# DISCRETE MATH[1] W3203 Quiz 1

open book

SOLUTIONS

____________________________________
Your Name (2 pts for legibly PRINTING your name)

| Problem | Points | Score |
|---|---|---|
| your name | 2 | |
| 1 | 13 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 25 | |
| 5 | 20 | |
| Total | 100 | |

SUGGESTION: Do the EASIEST problems first!

HINT: Some of the solution methods involve highschool math as well as new methods from this class.

[1] An example of the Reasonable Person Principle: A reasonable student expects to lose a lot of credit for neglecting to EXPLAIN an answer. Omit explanations at your own risk.

1 (13 pts). Decide whether these two propositions are equivalent.

SOLUTION: YES

| $r$ | $s$ | $t$ | $(s \vee r) \to (s \wedge \neg t)$ | $(r \to s) \wedge [t \to \neg(r \vee s)]$ |
|---|---|---|---|---|
| $T$ | $T$ | $T$ | $F$ | $F(\wedge F)$ |
| $T$ | $T$ | $F$ | $T(\to T)$ | $T$ |
| $T$ | $F$ | $T$ | $F$ | $F(F \wedge)$ |
| $T$ | $F$ | $F$ | $F$ | $F(F \wedge)$ |
| $F$ | $T$ | $T$ | $F$ | $F(\wedge F)$ |
| $F$ | $T$ | $F$ | $T(\to T)$ | $T$ |
| $F$ | $F$ | $T$ | $T(F \to)$ | $T$ |
| $F$ | $F$ | $F$ | $T(F \to)$ | $T$ |

2 (20 pts). Consider the quantified predicate

$$(\forall x \in D)(\exists y \in D)\left\lfloor (x \geq 1) \rightarrow \left( y^2 = x \right) \right\rfloor$$

where the domain D is a subset of the real numbers. Over which of the following possible domains D is this true?
EXPLAIN your answer.

2a (4). D = the real numbers.
SOL: TRUE. Every real x>1 has a real sq. root y.

2b (4). D = the rational numbers.
SOL: FALSE. Let x = 2. Then x>1, but $\sqrt{x}$ is irrational.

2c (4). D = the squares of the integers: 0, 1, 4, 9, 16, ... .
SOL: FALSE. Let x = 4. Then $\sqrt{x} = 2 \notin D$.

2d (4). D = the negative real numbers.
SOL: TRUE. The antecedent (x>1) is always false.

2e (4). D = {-1, 1}.
SOL: TRUE. x=1 is the only x≥1. $\sqrt{1} = \pm 1$.

3a (10 pts) To prove that $4n^3 \in \vartheta\left(n^{\underline{3}}\right)$, it is sufficient to verify the existential proposition $(\exists M)(\forall n>M)\left\lfloor 4n^3 \;<\; 9n^{\underline{3}}\right\rfloor$. Verify that existential proposition.

SOLUTION: (a sample proof)

*The ineq.* $4n^3 < 9n^{\underline{3}} = 9n^3 - 27n^2 + 18n$

*is equiv to* $5n^3 - 27n^2 + 18n \;>\; 0$.

*Now find M such that*

$5M^3 - 27M^2 + 18M \;>\; 0$

*and is* increasing.

*e.g., if* $5n^3 - 30n^2 + 25n \;>\; 7n$, *which is true if*

$n^2 - 6n + 5 \;>\; \frac{7}{5}$, *and thus for all* $n > 6$. *Choose* $M = 6$.

*Or test with* $M = 5$ *and prove* increasing.

3b (10). Now prove that $4n^{\underline{3}} \in \vartheta\left(n^3\right)$.

SOLUTION: (a sample proof)

*Suff to prove* $4n^3 - 12n^2 + 8n \;<\; 4n^3$

*simplifies to* $12n^2 - 8n > 0$

*or to* $3n - 2 \;>\; 0$. *Use* $M = 2$.

4a (6pts). A number is called ***perfect*** if it is the sum of its proper divisors. Decide whether 496 is a perfect number.

SOLUTION: $496 = 31 \bullet 2^4$

proper divisors are 1, 2, 4, 8, 16 (subtotal =31)
and 31•1, 31•2, 31•4, 31•8 (subtotal = 31•15).

Thus, 496 is perfect, because it equals the sum of its proper divisors.

4b (5pts). Convert the numeral $1728_{10}$ to base 7.

SOLUTION:

| | | | | | |
|---|---|---|---|---|---|
| 1728 | ÷ | 7 | = | 246 | *R*6 |
| 246 | ÷ | 7 | = | 35 | *R*1 |
| 35 | ÷ | 7 | = | 5 | *R*0 |
| 5 | ÷ | 7 | = | 0 | *R*5 |

Thus, $1728_{10} = 5016_7$

4c (7pts). Calculate $15^{49}$ mod 11.

SOLUTION:

$15^{49} \equiv 4^{49} \equiv 4^{9} \mod 11$

$\equiv 2^{18} \equiv 2^{8} \equiv 256 \mod 11$

$\equiv 3$

4d (5pts). Show that $1111_{12}$ is divisible by five.

SOLUTION: (sample proof)

$1 + 12 + 12^2 + 12^3 \equiv$

$1 + 2 + 2^2 + 2^3 \mod 5 \equiv$

$15 \mod 5 \equiv 0$

4e (2pts). Show that $11111111_{12}$ is divisible by five.

SOLUTION:

$1111_{12}$ divides $11111111_{12}$

5 divides $1111_{12}$ by (4d).

5a (6). Calculate gcd(429, 969) by prime power factorization.

SOLUTION:

429 = 3•11•13
969 = 3•17•19
gcd = 3

5b (8). Calculate gcd(429, 969) by the Euclidean algorithm.

SOLUTION:

| | | | |
|---|---|---|---|
| 969 | 429 | 2 | 111 |
| 429 | 111 | 3 | 96 |
| 111 | 96 | 1 | 15 |
| 96 | 15 | 6 | 6 |
| 15 | 6 | 2 | 3 |
| 6 | 3 | 2 | 0 |

gcd = 3

5c (1). Find integers M and N such that $0 < 43M + 23N < 23$.

SOLUTION:

43•(-1) + 23•2 = 3

5d (5). Find integers M and N such that $43M + 23N = 1$.

SOLUTION:

1 = 3•8 + 23•(-1)
= [43•(-1) + 23•2]•8 + 23•(-1)
= 43•(-8) + 23•15 = 1 M=-8, N=15