DISCRETE MATH[1] W3203 Quiz 1

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Your Name (2 pts for LEGIBLY PRINTING your name on this line)

Problem	Points	Score
your name	2	
1	10	
2	15	
3	18	
4	15	
5	20	
6	20	
Total	100	

SUGGESTION: Do the EASIEST problems first!

HINT: Some of the solution methods involve highschool math as well as new methods from this class.

[1]An example of the Reasonable Person Principle: A reasonable student expects to lose a lot of credit for neglecting to EXPLAIN an answer. Omit explanations at your own risk.

1 (13 pts). Use a truth table to decide whether this argument is valid.

$$\begin{array}{l} q \rightarrow p \\ \neg q \rightarrow r \\ \underline{\quad \neg p \quad} \\ \therefore \quad r \end{array}$$

p	q	r		
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

2a (15 pts). Decide which of the following functions are one-to-one.

2a (5). $f(x) = 3x \mod 7$ from the set $\{0,1,\ldots,6\}$ to itself.

2b (5). $f(x) = 4x \mod 6$ from the set $\{0,1,\ldots,5\}$ to itself.

2c (5). $f(x) = \lfloor \pi x \rfloor \mod 7$ from the set $\{0,1,\ldots,6\}$ to itself.

3 (18 pts). Prove that $\left|\left(\frac{-1}{2}\right)^{\underline{n}}\right|$ asymptotically dominates 2^n.

3a (6). First express $\left|\left(\frac{-1}{2}\right)^{\underline{n}}\right|$ as a product of n factors.

3b (6). Next find a value K for which $\left|\left(\frac{-1}{2}\right)^{\underline{K}}\right| > 2^K$.

3c (6). Prove that $\left|\left(\frac{-1}{2}\right)^{\underline{K+r}}\right| > 2^{K+r}$ for all $r > 0$.

4a (10pts). Convert 1246_7 to a base-6 numeral.
ZERO CREDIT unless your work is shown.

4b (5pts). Show that 667_{77} is divisible by 19.
ZERO CREDIT unless your work is shown.
Hint: First expand as a polynomial in 77, and then substitute
77= 4•19+1.

5a (6). Calculate gcd(3122, 833) by prime power factorization.

5b (8). Calculate gcd(3122, 833) by the Euclidean algorithm.

5c (6). Find integers M and N such that

$$3122M + 833N = \gcd(3122, 833).$$

Hint: divide both sides by gcd to simplify.

6a (10 pts). Calculate $147^{810} \bmod 13$.

6b (5 pts). Find the multiplicative inverse of 13 mod 17.

6c (5 pts). Find the multiplicative inverse of 41 mod 51.
Hint: Find M such that $41M + 51N = 1$.