

DISCRETE MATH¹ W3203 Quiz 2

open book

Your Name (2 pts for legibly PRINTING your name)

Problem	Points	Score
your name	2	
1	20	
2	20	
3	18	
4	20	
5	20	

Total 100

SUGGESTION: Do the EASIEST problems first!

HINT: Some of the solution methods involve highschool math as well as new methods from this class.

¹ An example of the Reasonable Person Principle: A reasonable student expects to lose a lot of credit for neglecting to EXPLAIN an answer. Omit explanations at your own risk.

1a (10 pts). Prove the following inequality:

$$2\sqrt{n+1} - \frac{1}{\sqrt{n+1}} > 2\sqrt{n} \quad \text{for all } n \geq 0.$$

1b (10 pts). Now prove the following:

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} < 2\sqrt{n} \quad \text{for all } n \geq 1.$$

2 (20 pts). Prove the following:

$$\sum_{k=0}^n k \binom{n}{k} = n2^{n-1} \quad \text{for } n \geq 0.$$

3 (18 pts) We define the set S of binary strings recursively:

B. $1 \in S$

R. if $s \in S$, then $s0, s1$ and s^{-1} (reverse of s) $\in S$

3a (5). Show step-by-step how to construct a string in S with three consecutive 1's.

3b (13). Prove that no string in S begins or ends in 111.

4 (20 pts). Consider the following recursion.

$$b_0 = 1$$

$$b_{n+1} = \binom{n}{0}b_n + \binom{n}{1}b_{n-1} + \binom{n}{2}b_{n-2} + \cdots + \binom{n}{n}b_0$$

4a (10). Calculate b_4 .

4b (10). Prove that $b_{n+1} \geq 2^n$.

5 (20 pts). A bakery sells four varieties of bagels: egg, onion, poppy, and sesame.

5a (10). How many different combinations of six bagels are possible?

5b (10). How many of the combinations in 5a contain at least four bagels of one kind?