

# DISCRETE MATH<sup>1</sup> W3203 Final Exam

open book

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Your Name (2 pts for LEGIBLY PRINTING your name on this line)

Problem	Points	Score
your name	2	
1	18	
2	20	
3	20	
4	25	
5	25	
6	30	
7	35	
8	25	
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Total	200	

**SUGGESTION:** Do the EASIEST problems first!

**HINT:** Some of the solution methods involve highschool math as well as new methods from this class.

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<sup>1</sup>An example of the Reasonable Person Principle: A reasonable student expects to lose a lot of credit for neglecting to EXPLAIN an answer. Omit explanations at your own risk.

1a (3 pts). For the following recursion, calculate  $a_3$ ,  $a_4$  and  $a_5$ .

$$a_0 = 0, \quad a_1 = 1, \quad a_2 = 2; \quad a_n = \frac{n}{n-1}a_{n-1} + \frac{n}{n-2}a_{n-2} \quad \text{for } n \geq 3$$

1b (15). Use induction to prove that  $a_n = nf_n$ , where  $f_n$  is the  $n^{\text{th}}$  number in the Fibonacci sequence  $0, 1, 1, 2, 3, 5, \dots$ .

2 (20 pts). Solve the following recursion:

$$a_0 = 2, \quad a_1 = 3; \quad 6a_n = 5a_{n-1} - a_{n-2}$$

3 (20 pts). Calculate the value of the general coefficient  $a_n$  in the

power series expansion  $\frac{2 - 3x}{1 - 10x + 21x^2} = \sum_{n=0}^{\infty} a_n x^n$

4 (25). For two positive integers, we write  $m \prec n$  if the sum of the (distinct) prime factors of the first is less than or equal to the product of the (distinct) prime factors of the second. For instance,  $75 \prec 14$ , because  $3 + 5 \leq 2 \cdot 7$ .

4a (5). Is this relation reflexive? Explain.

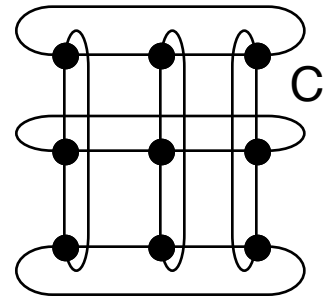
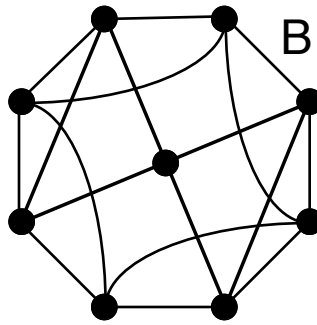
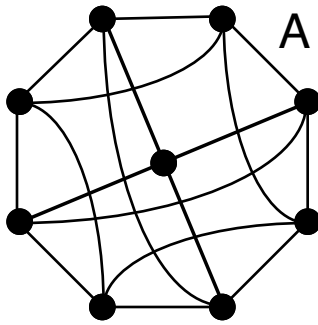
4b (10). Is this relation anti-symmetric? Explain.

4c (10). Is this relation transitive? Explain.

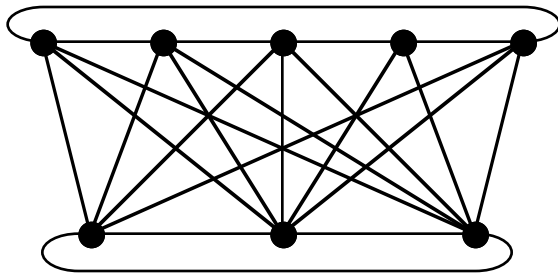
5a (10). Draw two non-isomorphic 5-vertex, 5-edge simple graphs with the same degree sequence.

5b (15). Draw all possible 7-vertex trees with maximum degree 3.

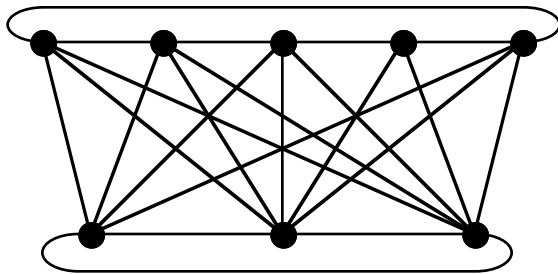
6 (30pts). Which pairs of these graphs are isomorphic. Explain.



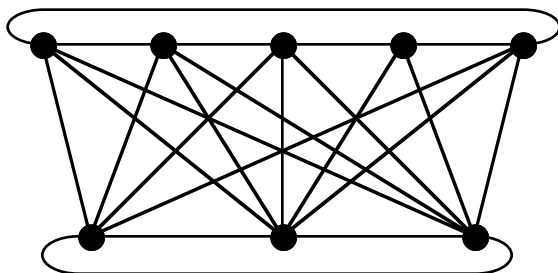
7a (10 pts). Calculate the chromatic number of this graph?



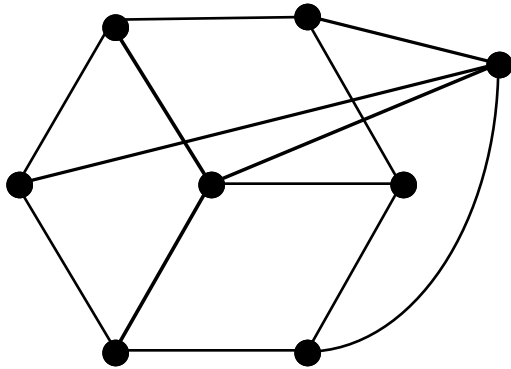
7b (15). Show three places where adding a single edge would increase the chromatic number.



7c (10). Show where to delete an edge to decrease the chromatic number.



8a (10 pts). Decide whether the following graph is planar.



8b (15 pts). Decide whether the following graph is planar.

