DISCRETE MATH\textsuperscript{1} W3203 Quiz 1

open book

\begin{center}
\begin{tabular}{lll}
Problem & Points & Score \\
your name & 2 & \\
1 & 13 & \\
2 & 20 & \\
3 & 20 & \\
4 & 25 & \\
5 & 20 & \\
\end{tabular}
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\begin{center}
Total 100
\end{center}

SUGGESTION: Do the EASIEST problems first!

HINT: Some of the solution methods involve highschool math as well as new methods from this class.

\textsuperscript{1} An example of the Reasonable Person Principle: A reasonable student expects to lose a lot of credit for neglecting to EXPLAIN an answer. Omit explanations at your own risk.
1 (13 pts). Decide whether these two propositions are equivalent.

\[
\begin{array}{ccc|c|c}
 r & s & t & (s \lor r) \to (s \land \neg t) & (r \to s) \land [t \to \neg(r \lor s)] \\
 T & T & T & & \\
 T & T & F & & \\
 T & F & T & & \\
 T & F & F & & \\
 F & T & T & & \\
 F & T & F & & \\
 F & F & T & & \\
 F & F & F & & \\
\end{array}
\]
2 (20 pts). Consider the quantified predicate
\[
(\forall x \in D)(\exists y \in D)\left( (x \geq 1) \rightarrow (y^2 = x) \right)
\]
where the domain \( D \) is a subset of the real numbers. Over which of the following possible domains \( D \) is this true? **EXPLAIN** your answer.

2a (4). \( D = \) the real numbers.

2b (4). \( D = \) the rational numbers.

2c (4). \( D = \) the squares of the integers: 0, 1, 4, 9, 16, ... .

2d (4). \( D = \) the negative real numbers.

2e (4). \( D = \{-1, 1\} \).
3a (10 pts) To prove that $4n^3 \in \vartheta(n^3)$, it is sufficient to verify the existential proposition $(\exists M)(\forall n > M)\left[4n^3 < 9n^3\right]$. Verify that existential proposition.

3b (10). Now prove that $4n^3 \in \vartheta(n^3)$. 
4a (6pts). A number is called **perfect** if it is the sum of its proper divisors. Decide whether 496 is a perfect number.

4b (5pts). Convert the numeral $1728_{10}$ to base 7.
4c (7pts). Calculate $15^{49} \mod 11$.

4d (5pts). Show that $1111_{12}$ is divisible by five.

4e (2pts). Show that $11111111_{12}$ is divisible by five.
5a (6). Calculate \( \gcd(429, 969) \) by prime power factorization.

5b (8). Calculate \( \gcd(429, 969) \) by the Euclidean algorithm.

5c (1). Find integers \( M \) and \( N \) such that \( 0 < 43M + 23N < 23 \).

5d (5). Find integers \( M \) and \( N \) such that \( 43M + 23N = 1 \).