COMS W3261 CS Theory: Problem Assignment 5 (revised)
Assigned November 29, 2016; due on Courseworks December 12, 2016

Instructions
- Problems 1-5 are each worth 20 points.
- Submit your solutions in pdf format on Courseworks/COMSW3261/Assignments by noon December 12, 2016. Late assignments will not be accepted.
- You can discuss these problems with others but your answers must be in your own words.

Problems

1. TAUT is the language of all encoded boolean expressions that are tautologies.
   (a) **Problem 1(a) has been withdrawn from HW5.** (It should have been to prove that TAUT is NP-hard under Cook reductions not Karp reductions which we have been using in this course.)
   (b) Prove that TAUT is co-NP-complete.
   (c) What would be the consequence of showing that TAUT is in NP?

2. The game PEBBLES is played on a $k \times n$ chessboard. Initially each square of the chessboard has a black pebble, or a white pebble, or no pebble. You play the game by removing pebbles one at a time. You win the game if you can end up with a board in which each column contains only pebbles of a single color and each row contains at least one pebble.
   (a) Show that the set of winnable PEBBLES games is in NP. Hint: Describe a nondeterministic polynomial-time algorithm to determine whether a given PEBBLES board is winnable.
   (b) Given a boolean expression $E$ in 3-CNF with $k$ clauses and $n$ variables, construct the following $k \times n$ board: If literal $x_i$ is in clause $c_j$, put a black pebble in column $x_i$, row $c_j$. If literal $\overline{x_i}$ is in clause $c_j$, put a white pebble in column $x_i$, row $c_j$. Show that $E$ is satisfiable if and only if this PEBBLES game is winnable.
   (c) What can you conclude from showing (a) and (b)?

3. Let $E$ be the lambda expression $(\lambda u. (\lambda x. u) u) ((\lambda y. (\lambda v. (\lambda w. v) w)) y)$.
   (a) Evaluate $E$ using normal order evaluation. Show the redex used in each step.
   (b) Evaluate $E$ using applicative order evaluation. Show the redex used in each step.

4. Let $E$ be the lambda expression $(\lambda x. (\lambda y. y)) ((\lambda z. zz) (\lambda z. zz))$.
   (a) Evaluate $E$ using normal order evaluation. Show the redex used in each step.
   (b) Evaluate $E$ using applicative order evaluation. Show the redex used in each step.
   (c) Does $E$ have a normal form? If so, what is it?

5. Let $G$ be the function definition $(\lambda f. \lambda x. f(f x))$. Evaluate the lambda expression $GG$. 


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