Each problem is worth 20 points. You can discuss these problems with others but your answers must be in your own words.

1. State whether each of the following languages is regular or not. If it is, give a regular expression for it. If it is not, briefly justify why it is not.
   (a) \( L_1 = \{ xyz \mid x, y, z \) are strings of a’s and b’s\}.
   (b) \( L_2 = \{ xyz \mid x, y, z \) are strings of a’s and b’s with an equal number of a’s and b’s\}.
   (c) \( L_3 = \{ a^p \mid p \) is a prime number\}.
   (d) \( L_4 = L_3^* \).
   (e) \( L_5 = \{ a^n \mid n \) is a composite number\}.

2. Using set notation describe the language denoted by the following regular expressions over the alphabet \( \{a, b\} \).
   (a) \( a \)
   (b) \( \epsilon \)
   (c) \( \emptyset \)
   (d) \( \emptyset^* \)
   (e) \( ((ab + ba)(aa + bb)^*(ab + ba)(aa + bb)^*)^*(aa + bb)^* \)

3. Define deterministic finite automaton precisely. Then draw a minimum-state DFA for each of the languages in problem (2) and briefly explain why each of your DFAs is minimum state.

4. Using the pumping lemma for regular languages, show that a proof that the language \( \{ a^n b^n \mid n \geq 0 \} \) is not regular can be framed as a five-step adverserial game: (1) we pick, (2) adversary picks, (3) we pick, (4) adversary picks, (5) we win by making a winning pick.

5. For two languages \( L \) and \( M \), let \( \text{insert}(L, M) = \{ xyz \mid xz \) is in L and y is in M\}. If \( L \) and \( M \) are regular languages, is \( \text{insert}(L, M) \) always regular? Briefly justify your answer.

6. Show that the two regular expressions \((a+b)^*\) and \((a^*b^*)^*\) are equivalent by constructing their minimum-state DFAs using the MYT algorithm, then the subset construction, and then the table-filling algorithm. Then show that the minimized automata are isomorphic up to renaming of states.

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