COMS W3261 CS Theory: Homework 1. Assigned Sep 25, 2017.
Answers in PDF due Oct 4, 2017 on Courseworks/COMSW3261/Assignments.

Each problem is worth 20 points. You can discuss these problems with others but your answers must be in your own words.

1. **State whether each of the following languages is regular or not. If it is, give a regular expression for it. If it is not, briefly justify why it is not.**

   (a) $L_1 = \{xyz \,|\, x, y, z$ are strings of a's and b's$\}$.

   (b) $L_2 = \{xyz \,|\, x, y, z$ are strings of a's and b's with an equal number of a's and b's$\}$.

   (c) $L_3 = \{a^p \,|\, p$ is a prime number$\}$.

   (d) $L_4 = L_3*$.

   (e) $L_5 = \{a^n \,|\, n$ is a composite number$\}$.

2. **Using set notation describe the language denoted by the following regular expressions over the alphabet $\{a, b\}$.**

   (a) a

   (b) ϵ

   (c) $\emptyset$

   (d) $\emptyset*$

   (e) $((ab + ba)(aa + bb)*(ab + ba)(aa + bb)*)*(aa + bb)*$

3. **Define *deterministic finite automaton* precisely. Then draw a minimum-state DFA for each of the languages in problem (2) and briefly explain why each of your DFAs is minimum state.**

4. **Using the pumping lemma for regular languages, show that a proof that the language $\{a^n b^n \,|\, n \geq 0\}$ is not regular can be framed as a five-step adverserial game: (1) we pick, (2) adversary picks, (3) we pick, (4) adversary picks, (5) we win by making a winning pick.**

5. **For two languages L and M, let *insert*$(L, M) = \{\ xyz \,|\, xz$ is in L and y is in $M\}$. If L and M are regular languages, is *insert*(L, M) always regular? Briefly justify your answer.**

6. **Show that the two regular expressions $(a+b)*$ and $(a*b*)*$ are equivalent by constructing their minimum-state DFAs using the MYT algorithm, then the subset construction, and then the table-filling algorithm. Then show that the minimized automata are isomorphic up to renaming of states.**

aho@cs.columbia.edu and *verma@cs.columbia.edu*