Instructions

- Problems 1-5 are each worth 20 points.
- Submit your solutions in pdf format on Courseworks/COMSW3261/Assignments by noon November 22, 2016. Late assignments will not be accepted.
- You can discuss these problems with others but your answers must be in your own words.

Problems

1. Let $A$, $B$, and $C$ be three languages. Prove that if there exist a reduction from $A$ to $B$ and a reduction from $B$ to $C$, then there is a reduction from $A$ to $C$.

2. Let $L$ be the language consisting of triples $(M_1, M_2, k)$, where $M_1$ and $M_2$ are two Turing machines (encoded by binary strings) and $k$ is an integer, such that $|L(M_1) \cup L(M_2)| \geq k$.
   a) Prove that $L$ is recursively enumerable.
   b) Prove that $L$ is not recursive.

   Hint: In the a) part of both Problems 2 and 3, the use of nondeterministic Turing machines may simplify the proof (though there are ways using deterministic Turing machines only).

3. A useless state in a Turing machine is one that is never entered on any input string. Let $L$ be the language consisting of Turing machines $M$ with no useless state.
   a) Prove that $L$ is recursively enumerable.
   b) Prove that $L$ is not recursive.

4. Show that the PCP is decidable over a unary alphabet, i.e., over the alphabet $\Sigma = \{1\}$. (The algorithm is indeed a very efficient one.)

5. Prove that P is closed under complement. Discuss why the same argument fails to show that NP is closed under complement.