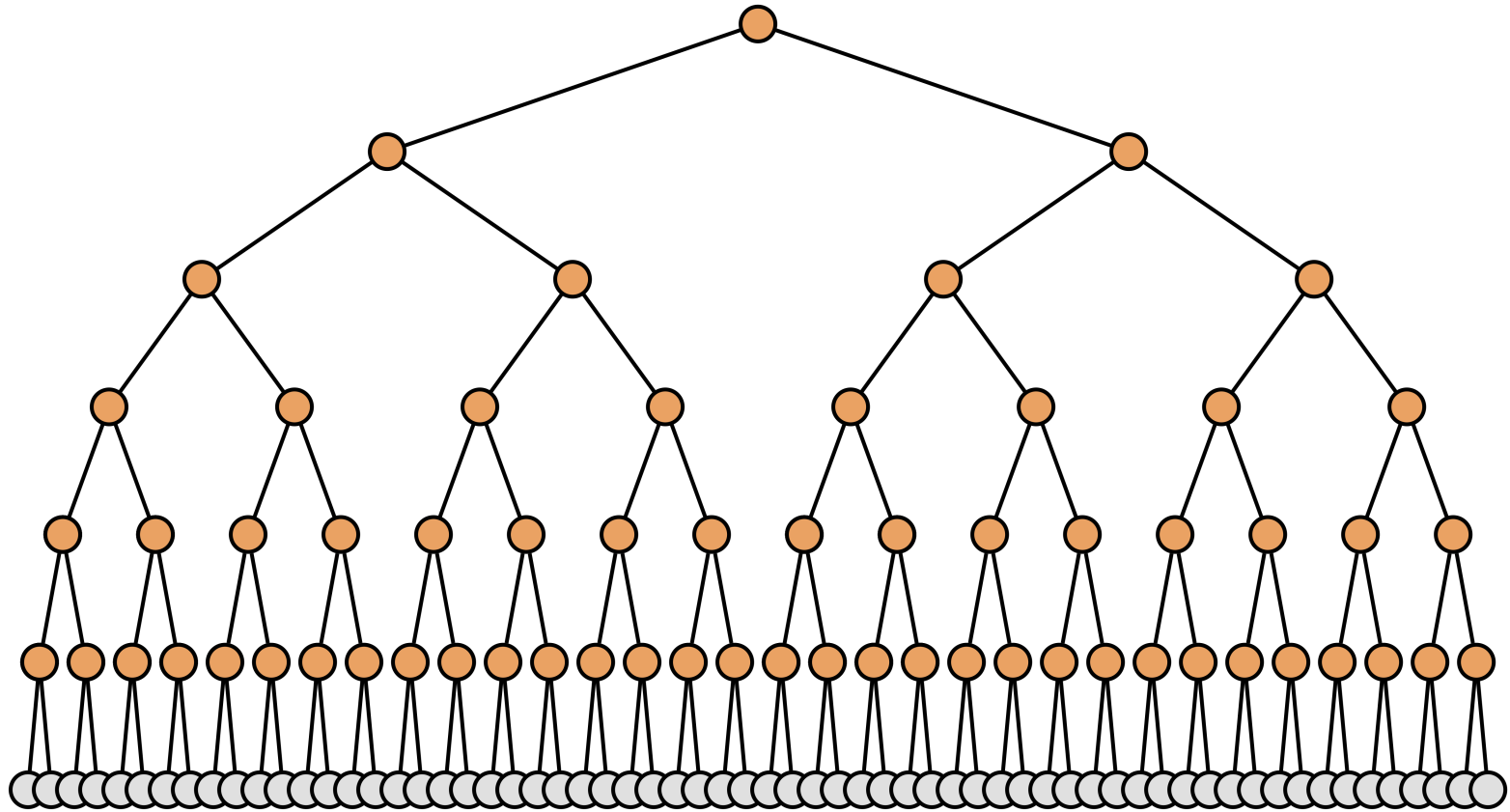
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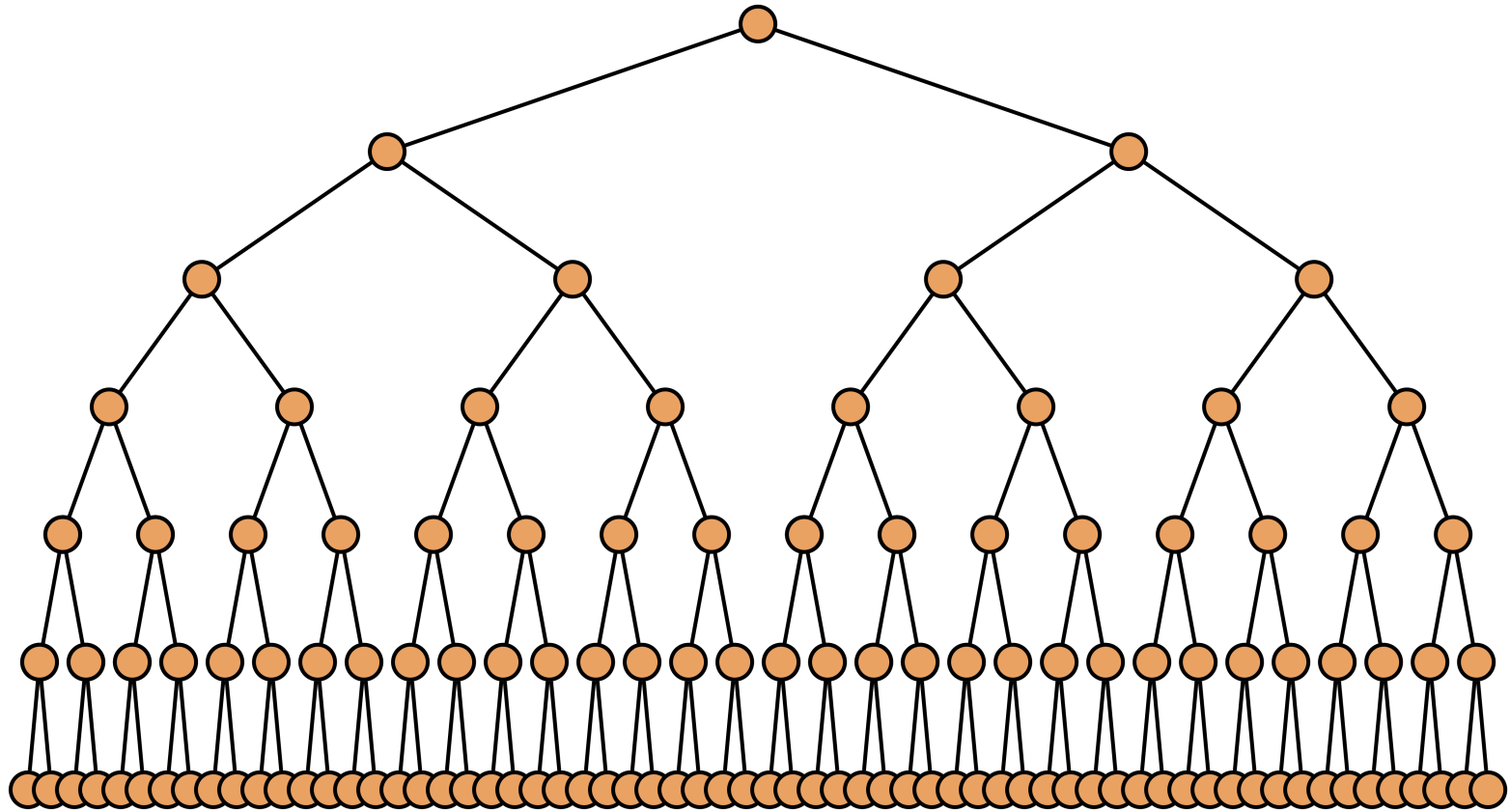
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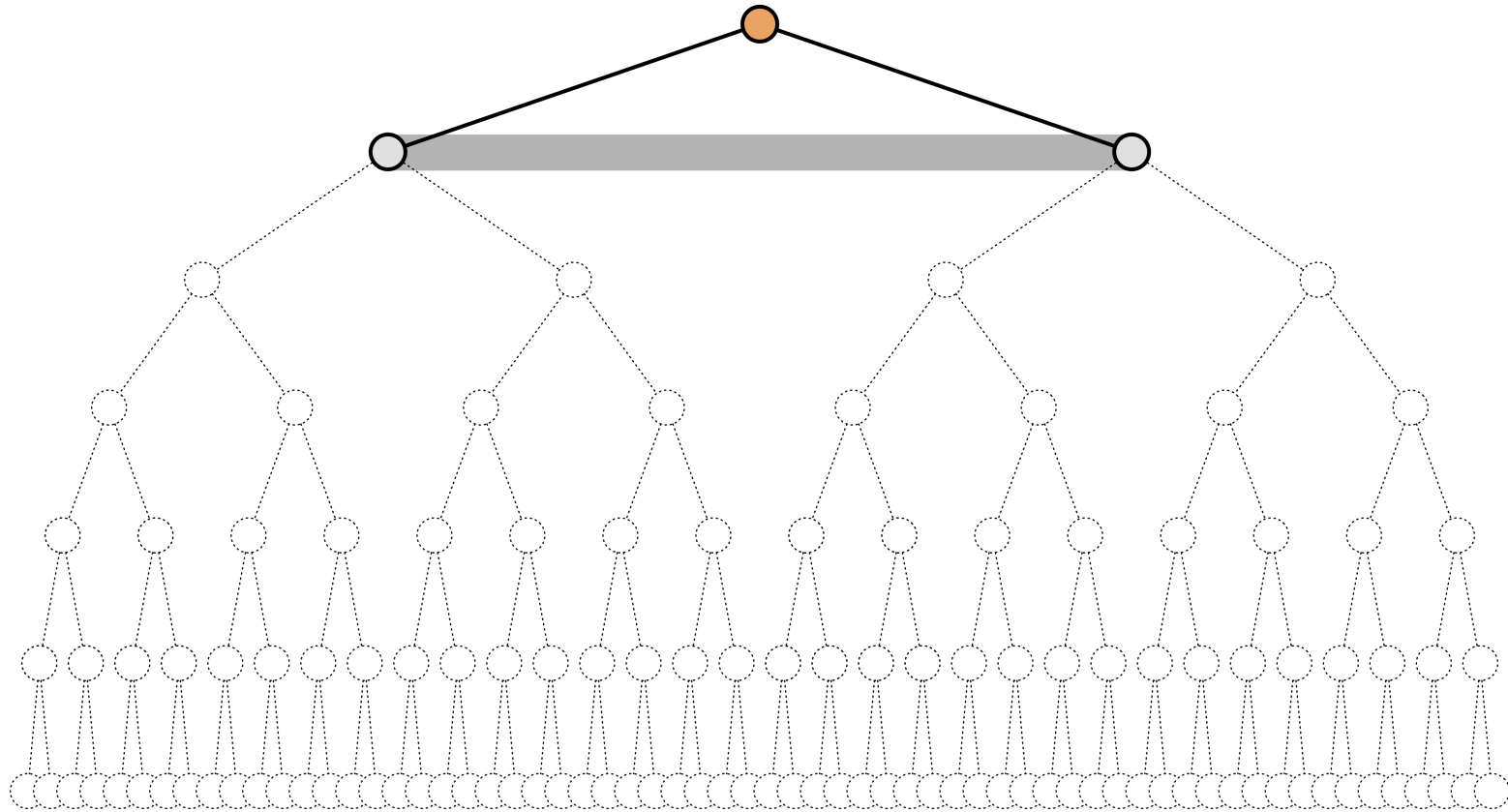
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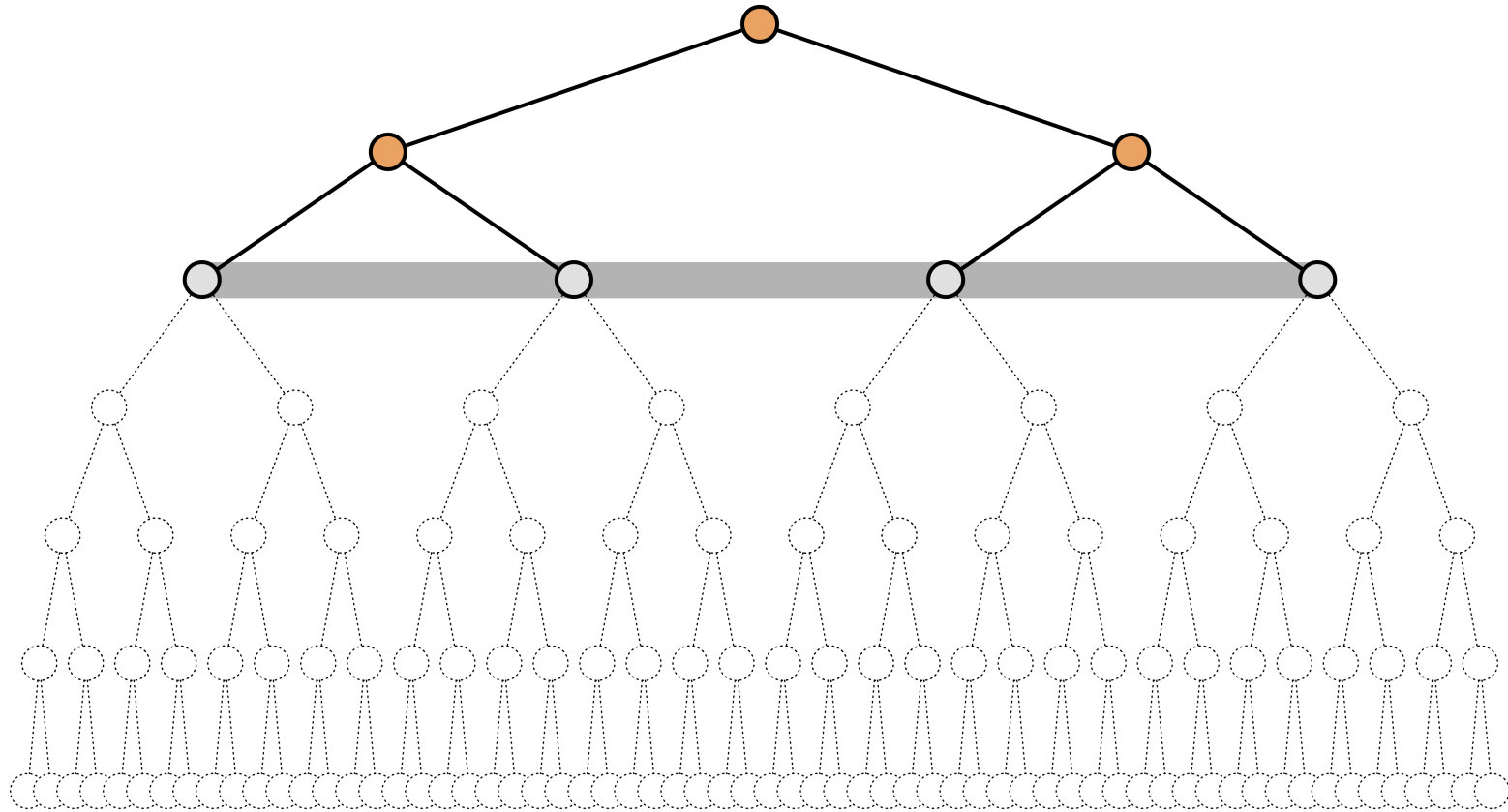
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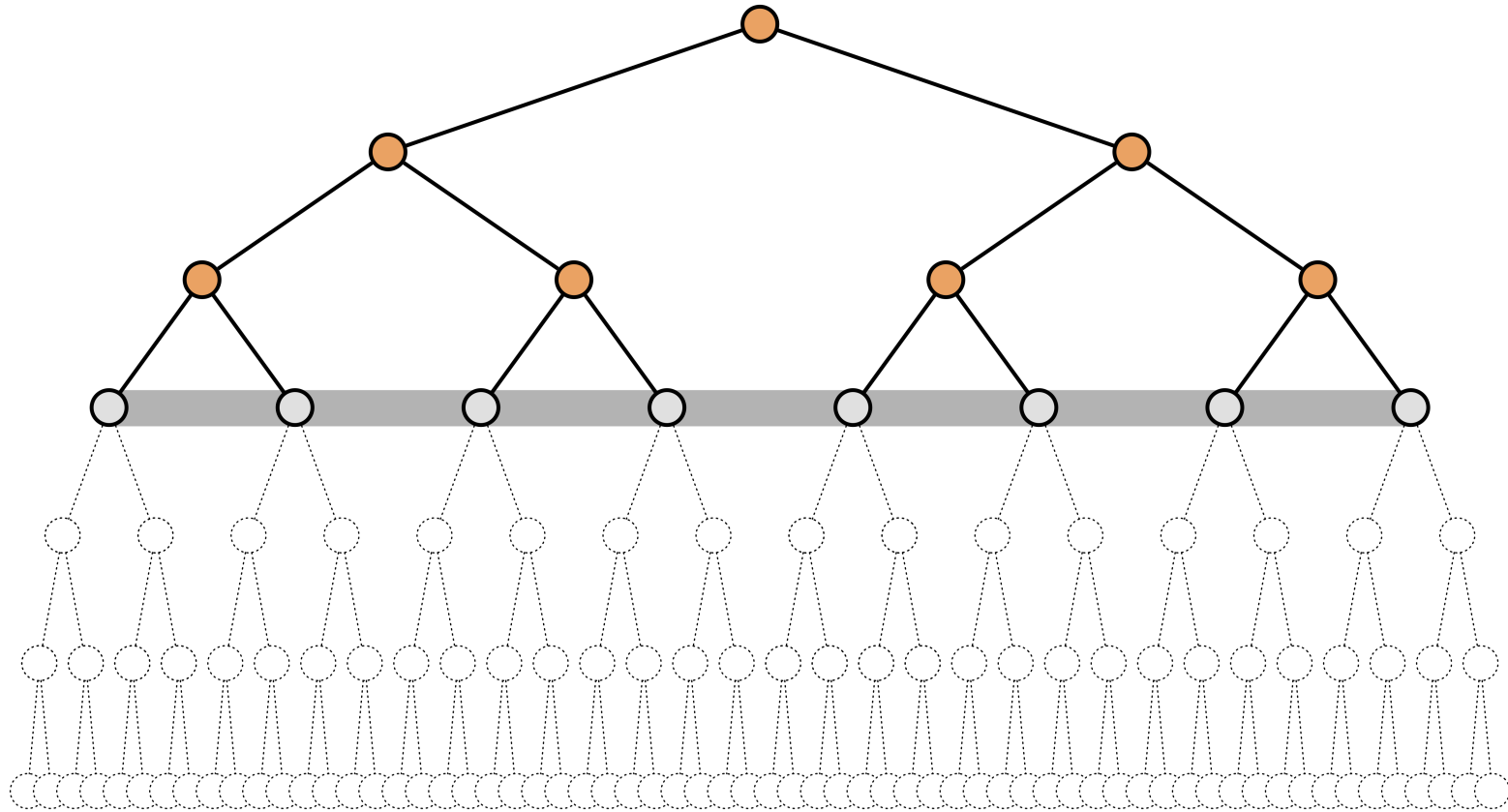
The frontier consists of nodes of similar depth (horizontal).
Search proceeds by exhausting one layer at a time.

BFS



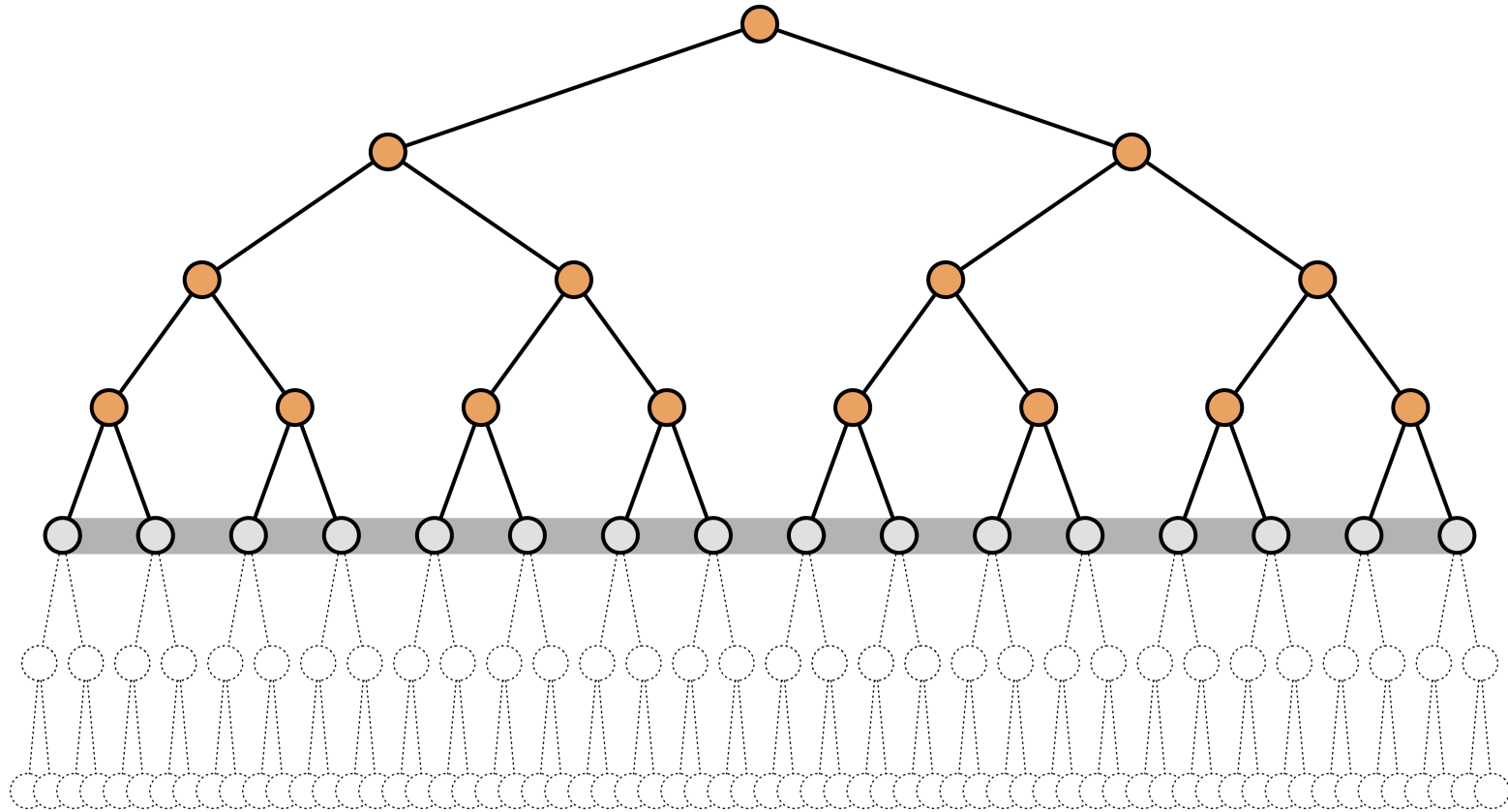
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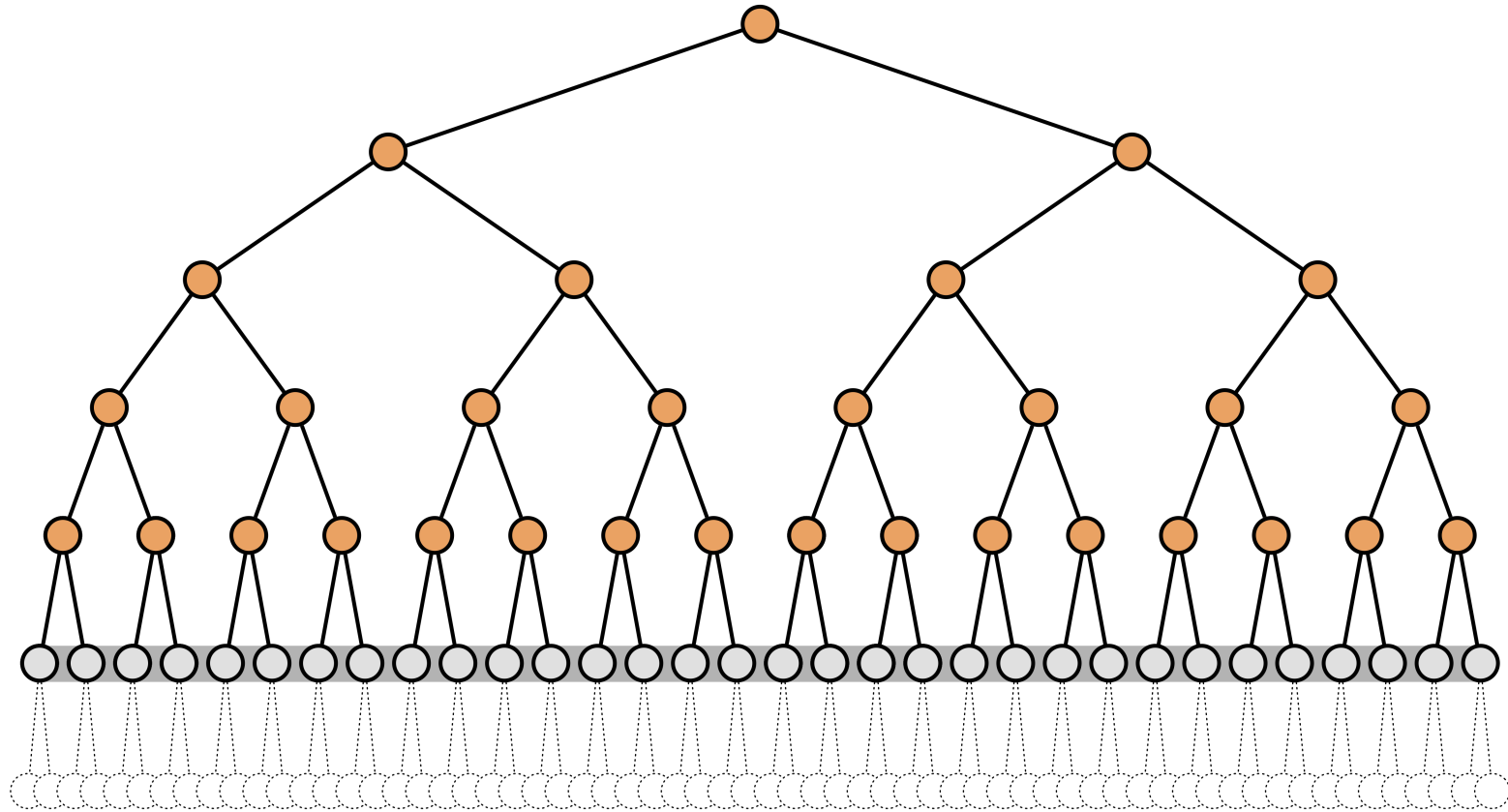
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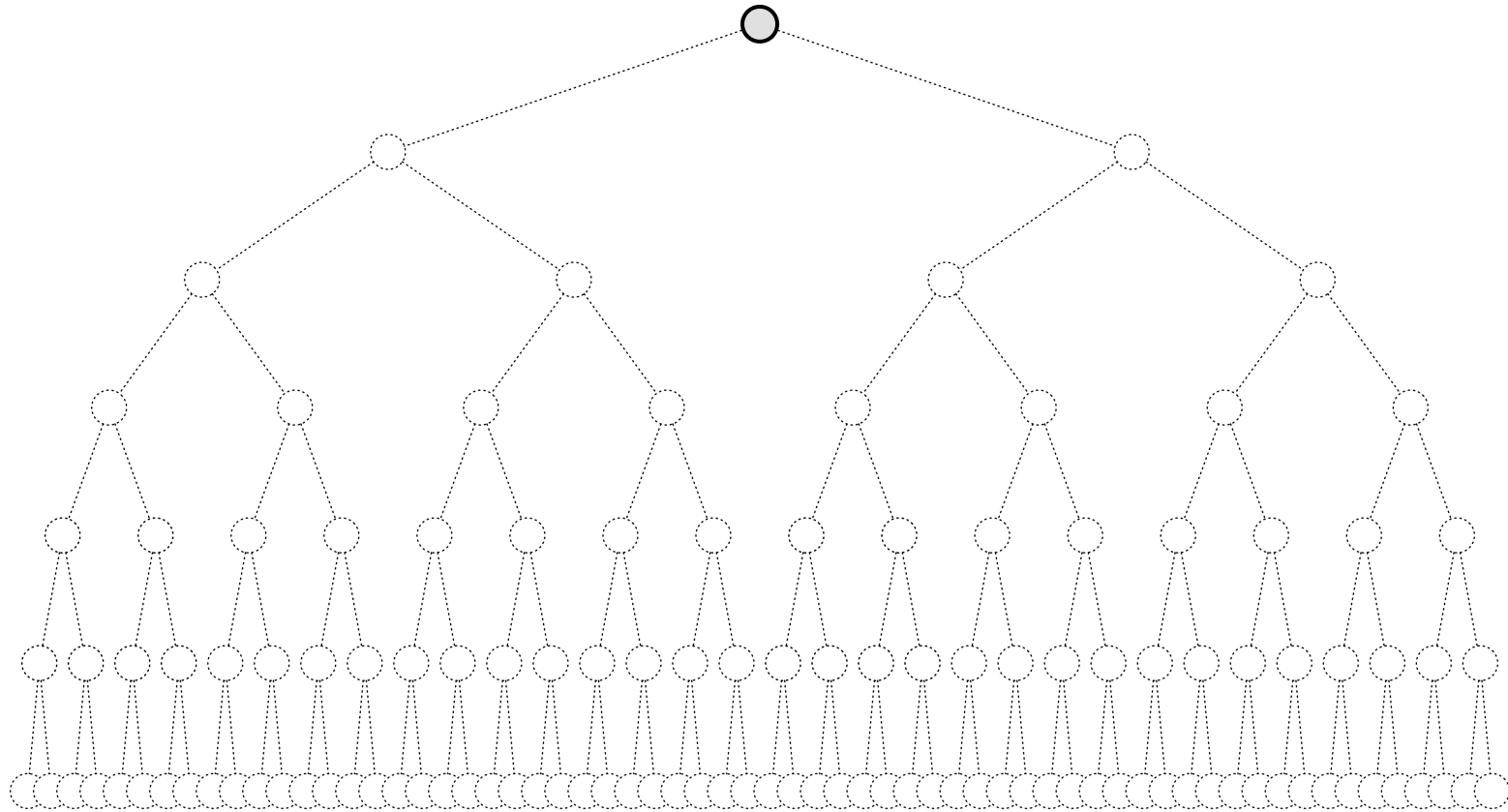
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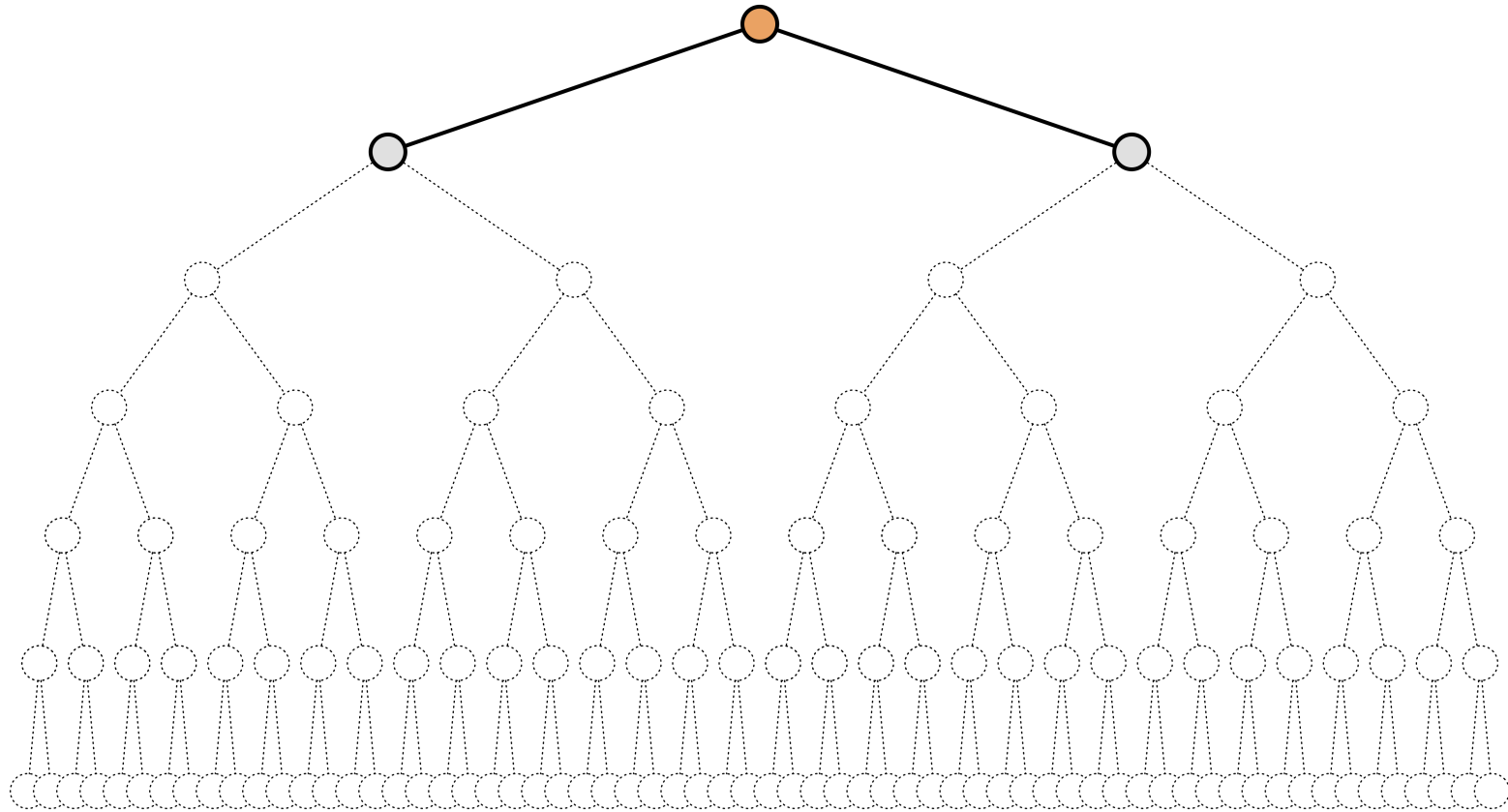
UCS



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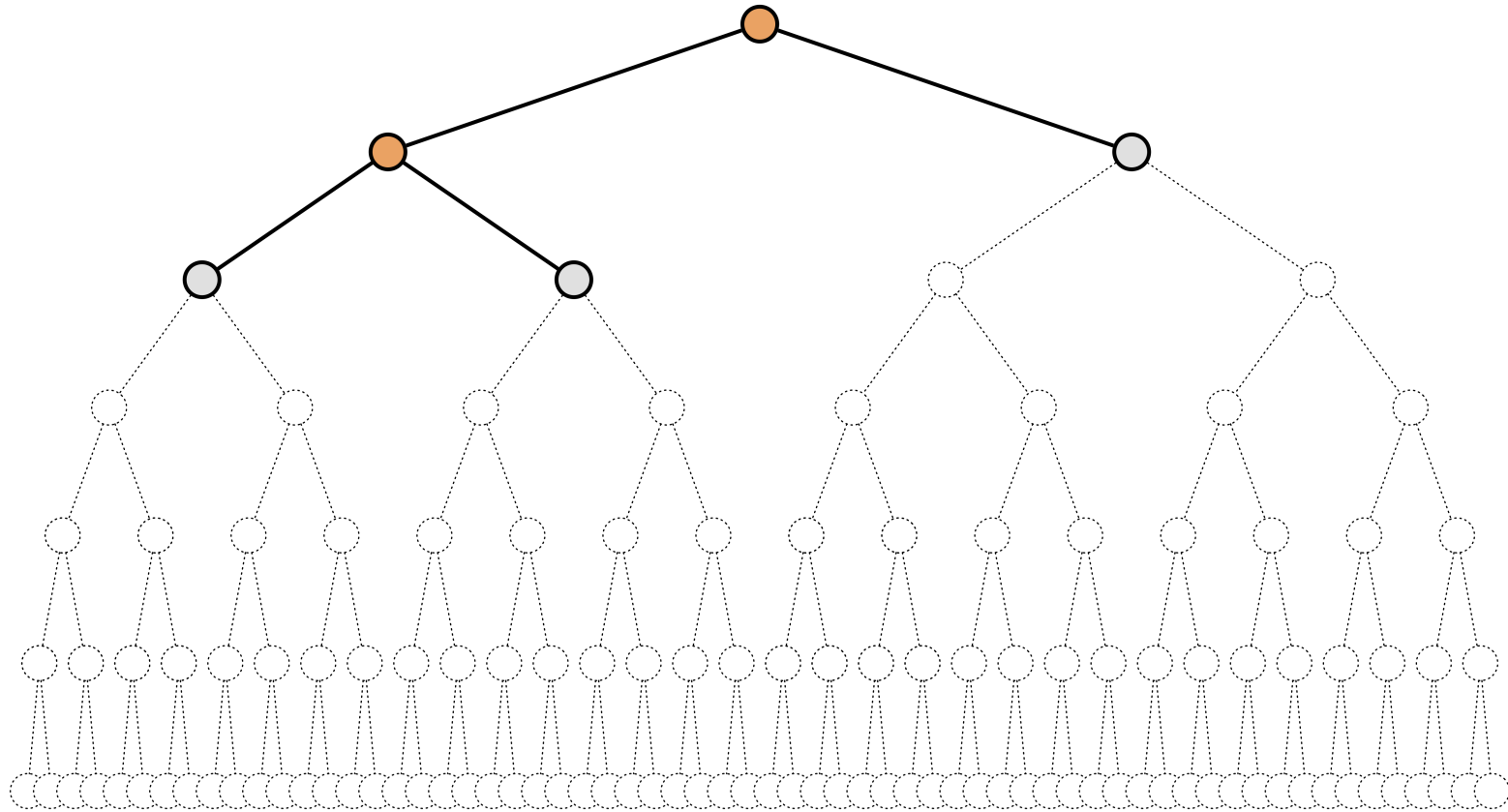
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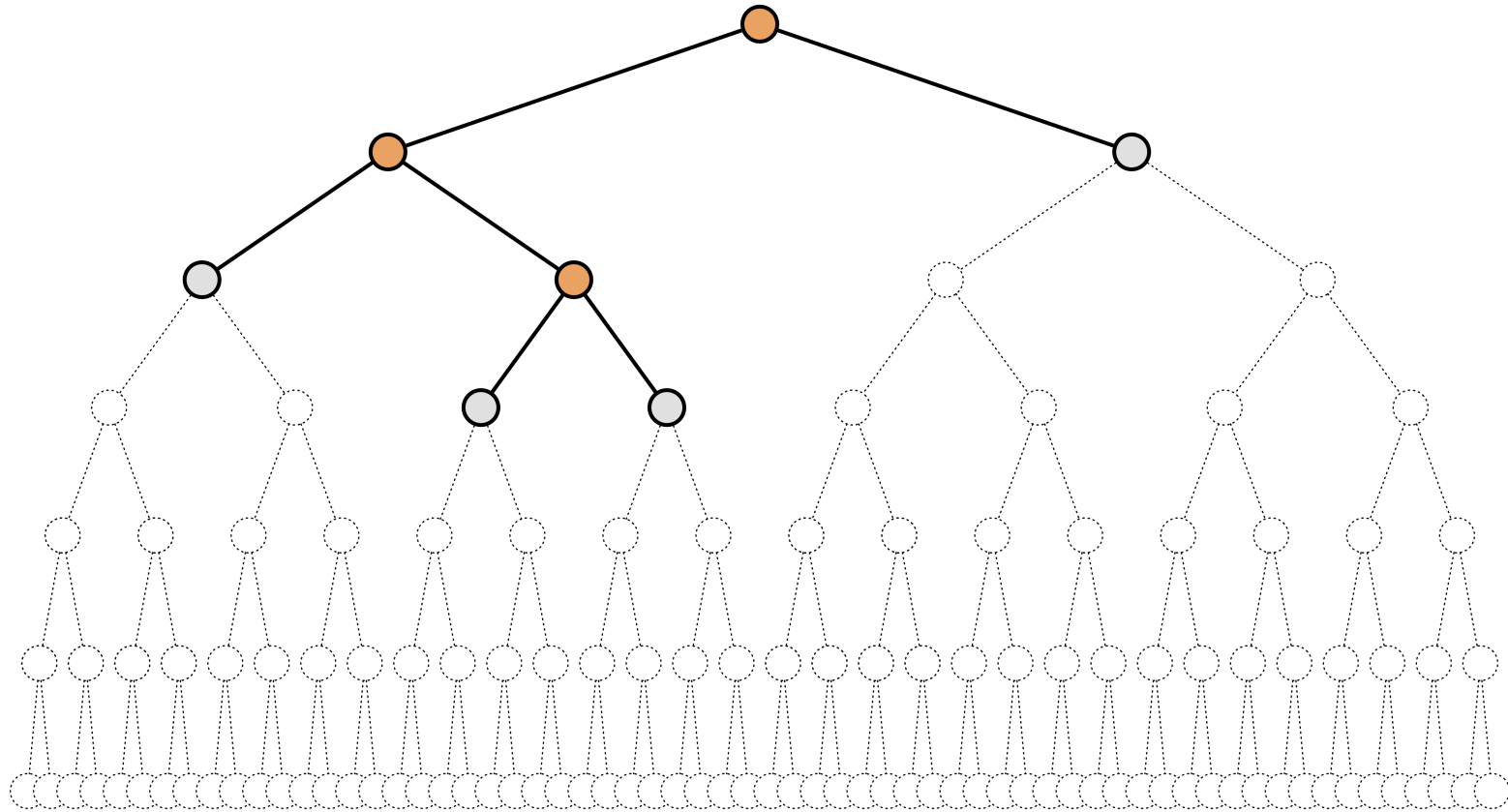
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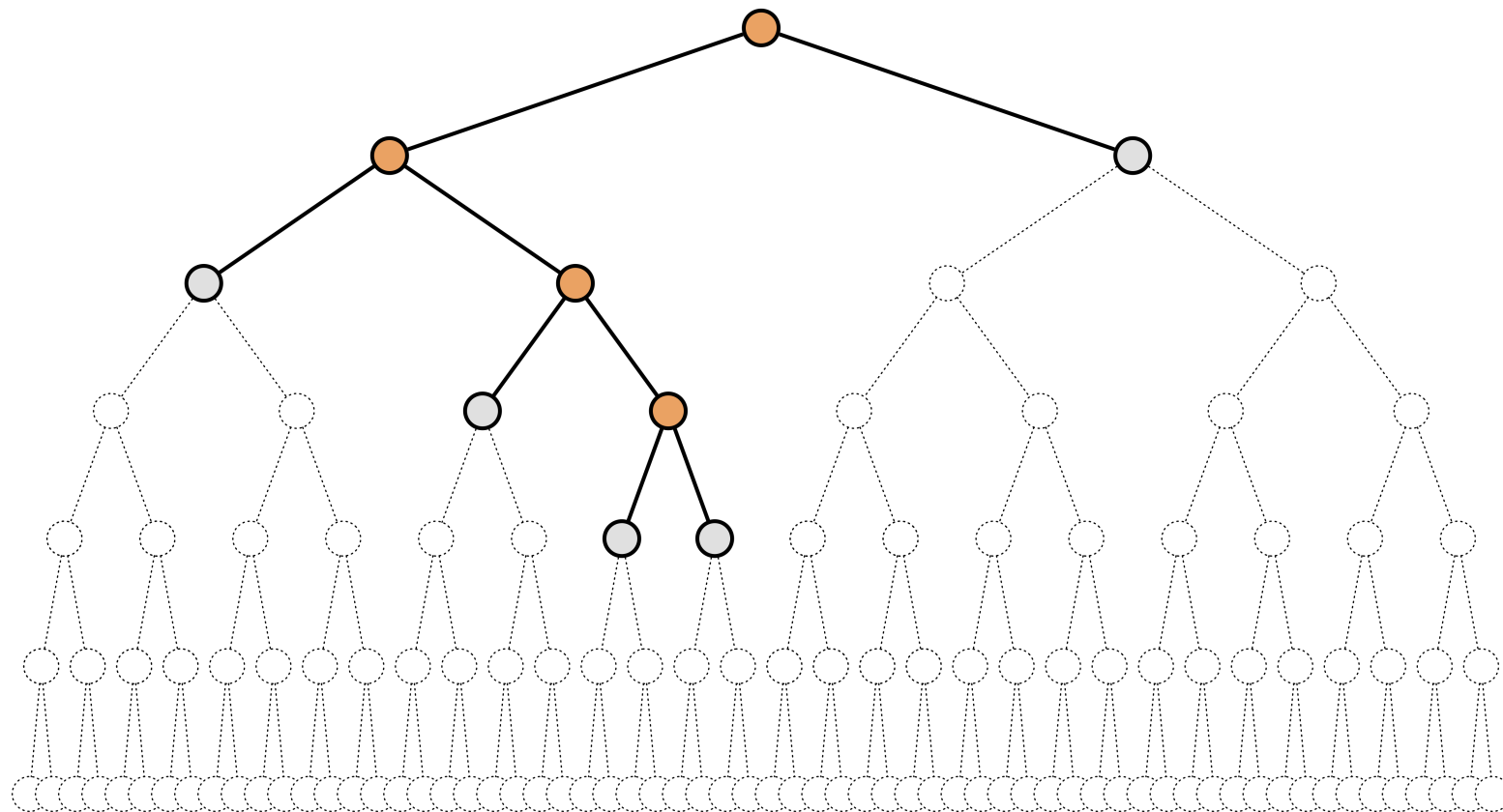
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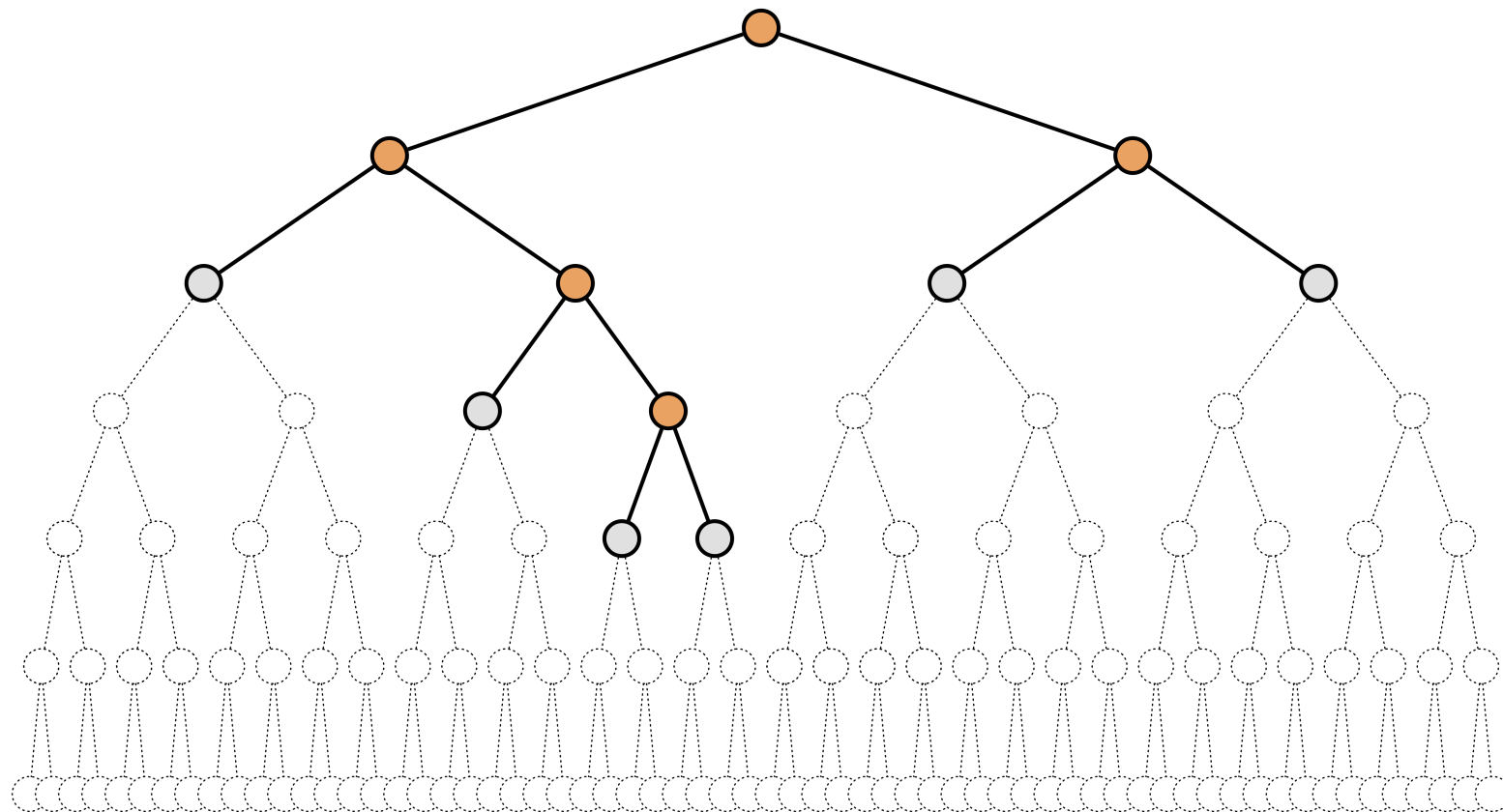
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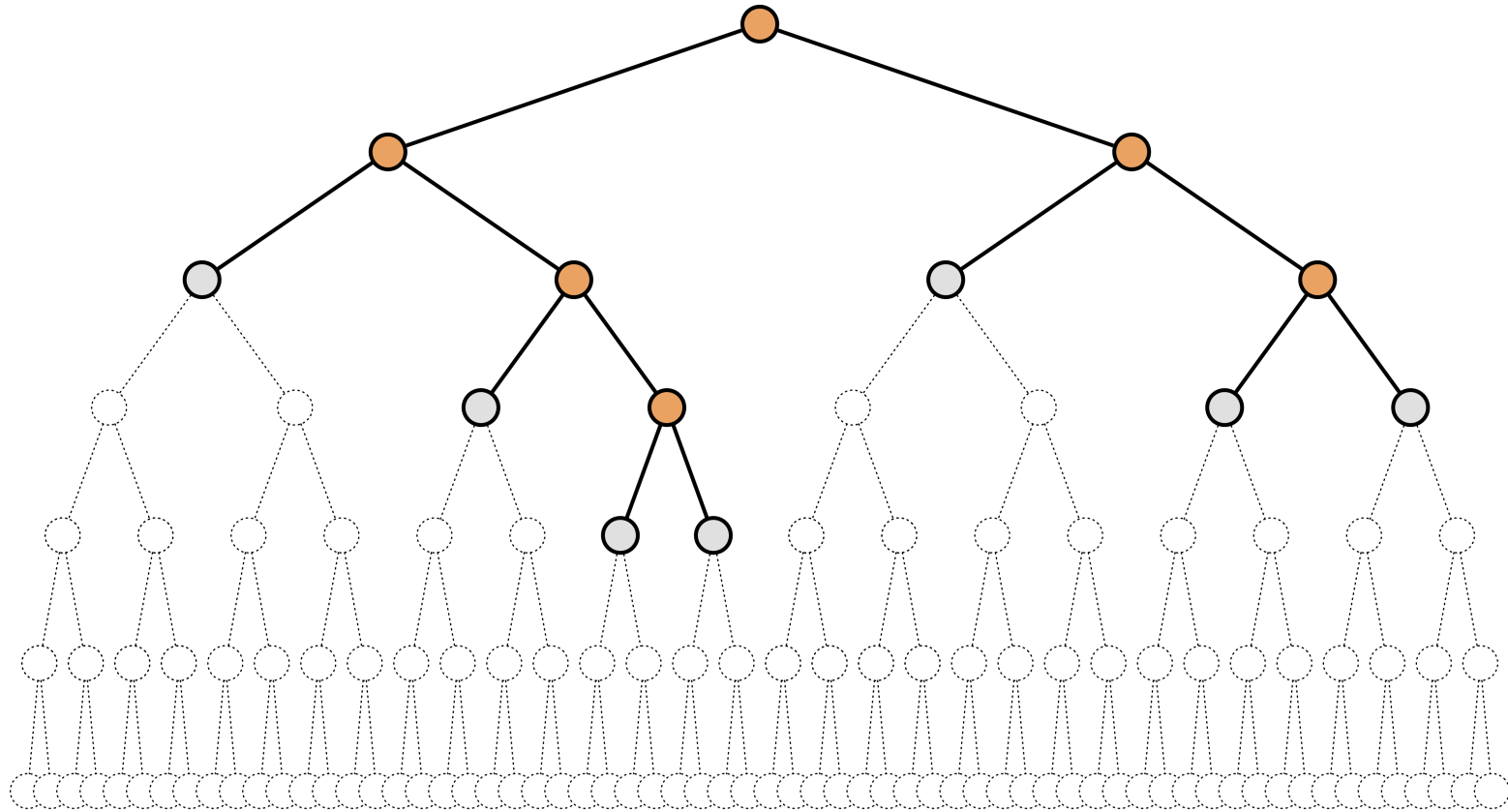
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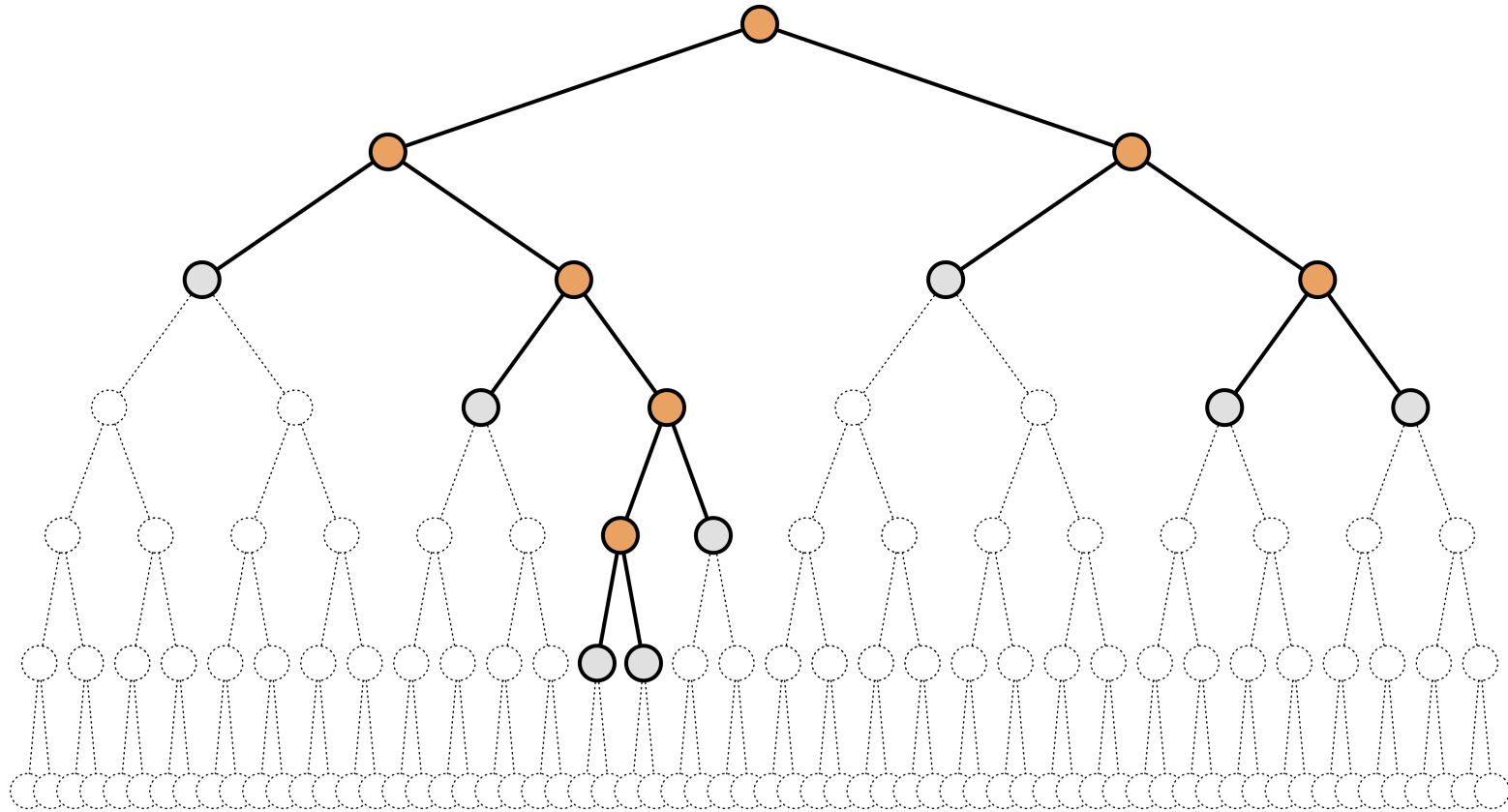
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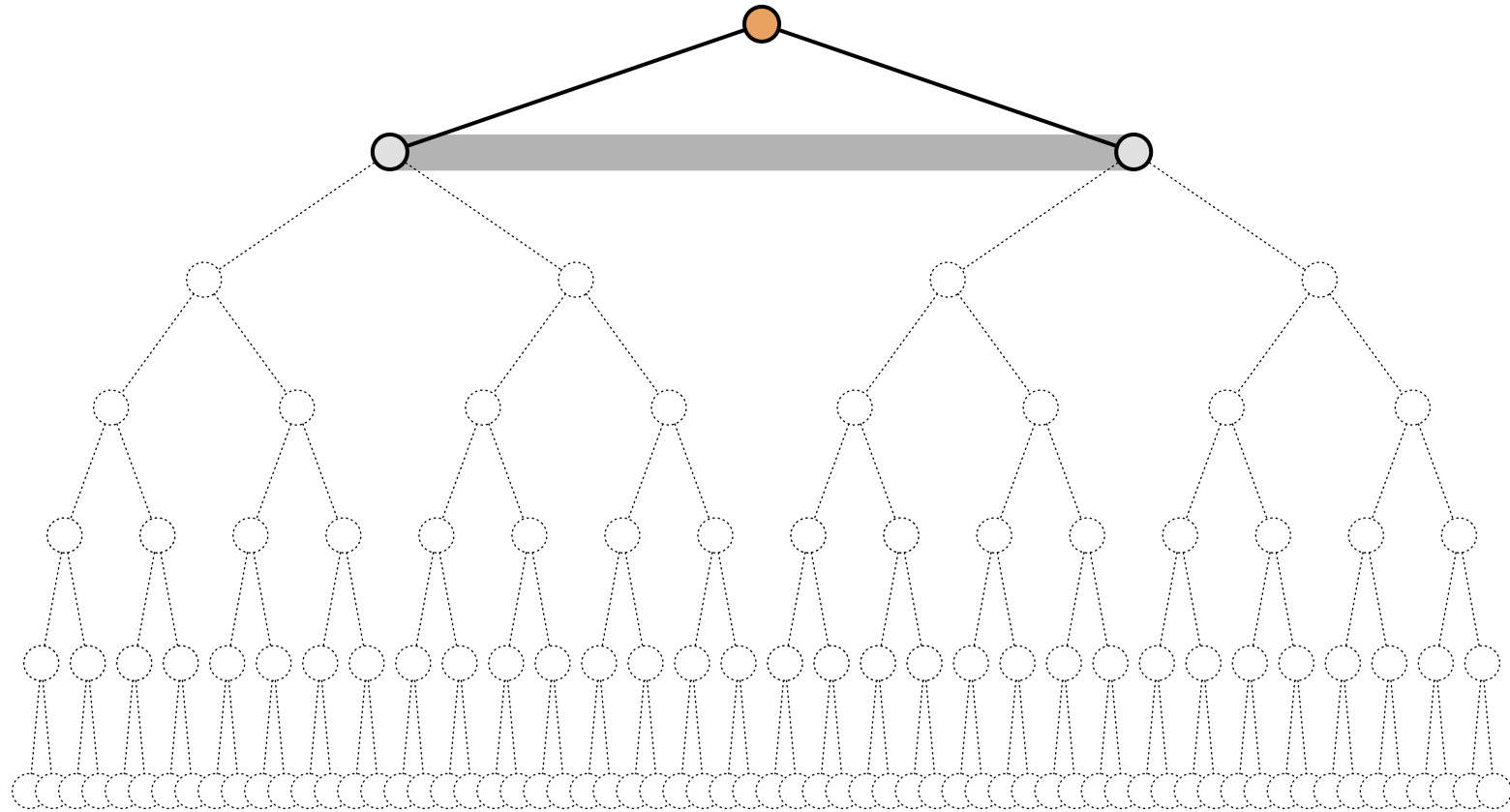
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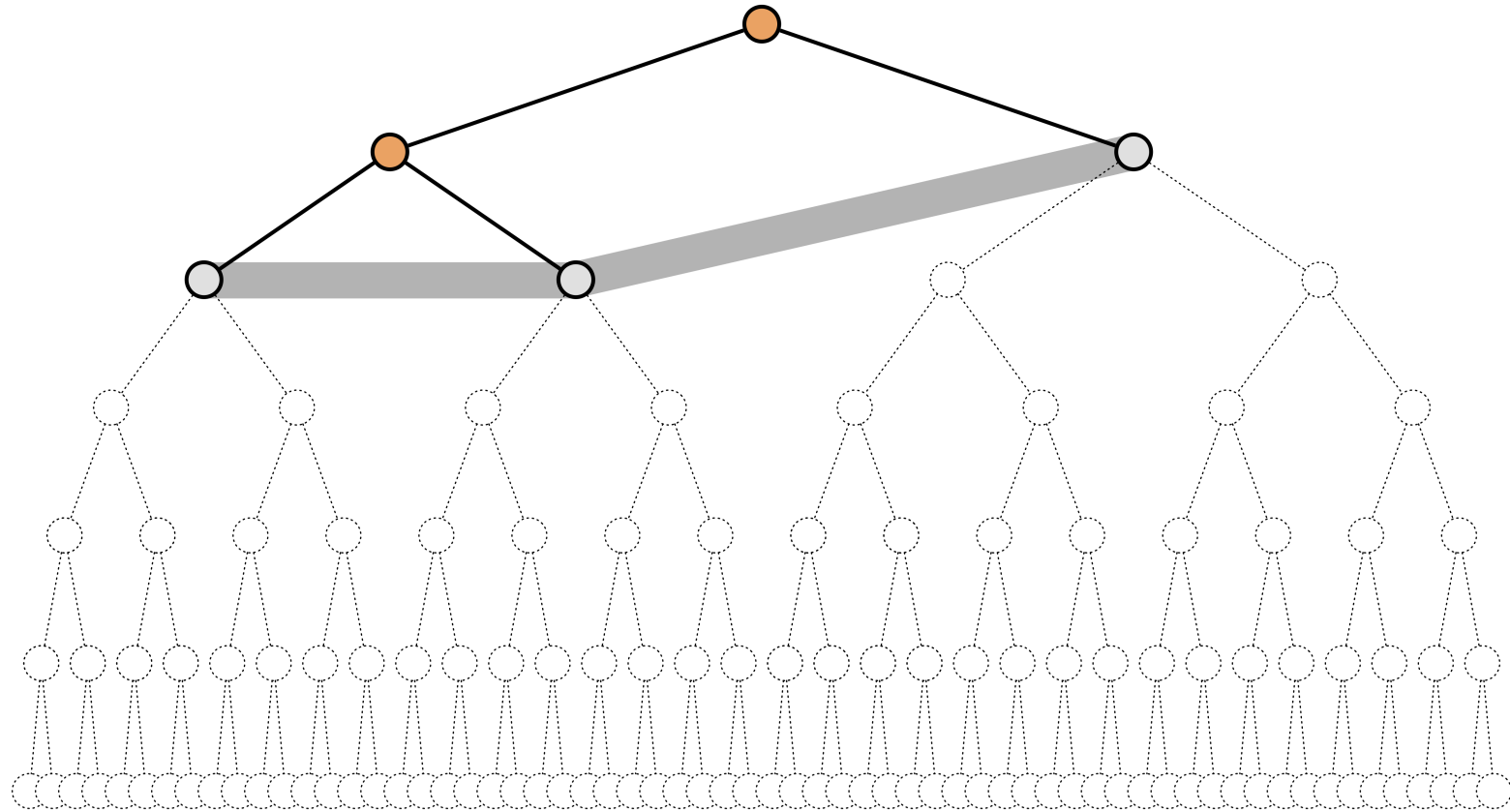
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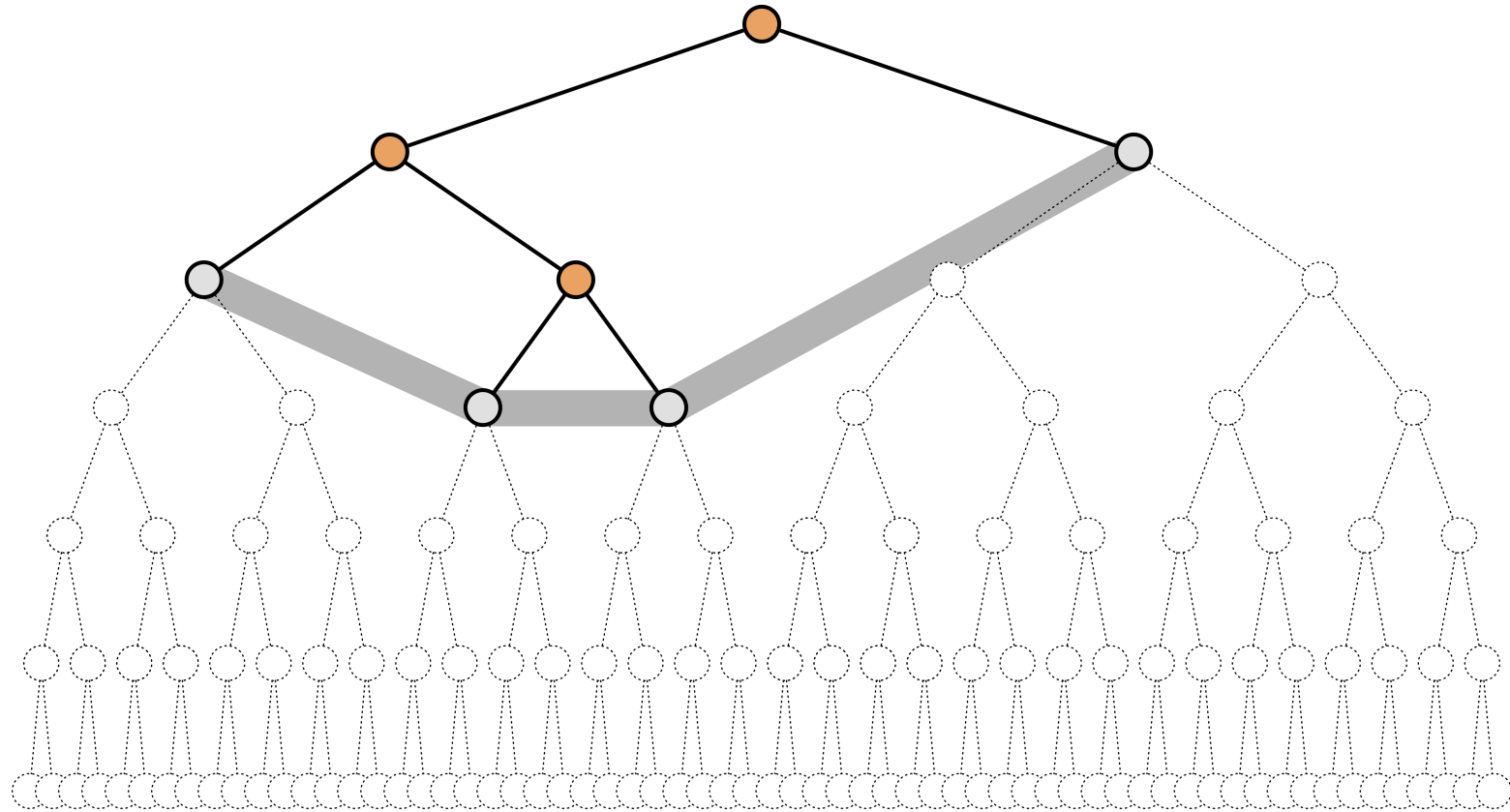
The frontier consists of nodes of various depths (jagged).
Search proceeds by expanding the lowest-cost nodes.

UCS



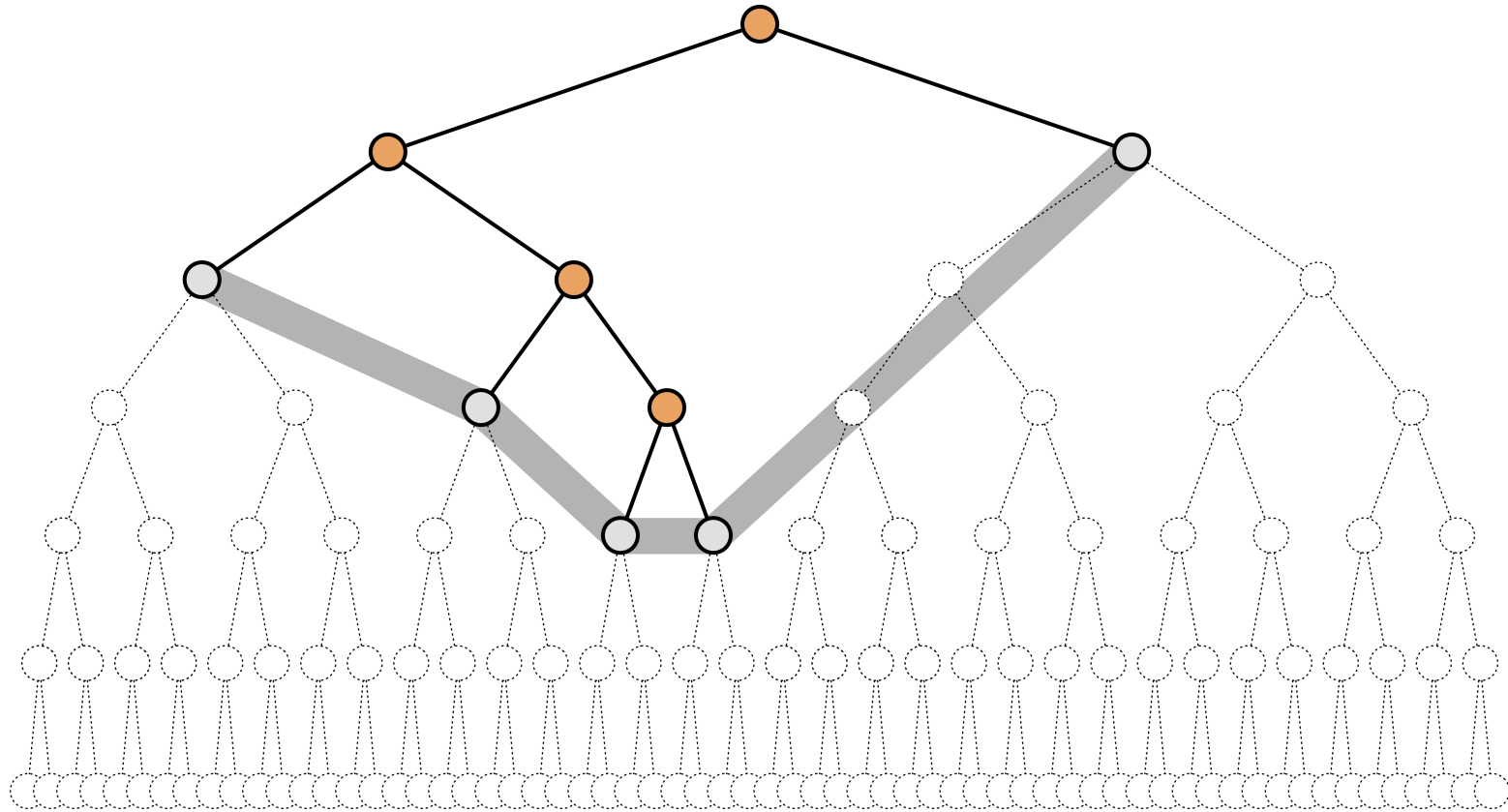
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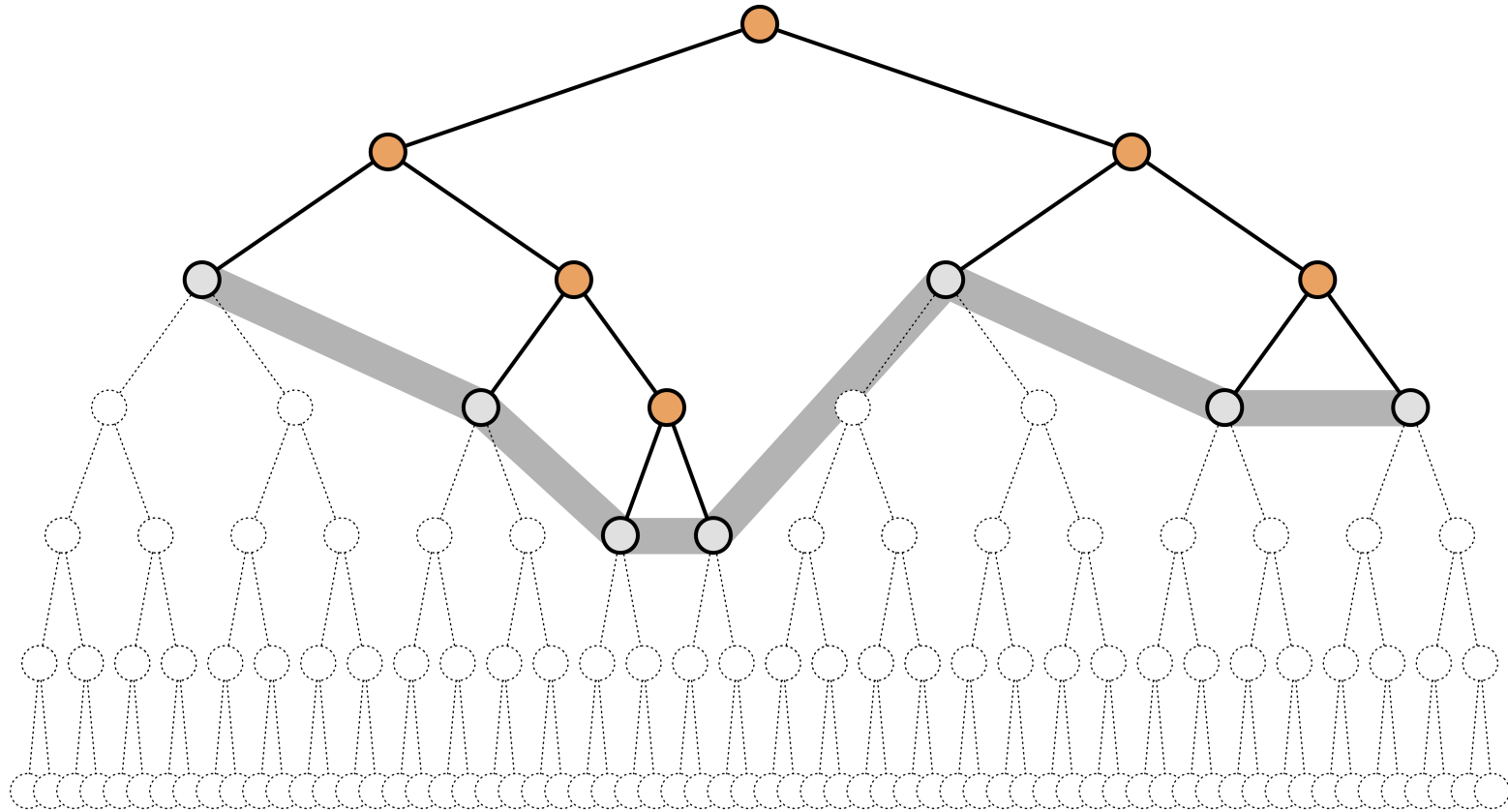
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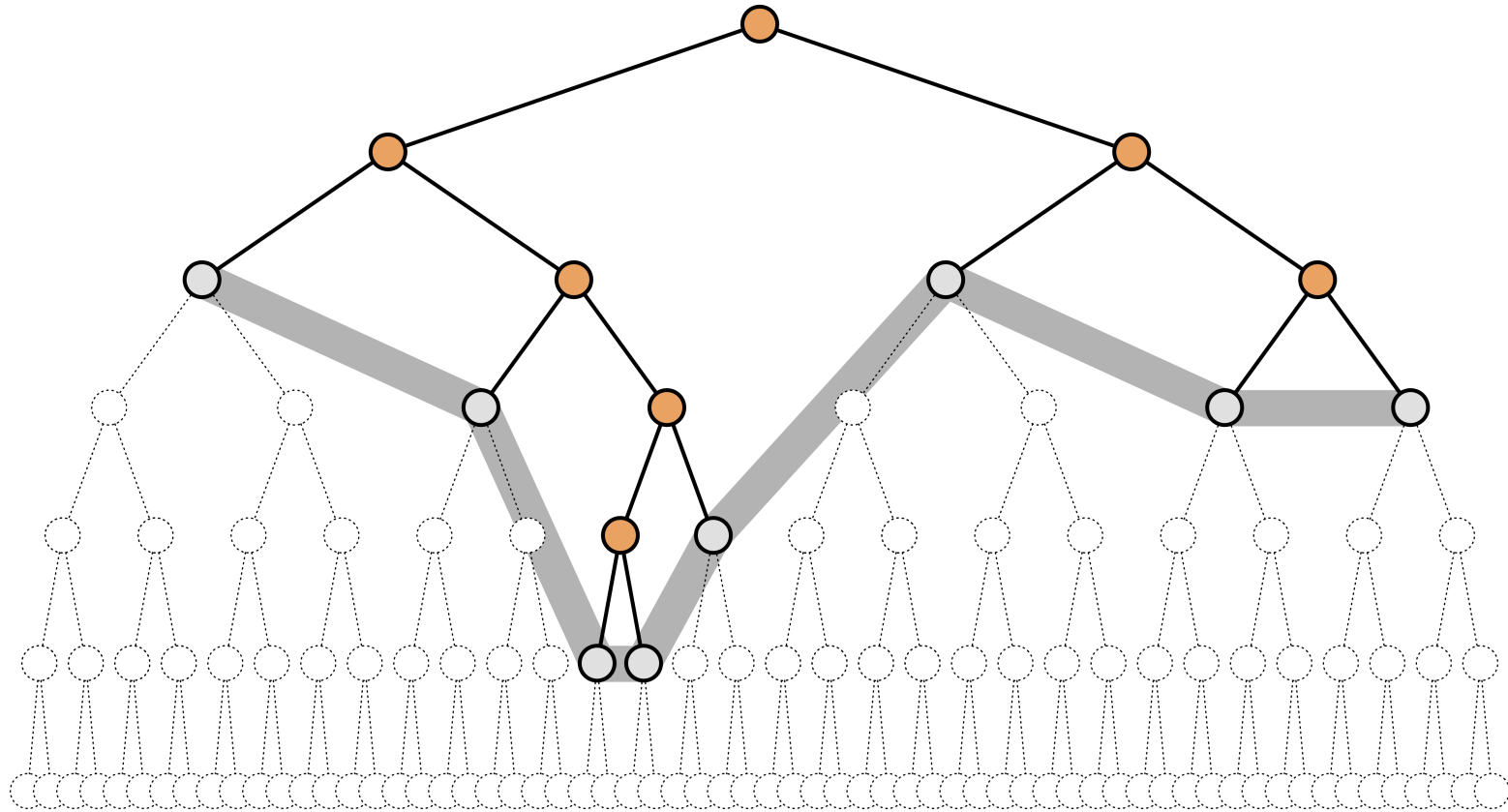
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Recap

We can organize the algorithms into pairs where the first proceeds by layers, and the other proceeds by subtrees.

(1) Iterate on Node Depth:

- BFS searches layers of increasing node depth.
- IDS searches subtrees of increasing node depth.

Recap

We can organize the algorithms into pairs where the first proceeds by layers, and the other proceeds by subtrees.

(1) Iterate on Node Depth:

- BFS searches layers of increasing node depth.
- IDS searches subtrees of increasing node depth.

(2) Iterate on Path Cost + Heuristic Function:

- A* searches layers of increasing path cost + heuristic function.
- IDA* searches subtrees of increasing path cost + heuristic function.

Recap

Which cost function?

- UCS searches layers of increasing path cost.
- Greedy best first search searches layers of increasing heuristic function.
- A* search searches layers of increasing path cost + heuristic function.

Credit

- Artificial Intelligence, A Modern Approach. Stuart Russell and Peter Norvig. Third Edition. Pearson Education.

<http://aima.cs.berkeley.edu/>