Compiling Parallel Algorithms to Memory Systems

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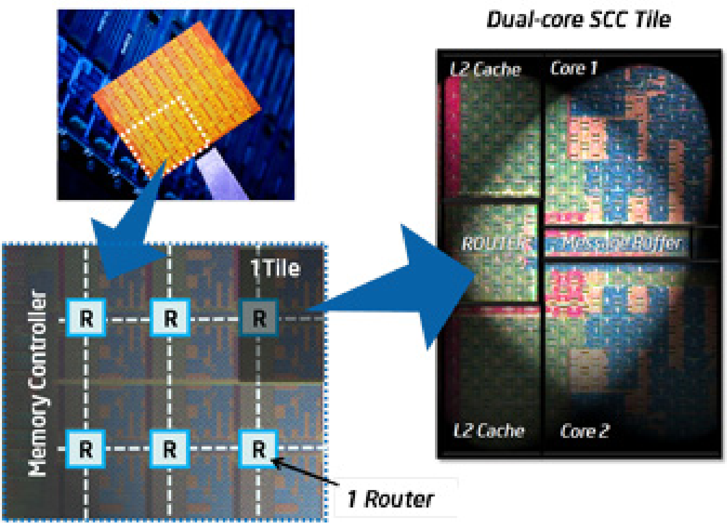
$$(\lambda x.?)f = \text{FPGA}$$

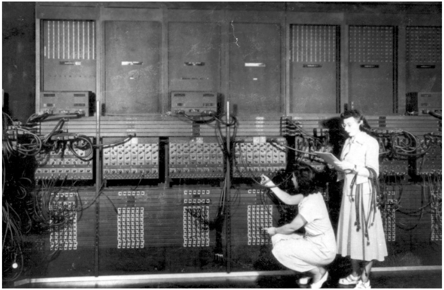
Parallelism is the Big Question

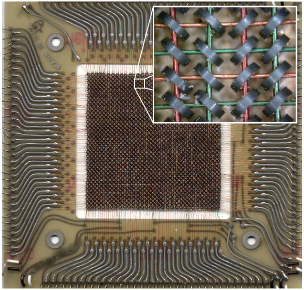
Massive On-Chip Parallelism is Inevitable



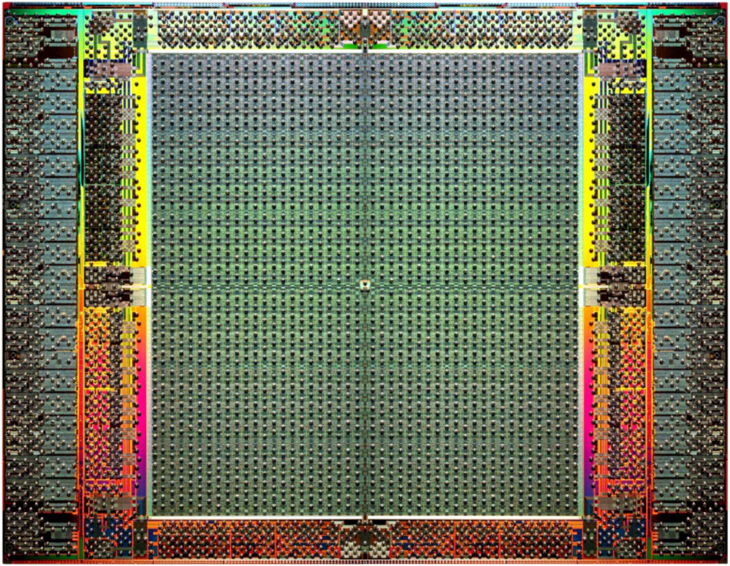
Intel’s 48-core “Single Chip Cloud Computer”

The Future is Wires and Memory



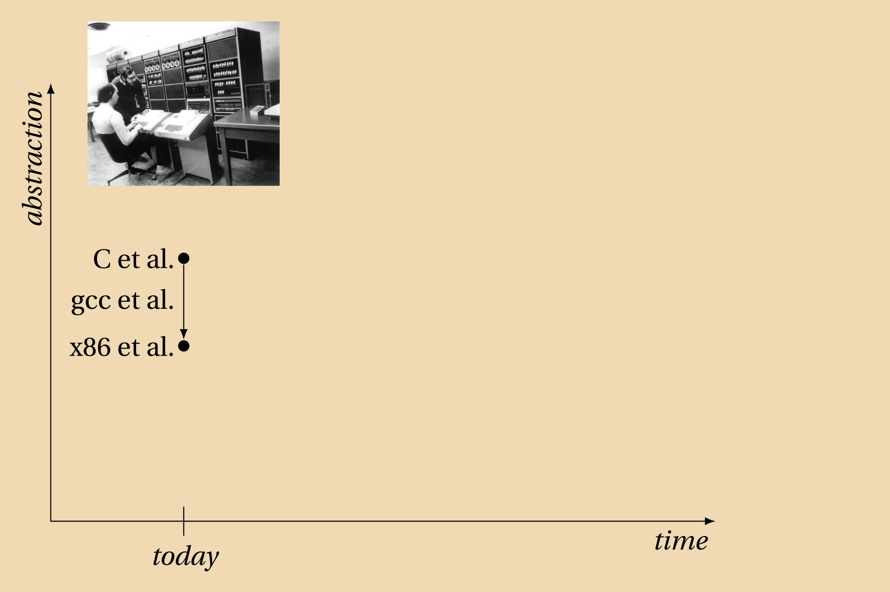


...and it's Already Here



Altera Stratix IV FPGA

What We are Doing About It



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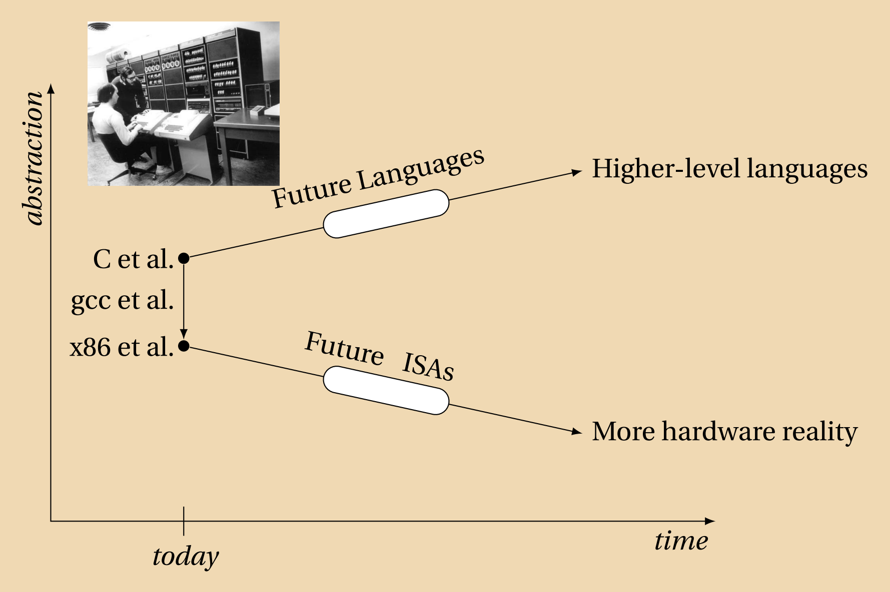

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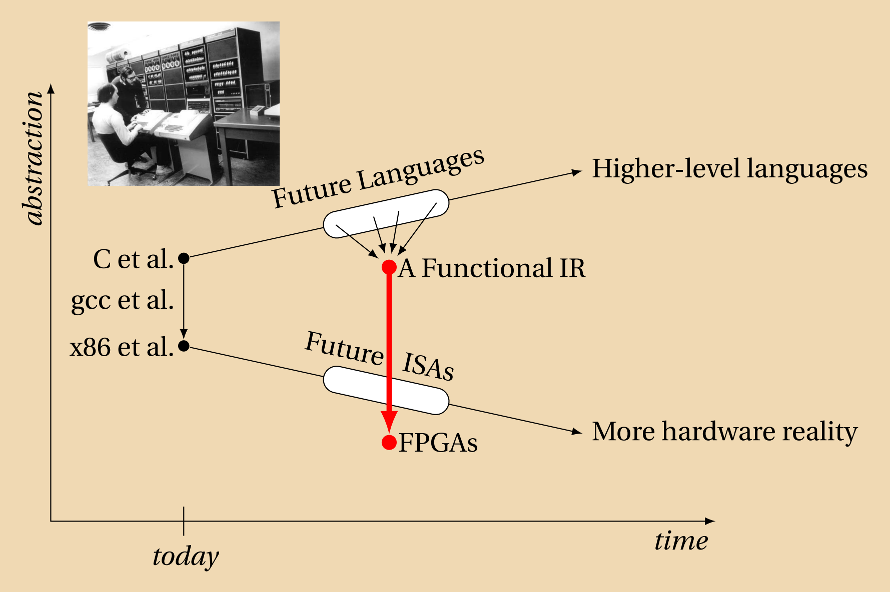
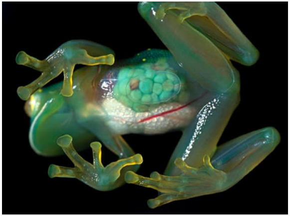


Why Functional Specifications?

- Referential transparency/side-effect freedom make formal reasoning about programs vastly easier

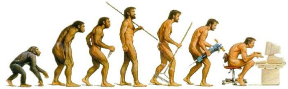
- Inherently concurrent and race-free (Thank Church and Rosser). If you want races and deadlocks, you need to add constructs.

- Immutable data structures makes it vastly easier to reason about memory in the presence of concurrency



Why FPGAs?

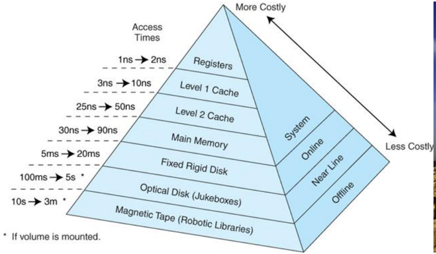
- We do not know the structure of future memory systems
  Homogeneous/Heterogeneous?
  Levels of Hierarchy?
  Communication Mechanisms?



- We do not know the architecture of future multi-cores
  Programmable in Assembly/C?
  Single- or multi-threaded?

Use FPGAs as a surrogate. Ultimately too flexible, but representative of the long-term solution.

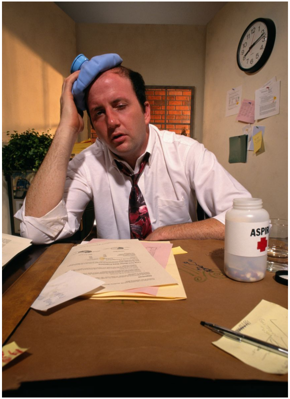
The Memory Hierarchy is the Interesting Part



Multiprocessor Memory is a Headache

- Cache Coherency
- Write buffers
- Sequential Memory Consistency
- Memory barriers
- Data Races
- Atomic operations

Immutable data structures simplify these



The Practical Question

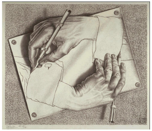
How do we synthesize hardware
from pure functional languages
for FPGAs?

Control and datapath are easy; the memory system is interesting.

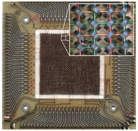
To Implement Real Algorithms in Hardware, We Need

Structured, recursive data types

Recursion to handle recursive data types



Memories



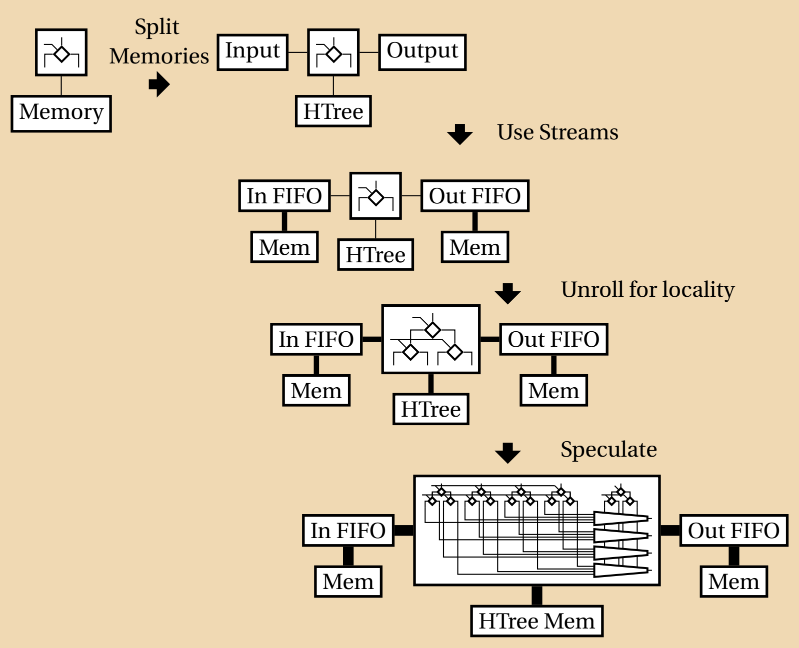
Memory Hierarchy

Example: Huffman Decoder in Haskell

```
data HTree = Branch HTree HTree
           | Leaf Char

decode :: HTree -> [Bool] -> [Char] -- Huffman tree & bitstream to symbols

decode table str = decoder table str
  where
    decoder (Leaf s) i = s : (decoder table i) -- Identified symbol; start again
    decoder _ [] = []
    decoder (Branch f _) (False:xs) = decoder f xs -- 0: follow left branch
    decoder (Branch _ t) (True:xs)  = decoder t xs -- 1: follow right branch
```

Three data types: Input bitstream, output character stream, and Huffman tree

Planned Optimizations



One Way to Encode the Types

Huffman tree nodes: (19 bits)

0	8-bit character	(unused)	Leaf Char
1	9-bit tree ptr.	9-bit tree ptr.	Branch Tree Tree

Boolean input stream: (10 bits)

0	(unused)		Nil
1	bit	8-bit tail pointer	Cons Bool List

Character output stream: (19 bits)

0	(unused)		Nil
1	8-bit character	10-bit tail pointer	Cons Char List

Intermediate Representation Desiderata

Mathematical formalism convenient for performing “parallelizing” transformations, a.k.a. parallel design patterns

- Pipeline
- Speculation
- Multiple workers
- Map-reduce

Intermediate Representation: Recursive "Islands"

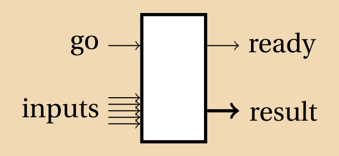
program	::= *island**	
island	::= **island** *name arg** = *expr state**	Group of states w/ stack
state	::= *label arg** = *expr*	Arguments & expression
expr	::= *name var**	Apply a function
	| **let** (*var* = *expr*)+ **in** *expr*	Parallel evaluation
	| **case** *var* **of** (*pattern* -> *expr*)+	Multiway conditional
	| *var*	
	| *literal*	
	| **recurse** *label var** (*var**)	Explicit continuation
	| **return** *var*	
	| **goto** *label var**	Branch to another state
pattern	::= *name var** | *literal* | _	Constructor/literal/def.

Huffman as a Recursive Island

```
data HTree = Branch HTree HTree
           | Leaf Char

decode :: HTree -> [Bool] -> [Char]

decode table str = decoder table str
  where
    decoder (Leaf s) i =
        s : (decoder table i)
    decoder _ [] = []
    decoder (Branch f _) (False:xs) =
        decoder f xs
    decoder (Branch _ t) (True:xs) =
        decoder t xs
```

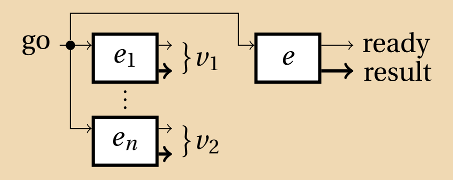
```
island decoder treep ip =
   let r = dec treep treep ip in return r

island dec treep statep ip =
  let i = fetchi ip
      state = fetcht statep in
  case state of
  Leaf a -> recurse s1 a (treep treep ip)
  Branch f t ->
    case i of
    Nil -> let np = Nil in return np
    Cons x xsp ->
      case x of
      True -> goto dec treep t xsp
      False -> goto dec treep f xsp

s1 a rp = let rrp = Cons a rp
          in return rrp
```

The Basic Translation Template

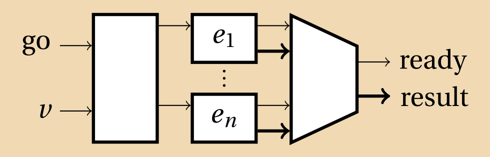


Strobe-based interface: *go* indicates inputs are valid; *ready* pulses once when result is valid.

Translating Let and Case

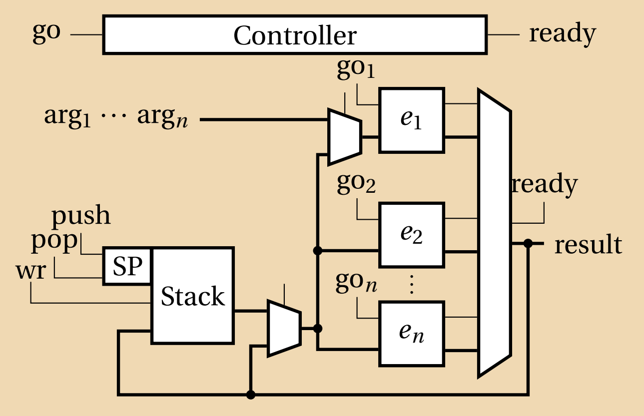


Let makes new values available to an expression.



Case invokes one of its sub-expressions, then synchronizes.

Translating an Island



Each island consists of expressions for each state, its own stack, and a controller that manages the stack and invokes the states.

Constructors and Memory

A constructor is a function that stores data in memory.

$$\text{constructor } \alpha :: \alpha \to \text{Ptr } \alpha$$

Memory access functions turn pointers into data.

$$\text{fetch } \alpha :: \text{Ptr } \alpha \to \alpha$$

Memory stores return an address, not take one as an argument

Constructor is responsible for memory management.

By default, each data type gets its own memory.

Duplication for Performance

$$
\begin{aligned}
fib\ 0 &= 0 \\
fib\ 1 &= 1 \\
fib\ n &= fib\ (n-1) + fib\ (n-2)
\end{aligned}
$$

Duplication for Performance

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\end{aligned}
$$

After duplicating functions:

$$
\begin{aligned}
&fib\ 0 = 0\\
&fib\ 1 = 1\\
&fib\ n = fib'\ (n-1) + fib''\ (n-2)
\end{aligned}
$$

$$
\begin{aligned}
&fib'\ 0 = 0\\
&fib'\ 1 = 1\\
&fib'\ n = fib'\ (n-1) + fib'\ (n-2)
\end{aligned}
$$

$$
\begin{aligned}
&fib''\ 0 = 0\\
&fib''\ 1 = 1\\
&fib''\ n = fib''\ (n-1) + fib''\ (n-2)
\end{aligned}
$$

Here, *fib'* and *fib''* may run in parallel.

Unrolling Recursive Data Structures

Like a “blocking factor,” but more general. Idea is to create larger memory blocks that can be operated on in parallel.

Original Huffman tree type:

data *Htree* = *Branch Htree HTree* | *Leaf* **Char**

Unrolled Huffman tree type:

data *Htree* = *Branch Htree' HTree'* | *Leaf* **Char**
data *Htree'* = *Branch' Htree'' HTree''* | *Leaf'* **Char**
data *Htree''* = *Branch'' Htree HTree* | *Leaf''* **Char**

Recursive instances must be pointers; others can be explicit.

Functions must be similarly modified to work with the new types.

Acknowledgements

Project started while at MSR Cambridge

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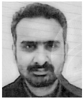

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