The Par Monad: Dataflow Parallelism

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The Par Monad

spawn and spawnP

parMap and parMapM

Example: Shortest Paths in a Graph

A Sequential Implementation

Parallelizing Floyd-Warshall

The Par Monad: Dataflow Parallelism

In Control.Monad.Par,

```haskell
newtype Par a = ...
instance Applicative Par ...
instance Monad Par ...

runPar :: Par a -> a
fork :: Par () -> Par ()

data IVar a = ...
new :: Par (IVar a)
put :: NFData =>
       IVar a -> a -> Par ()
get :: IVar a -> Par a
```

put forces evaluation of its argument (NFData)

```haskell
runPar $ do -- parmonad.hs
  i <- new                 -- Create IVar
  j <- new
  fork (put i (fib n)) -- Write result
  fork (put j (fib m))
  a <- get i               -- Wait for result
  b <- get j
  return (a+b)
```

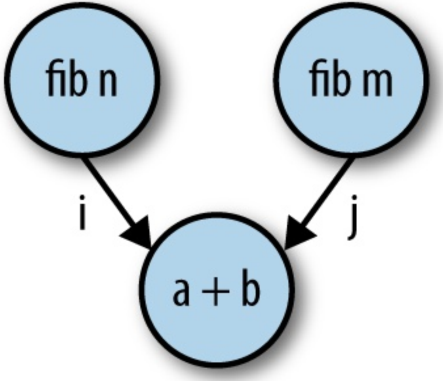
An IVar is a write-once variable

get waits for data to be *put*

Multiple *puts* to the same IVar cause a runtime error

Restrict each IVar to a single Par

The Par Monad: Dataflow Parallelism



Marlow, Fig. 4-1

```
runPar $ do -- parmonad.hs
  i <- new                -- Create IVar
  j <- new
  fork (put i (fib n)) -- Write result
  fork (put j (fib m))
  a <- get i              -- Wait for result
  b <- get j
  return (a+b)
```

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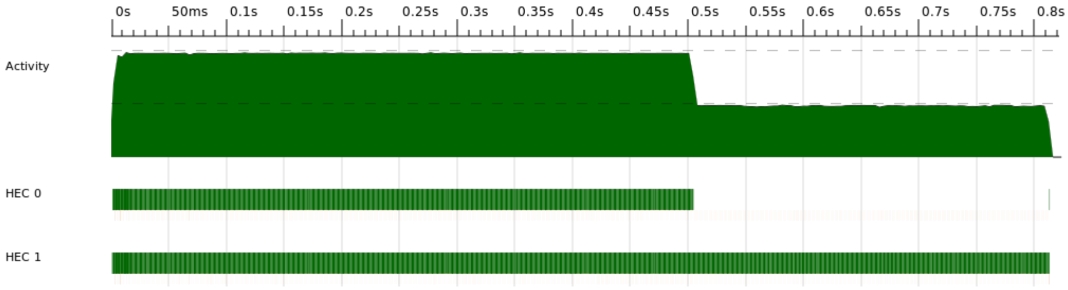
Running Marlow's parmonad.hs Example

```
$ stack ghc -- -O2 -threaded -rtsopts -eventlog parmonad.hs
$ ./parmonad 34 35 +RTS -N2 -ls
24157817
```



Works OK with `-N2`; obvious load-balancing problem. `-N8` slows it down

Control.Monad.Par.{spawn,spawnP}: Fork and Return New IVar

```
spawn :: NFData a => Par a -> Par (IVar a) -- Spawn a process
spawn p = do i <- new                      -- Create a new IVar i
             fork $ do x <- p              -- Run p
                       put i x             -- Put the result in i
             return i                      -- Return the IVar i

spawnP :: NFData a -> a -> Par (IVar a) -- Evaluate pure expression
spawnP = spawn . return
```

```
runPar $ do
  i <- spawnP (fib n) -- Start fib n in parallel with
  j <- spawnP (fib m) -- Start fib m
  a <- get i          -- Wait for fib n to finish
  b <- get j          -- Wait for fib m to finish
  return (a+b)
```

Control.Monad.Par.{parMapM,parMap}

parMapM applies a function that works in the monad:

```
parMapM :: NFData b => (a -> Par b) -> [a] -> Par [b]
parMapM f as = do
  ibs <- mapM (spawn . f) as   -- Run each in parallel
  mapM get ibs                 -- Wait for all list elements
```

parMap is similar but applies a pure function:

```
parMap :: NFData b => (a -> b) -> [a] -> Par [b]
parMap f as = do
  ibs <- mapM (spawn . return . f) as
  mapM get ibs
```

Actual implementations in Control.Monad.Par are more general

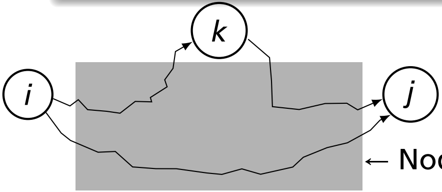
The Floyd-Warshall Shortest Paths Algorithm

The edge in g from i to j has `weight` $g\ i\ j$

Vertices are numbered $0 \ldots n$

In pseudocode,

```
shortestPath :: Graph -> Vertex -> Vertex -> Vertex -> Weight
shortestPath g i j 0 = weight g i j
shortestPath g i j k = min (shortestPath g i j (k-1))
                           (shortestPath g i k (k-1) +
                            shortestPath g k j (k-1))
```



$\leftarrow$ Nodes $< k$

Like *Fibonacci*, a recursive definition that should be implemented bottom-up with results recorded.

$O(n^3)$ overall

Sparse Graph Representation: Maps of Maps

Marlow's code in fwsparse/SparseGraph.hs

```
import qualified Data.IntMap.Strict as Map

type Vertex = Int
type Weight = Int

type Graph = IntMap (IntMap Weight)

weight :: Graph -> Vertex -> Vertex -> Maybe Weight
weight g i j = do
  jmap <- Map.lookup i g
  Map.lookup j jmap
```

IntMap is tuned to better work with Int keys

$O(n^3)$ Sequential Implementation

```
shortestPaths :: [Vertex] -> Graph -> Graph
shortestPaths vs g = foldl' update g vs where -- For each vertex k
  update g k = Map.mapWithKey shortmap g where -- For each vertex i
   shortmap i jmap = foldr shortest Map.empty vs -- Shortest from i
    where
     shortest j m = case (old,new) of -- Update path from i to j via k
       (Nothing, Nothing) -> m                    -- No path
       (Nothing, Just w ) -> Map.insert j w m     -- Found a new path
       (Just w,  Nothing) -> Map.insert j w m     -- Existing path only
       (Just w1, Just w2) -> Map.insert j (min w1 w2) m -- Best
      where
       old = Map.lookup j jmap        -- Previous i → j path
       new = do w1 <- weight g i k -- i → k
                w2 <- weight g k j -- k → j
                return (w1+w2)
```

Running Sequential Floyd-Warshall

Random graph with 1000 vertices and 800 nodes:

```
$ stack ghc -- -O2 -rtsopts fwsparse.hs
$ ./fwsparse 1000 800 +RTS -s
  Total   time   14.531s  ( 14.575s elapsed)
```

Fundamentally, three nested loops:

```
shortestPaths vs g = foldl' update g vs where
  update g k = Map.mapWithKey shortmap g where
   shortmap i jmap = foldr shortest Map.empty vs
```

Two are folds, which are difficult to parallelize unless operation is associative

However, mapWithKey is a map

Parallelizing Floyd-Warshall

mapWithKey is an unusual map over an IntMap.

We need a map that runs in the Par monad. Fortunately, IntMap provides

```
traverseWithKey :: Applicative t =>
    (Key -> a -> t b) -> IntMap a -> t (IntMap b)
```

and in Traversable,

```
traverse :: (Traversable t, Applicative f) =>
      (a -> f b) -> t a -> f (t b)
```

So we can update our *update* function to spawn *shortmap* in parallel:

```
  update g k = runPar $ do
    m <- Map.traverseWithKey
         (\i jmap -> spawnP (shortmap i jmap)) g
    traverse get m -- get each IVar in the IntMap
```

Running Parallel Floyd-Warshall

```
$ stack ghc -- -O2 -threaded -rtsopts -eventlog fwsparse1.hs
$ ./fwsparse1 1000 800 +RTS -s -N1
    Total   time    6.091s  (  6.150s elapsed)
$ ./fwsparse1 1000 800 +RTS -s -N8
    Total   time   12.071s  (  1.832s elapsed)
```

A 3.4× speedup on 8 cores, but we beat the sequential version (?)

Note the total time increased substantially (parallel overhead), but the elapsed time decreased anyway