DISCRETE MATH[1] W3203 Quiz 1

open book

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Your Name (2 pts for legibly PRINTING your name)

Problem	Points	Score
your name	2	
1	13	
2	20	
3	20	
4	25	
5	20	
Total	100	

SUGGESTION: Do the EASIEST problems first!

HINT: Some of the solution methods involve highschool math as well as new methods from this class.

[1] An example of the Reasonable Person Principle: A reasonable student expects to lose a lot of credit for neglecting to EXPLAIN an answer. Omit explanations at your own risk.

1 (13 pts). Decide whether these two propositions are equivalent.

r	s	t	$(s \vee r) \rightarrow (s \wedge \neg t)$	$(r \rightarrow s) \wedge [t \rightarrow \neg(r \vee s)]$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

2 (20 pts). Consider the quantified predicate

$$(\forall x \in D)(\exists y \in D)\left\lfloor (x \geq 1) \rightarrow \left(y^2 = x \right) \right\rfloor$$

where the domain D is a subset of the real numbers. Over which of the following possible domains D is this true?
EXPLAIN your answer.

2a (4). D = the real numbers.

2b (4). D = the rational numbers.

2c (4). D = the squares of the integers: 0, 1, 4, 9, 16, ...

2d (4). D = the negative real numbers.

2e (4). D = {-1, 1}.

3a (10 pts) To prove that $4n^3 \in \vartheta\left(n^3\right)$, it is sufficient to verify the existential proposition $(\exists M)(\forall n>M)\left\lfloor 4n^3 \;<\; 9n^3 \right\rfloor$. Verify that existential proposition.

3b (10). Now prove that $4n^3 \in \vartheta\left(n^3\right)$.

4a (6pts). A number is called ***perfect*** if it is the sum of its proper divisors. Decide whether 496 is a perfect number.

4b (5pts). Convert the numeral 1728_{10} to base 7.

4c (7pts). Calculate 15^{49} mod 11.

4d (5pts). Show that 1111_{12} is divisible by five.

4e (2pts). Show that 11111111_{12} is divisible by five.

5a (6). Calculate gcd(429, 969) by prime power factorization.

5b (8). Calculate gcd(429, 969) by the Euclidean algorithm.

5c (1). Find integers M and N such that $0 < 43M + 23N < 23$.

5d (5). Find integers M and N such that $43M + 23N = 1$.