DISCRETE MATH[1] W3203 FINAL EXAM

open book

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Your Name (2 pts for legibly PRINTING your name)

Problem	Points	Score
your name	2	
1	23	
2	20	
3	25	
4	25	
5	30	
6	30	
7	20	
8	25	
Total	200	

SUGGESTION: Do the EASIEST problems first!

HINT: Some of the solution methods involve highschool math as well as new methods from this class.

[1] An example of the Reasonable Person Principle: A reasonable student expects to lose a lot of credit for neglecting to EXPLAIN an answer. Omit explanations at your own risk.

1 (23 pts). Consider the following recurrence system:

$$a_0 = 0; \quad a_1 = 1;$$

$$a_n = 4a_{n-2} + 1$$

1a (2). Calculate a_2, a_3, a_4, *and* a_5.

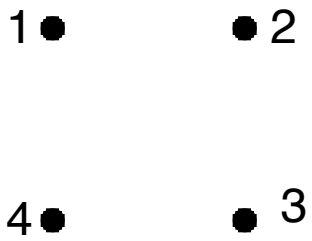
1b (21). Find a closed form for a_n.

2 (20 pts). Consider this binary relation on $\{1, 2, 3, 4\}$:

$$R = \{(2,1),(2,3),(2,4),(3,3),(4,3)\}$$

2a (5). Represent R as a matrix.

2b (5). Represent R as a digraph.



2c (10). Calculate the relation R^2.

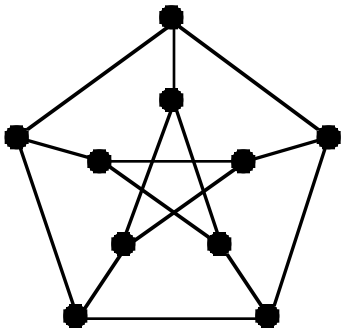
3a (10 pts). Give an example of a relation R on the set

$$S = \{a,\ b,\ c,\ d\}$$

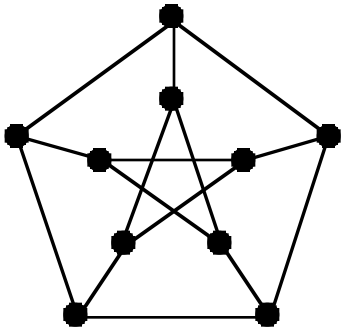
such that R is both an equivalence relation and a partial ordering.

3b (15 pts). Prove that there is only one such relation R.

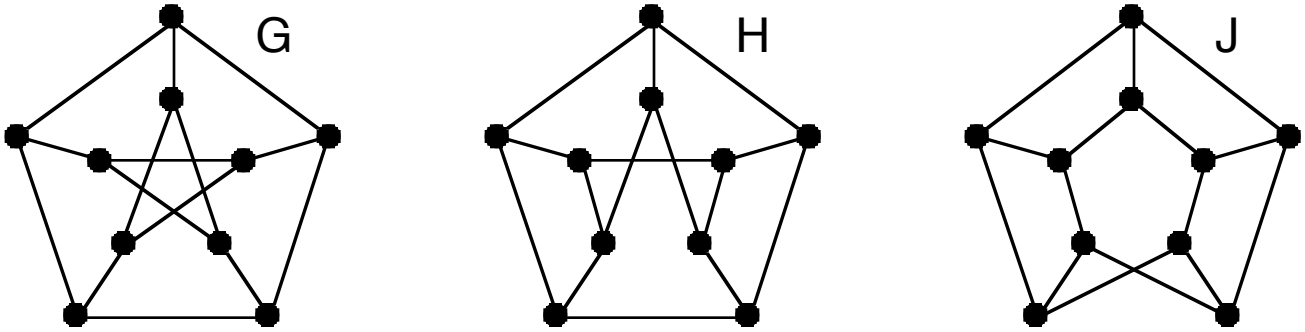
4a (10). The *diameter* of a graph is the maximum, taken over all vertex pairs u, v of the distance between u and v. Calculate the diameter of the Petersen graph.



4b (15). Calculate the chromatic number of the Petersen graph.



5 (30 pts). Consider the following three graphs:

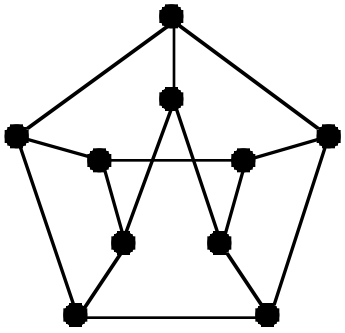


5a (10). Decide whether G and H are isomorphic. Explain.

5b (10). Decide whether G and J are isomorphic. Explain.

5c (10). Decide whether H and J are isomorphic. Explain.

6a (15). Mark two edges in the following graph such that it remains nonplanar after both are deleted. Prove your answer.



6b (15). Let G be the edge-complement of a 10-vertex, 3-regular simple graph. Prove that G is non-planar.

7 (20). Consider the following recurrence:

$$a_0 = 0; \quad a_n = n^2 - na_{n-1}$$

7a (3). Calculate a_1, a_2, *and* a_3.

7c (17). Prove that $a_n = n$.

8 (25 pts). A bowl contains 50 fair coins {with p(H) = 0.5} and 50 standard loaded coins {with p(H) = 0.8}. A coin is drawn at random.

8a (10). The coin is tossed once. What is the probability of the outcome H?

8b (15). If the outcome of the toss is H, what is the probability that the coin selected was loaded?