

# Maze Generation: Recursive Division

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## 1 Introduction

This paper presents a parallelized maze generator implemented in Haskell. There are several algorithms to generate a maze, such as depth-first search, Kruskal's algorithm, Prim's algorithm. However, most of the algorithms are hard to run in parallel. In order to demonstrate the parallelization techniques in Haskell, we decided to use another algorithm, which is known as the Recursive Division algorithm. It is the fastest maze generation algorithm without directional biases thanks to its parallelism, each sub-maze could be processed at finer and finer levels of detail. However, compared to other more convoluted maze generation algorithms, the output maze usually contains long straight walls crossing the space and looks less complicated.

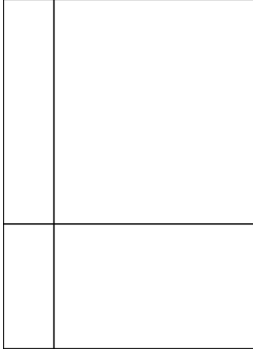
## 2 Recursive Division Algorithm

The Recursive Division works as follows:

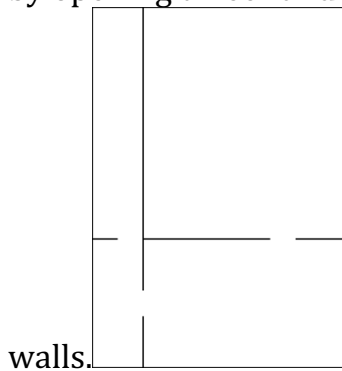
1. Initialize the maze with no walls inside, i.e. only a frame.



2. Divide the frame into four separate spaces by adding two randomly positioned walls, one vertically and one horizontally.



3. Randomly selected three from the four spaces and connected them together by opening three randomly positioned passages on the

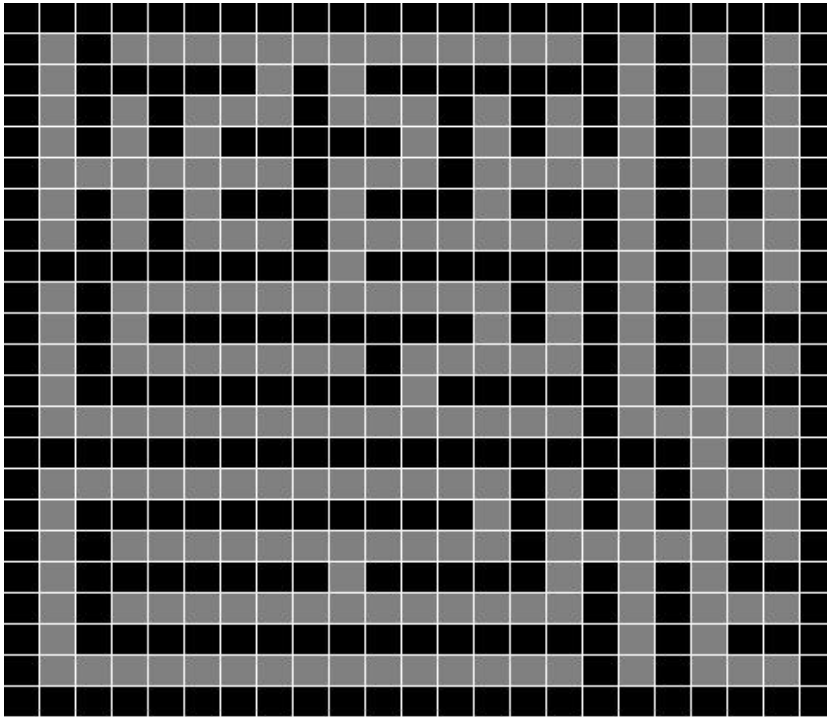


walls.

4. Recursively repeat the process on the sub maze until all sub maze can't be divided anymore.

## 2.1 Data Types

Maze could be abstracted as a grid, each passage in the maze could be seen as a cell in the grid, but there are usually two ways to represent the walls in the maze. One is using lines in the grid to represent the walls, whereas the other way represents walls also as cells. In our implementation, we represent both passage and wall using the cells.



We defined two main data types for our implementation, Maze and Wall. The Maze type is the abstract of the generated maze, with the property width and height represented by m and n. It also contains a set of Wall types, representing the walls in the maze.

```
data Maze = Maze {
  m :: Int, -- number of matrix rows
  n :: Int, -- number of matrix cols
  walls :: Set W.Wall
}

instance Show Maze where
  show (Maze m' n' walls') =
    unlines maze
  where
    maze = [rowToStrng r | r <- [0 .. m'-1]]
    rowToStrng r = [if isWall r c then '#' else ' ' | c <- [0 .. n'-
1]]
    isWall r c = W.Wall r c `elem` walls'

instance NFData Maze where
  rnf (Maze m' n' walls') = rnf m' `seq` rnf n' `seq` rnf walls'
```

The data type `Wall` has the properties `r` and `c` representing the row and column number of this wall block respectively.

```
data Wall = Wall {
  r :: Int,
  c :: Int
} deriving (Show, Eq, Ord)

instance NFData Wall where
  rnf (Wall r c) = rnf r `seq` rnf c
```

## 2.2 Implementation

The entry point of the algorithm is the function `generateWalls`, which generates walls in the area defined by four integers  $(x_0, y_0)$ ,  $(x_1, y_1)$  representing the top left and bottom right corners.

In this function, we split the given area into four sub mazes and solved each of them individually. In the end, we return the result by combining the four solutions.

```
-- Given an frame, generate walls inside the frame
-- top left wall cell: (tr, tc)
-- bottom right wall cell: (br, bc)
generateWalls :: RandomGen g => Int -> Int -> Int -> Int -> g ->
Set W.Wall
generateWalls tr tc br bc g
  | bc - tc < 4 || br - tr < 4 = empty
  | otherwise = walls
  `union` topLeft
  `union` topRight
  `union` bottomLeft
  `union` bottomRight
where
  (g1, g2) = split g
  (g3, g4) = split g1
  (g5, g6) = split g2
  (g7, g8) = split g3
  (randomRow, _) = pickRandom [(tr + 2), (tr + 4)..(br - 2)]
g1
  (randomCol, _) = pickRandom [(tc + 2), (tc + 4)..(bc - 2)]
g2
  walls = verticalWalls `union` horizontalWalls `difference`
holes
```

```
verticalWalls = fromList [W.Wall {W.r = r, W.c = randomCol}
| r <- [(tr + 1)..(br - 1)]]
horizontalWalls = fromList [W.Wall {W.r = randomRow, W.c =
c} | c <- [(tc + 1)..(bc - 1)]]
topLeft = generateWalls tr tc randomRow randomCol g3
topRight = generateWalls tr randomCol randomRow bc g4
bottomLeft = generateWalls randomRow tc br randomCol g5
bottomRight = generateWalls randomRow randomCol br bc g6
(holes, _) = getHoles tr tc br bc randomRow randomCol g7
```

## 2.3 Results

Below are the screenshots of the visualization of the generated maze, the program also accepts the third parameter, which is a seed integer provided by users. The idea is when the given seed is the same, the maze generated will be the same as well.



### 3 Parallelization

For the parallelization, we firstly checked our maze generation algorithms. Because we used recursive division, this algorithm keeps dividing the frames with a horizontal wall and a vertical wall. During this process, we find the possibility of parallelization for the algorithm.

The first step we conducted our experiment is that we run the algorithm in a sequential way to generate a 5000 \* 5000 maze and check the result.

```
Registering library for maze-generator-0.1.0.0..
44,440,349,848 bytes allocated in the heap
19,788,555,000 bytes copied during GC
 2,930,768,648 bytes maximum residency (19 sample(s))
  7,742,712 bytes maximum slop
    6426 MiB total memory in use (0 MB lost due to fragmentation)

Gen 0      42674 colls,    0 par   15.930s  16.283s   0.0004s   0.0110s
Gen 1       19 colls,    0 par   12.450s  16.819s   0.8852s   6.9059s

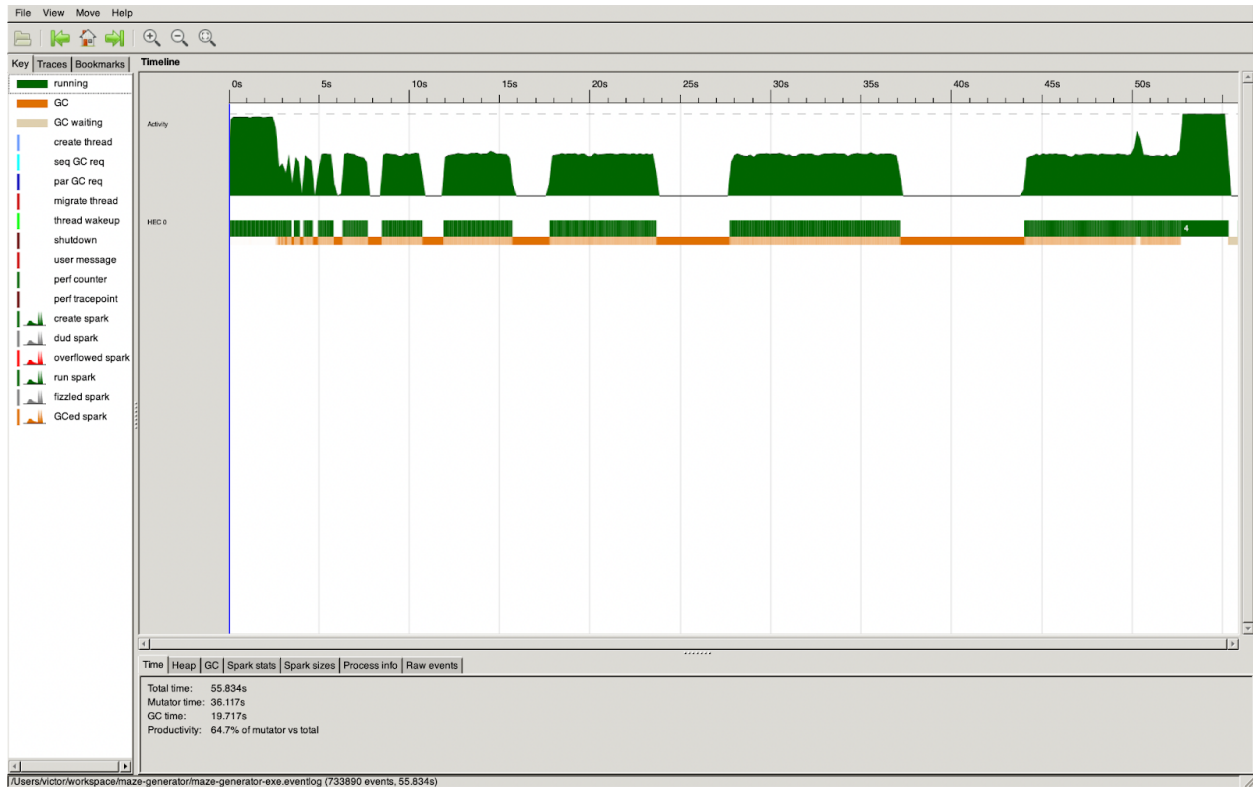
TASKS: 4 (1 bound, 3 peak workers (3 total), using -N1)

SPARKS: 0 (0 converted, 0 overflowed, 0 dud, 0 GC'd, 0 fizzled)

INIT   time    0.000s ( 0.004s elapsed)
MUT   time   20.831s ( 22.721s elapsed)
GC    time   28.380s ( 33.102s elapsed)
EXIT   time    0.000s ( 0.007s elapsed)
Total time   49.211s ( 55.834s elapsed)

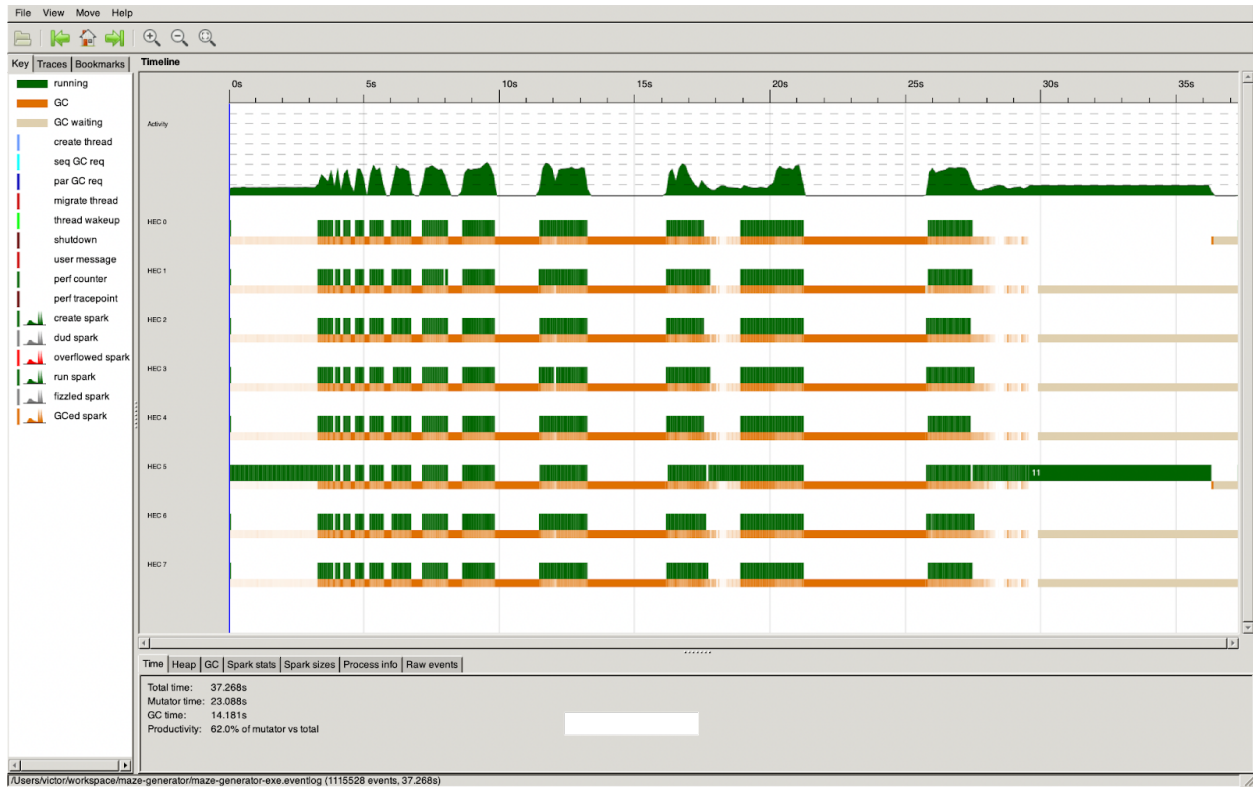
Alloc rate   2,133,407,274 bytes per MUT second

Productivity 42.3% of total user, 40.7% of total elapsed
```



After finding the total time is about 56s, we tried to use the first method to implement the parallelization. The first method is using par in the Control.Parallel package.





We can find that using par and running the code on 8 cores, that total time decreased to about 37s.

Then, we tried the Strategy and used runEval and rpar to do the parallelization. The result is below.

```

11K 33,000,000,000 bytes allocated in the heap
28,541,084,408 bytes copied during GC
3,522,292,104 bytes maximum residency (20 sample(s))
9,885,304 bytes maximum slop
8250 MiB total memory in use (0 MB lost due to fragmentation)

Gen 0      14285 colls, 14285 par    45.701s   6.420s   0.0004s   0.0038s
Gen 1       20 colls,   19 par    63.360s   7.592s   0.3796s   3.6877s

Parallel GC work balance: 81.67% (serial 0%, perfect 100%)

TASKS: 18 (1 bound, 17 peak workers (17 total), using -N8)

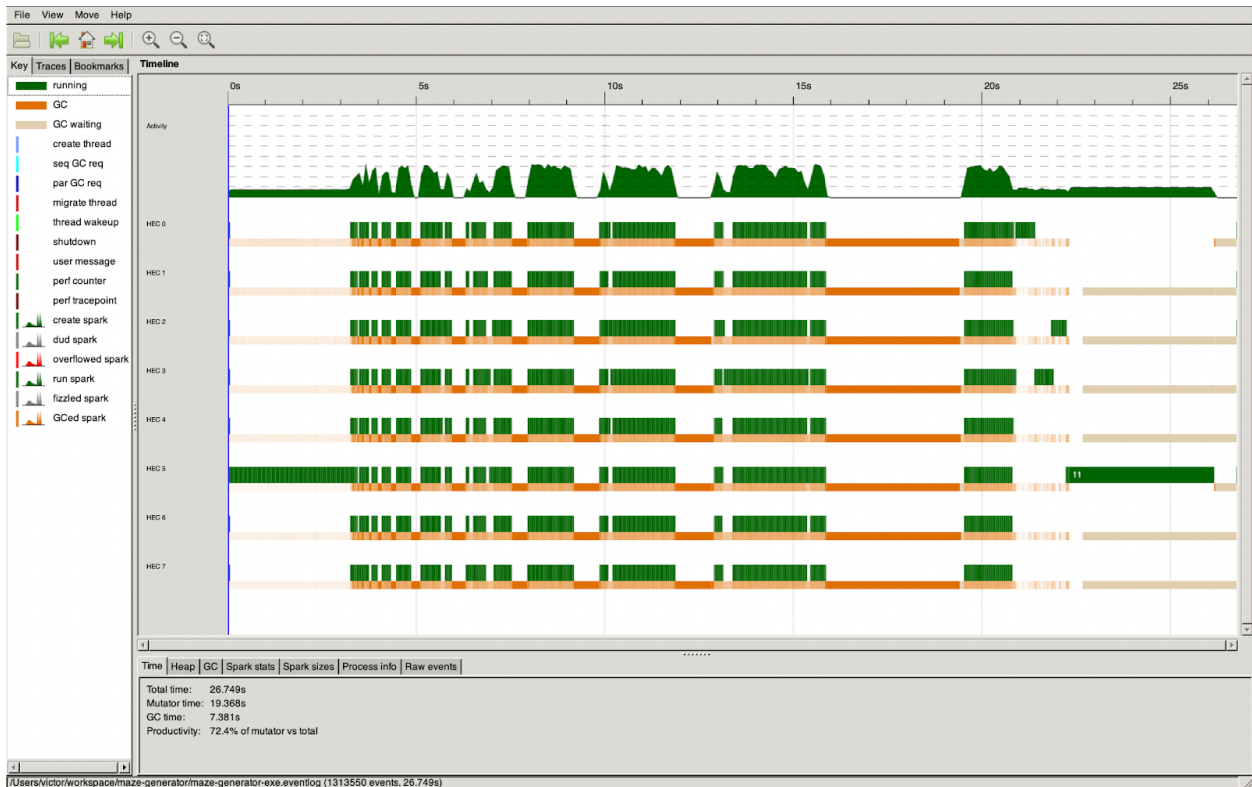
SPARKS: 3982630 (64821 converted, 0 overflowed, 0 dud, 3032758 GC'd, 885051 fizzled)

INIT   time    0.001s ( 0.006s elapsed)
MUT    time   42.418s (12.721s elapsed)
GC     time  109.061s (14.012s elapsed)
EXIT   time    0.000s ( 0.011s elapsed)
Total  time  151.480s (26.749s elapsed)

Alloc rate  1,367,530,482 bytes per MUT second

Productivity 28.0% of total user, 47.6% of total elapsed

```



We can see that using the rpar and running the code on 8 cores, the total time decreased to about 26s, far from the original sequential time of 56s. But, we still have a problem that the GC time is too much and there are a lot of unnecessary sparks generated during the code running. To solve this problem, we tried to control

the depth to parallel to a certain depth. We implemented the depth control like below.

```
generateHWalls 0 tr tc br bc seed = generateWallsOriginal tr tc br bc seed
generateWalls deep tr tc br bc seed
-- | trace ("generateHWalls for the space: " ++ show (tx, ty, bx, by)) False =
undefined
-- | trace ("holes: " ++ show holes) False = undefined
| bc - tc < 4 || br - tr < 4 = (empty, seed)
| otherwise = (walls
`union` runEval (rpar (topLeft `union` topRight))
`union` runEval (rpar (bottomLeft `union` bottomRight))
, newSeed5)
where
  (randomRow, newRowSeed) = pickRandom [(tr + 2), (tr + 4)..(br - 2)] seed
  (randomCol, newColSeed) = pickRandom [(tc + 2), (tc + 4)..(bc - 2)]
newRowSeed
  walls = verticalWalls `union` horizontalWalls `difference` holes
  verticalWalls = fromList [W.Wall {W.r = r, W.c = randomCol} | r <- [(tr + 1)..(br -
1)]]
  horizontalWalls = fromList [W.Wall {W.r = randomRow, W.c = c} | c <- [(tc +
1)..(bc - 1)]]
  (topLeft, newSeed1) = generateWalls (deep - 1) tr tc randomRow randomCol seed
  (topRight, newSeed2) = generateWalls (deep - 1) tr randomCol randomRow bc
seed
  (bottomLeft, newSeed3) = generateWalls (deep - 1) randomRow tc br randomCol
seed
  (bottomRight, newSeed4) = generateWalls (deep - 1) randomRow randomCol br
bc seed
  (holes, newSeed5) = getHoles tr tc br bc randomRow randomCol seed
```

```
generateWallsOriginal tr tc br bc seed
-- | trace ("generateHWalls for the space: " ++ show (tx, ty, bx, by)) False =
undefined
-- | trace ("holes: " ++ show holes) False = undefined
| bc - tc < 4 || br - tr < 4 = (empty, seed)
| otherwise = (walls
`union` runEval (rpar (topLeft `union` topRight))
`union` runEval (rpar (bottomLeft `union` bottomRight))
, newSeed5)
where
  (randomRow, newRowSeed) = pickRandom [(tr + 2), (tr + 4)..(br - 2)] seed
```

```

(randomCol, newColSeed) = pickRandom [(tc + 2), (tc + 4)..(bc - 2)]
newRowSeed
walls = verticalWalls `union` horizontalWalls `difference` holes
verticalWalls = fromList [W.Wall {W.r = r, W.c = randomCol} | r <- [(tr + 1)..(br -
1)]]
horizontalWalls = fromList [W.Wall {W.r = randomRow, W.c = c} | c <- [(tc +
1)..(bc - 1)]]
(topLeft, newSeed1) = generateWallsOriginal tr tc randomRow randomCol seed
(topRight, newSeed2) = generateWallsOriginal tr randomCol randomRow bc seed
(bottomLeft, newSeed3) = generateWallsOriginal randomRow tc br randomCol
seed
(bottomRight, newSeed4) = generateWallsOriginal randomRow randomCol br bc
seed
(holes, newSeed5) = getHoles tr tc br bc randomRow randomCol seed

```

After control the depth of the code, we found that the sparks generated decreased a lot.

```

(base) victor@victordembo maze-generator % stack run maze-generator-exe 4 5000 5000 6 -- +RTS -N8 -ls -s
48,551,710,512 bytes allocated in the heap
20,060,812,712 bytes copied during GC
2,863,613,448 bytes maximum residency (15 sample(s))
8,231,416 bytes maximum slop
6063 MiB total memory in use (0 MB lost due to fragmentation)

Gen 0      19770 colls, 19770 par   Tot time (elapsed)  Avg pause  Max pause
Gen 1       15 colls,   14 par   30.812s   3.810s   0.2540s   1.9843s

Parallel GC work balance: 65.14% (serial 0%, perfect 100%)

TASKS: 18 (1 bound, 17 peak workers (17 total), using -N8)

SPARKS: 170 (106 converted, 0 overflowed, 0 dud, 0 GC'd, 64 fizzled)

INIT   time   0.000s ( 0.005s elapsed)
MUT   time 25.587s (10.256s elapsed)
GC    time 75.818s ( 9.595s elapsed)
EXIT   time 0.000s ( 0.011s elapsed)
Total time 101.406s (19.868s elapsed)

Alloc rate 1,897,488,818 bytes per MUT second

Productivity 25.2% of total user, 51.6% of total elapsed

```

## 5 Code Listing

### Main.hs

```

module Main where
import System.Exit(die);
import System.Environment (getArgs, getProgName)

```

```

import System.Random (getStdGen)
import System.CPUTime
import Generator
import Control.DeepSeq

main :: IO Integer
main = do
  args <- getArgs
  seed <- getStdGen
  case args of
    [deep, width, height] -> do
      start <- getCPUTime
      let r = mazeGenerator (read deep) (read width) (read
height) seed
      end <- r `deepseq` getCPUTime
      return (end - start)
    _ -> do
      progName <- getProgName
      die $ "Usage: " ++ progName ++ " <width> <height>"

```

## Generator.hs (using rpar to parallelize)

```

module Generator where

import qualified Maze as M
import qualified Wall as W
import Debug.Trace
import Data.Bifunctor
import Control.Monad
import Data.Array.IO
import Data.Set (Set, fromList, union, empty, difference)
import System.Random
import Control.Parallel.Strategies

-- mazeGenerator :: RandomGen g => Int -> Int -> g -> M.Maze
mazeGenerator deep width height seed =
  M.Maze {
    M.m = m,
    M.n = n,

```

```

    M.walls = initializeFrame m n `union` fst (generateWalls deep
0 0 (m - 1) (n - 1) seed)
    -- M.walls = initializeFrame m n
}
where
    m = width * 2 + 1 -- number of matrix rows
    n = height * 2 + 1 -- number of matrix cols

```

```

initializeFrame :: Int -> Int -> Set W.Wall
initializeFrame m n = fromList frame

```

```

where
    frame = [ W.Wall {W.r = r, W.c = c} |
        r <- [0..m - 1],
        c <- [0..n - 1],
        r == 0 || r == m - 1 || c == 0 || c == n - 1,
        (r, c) /= (0, 1), -- entrance
        (r, c) /= (m - 1, n - 2)] -- exit

```

```

-- Given an frame, generate walls inside the frame
-- top left wall cell: (tr, tc)
-- bottom right wall cell: (br, bc)

```

```

generateHWalls 0 tr tc br bc seed = generateWallsOriginal tr tc
br bc seed

```

```

generateWalls deep tr tc br bc seed
    -- | trace ("generateHWalls for the space: " ++ show (tx, ty,
bx, by)) False = undefined
    -- | trace ("holes: " ++ show holes) False = undefined
    | bc - tc < 4 || br - tr < 4 = (empty, seed)
    | otherwise = (walls
        `union` runEval (rpar (topLeft `union` topRight))
        `union` runEval (rpar (bottomLeft `union` bottomRight))
        , newSeed5)

```

```

where
    (randomRow, newRowSeed) = pickRandom [(tr + 2), (tr + 4)..(br
- 2)] seed
    (randomCol, newColSeed) = pickRandom [(tc + 2), (tc + 4)..(bc
- 2)] newRowSeed
    walls = verticalWalls `union` horizontalWalls `difference`
holes

```

```

    verticalWalls = fromList [W.Wall {W.r = r, W.c = randomCol} |
r <- [(tr + 1)..(br - 1)]]
    horizontalWalls = fromList [W.Wall {W.r = randomRow, W.c = c}
| c <- [(tc + 1)..(bc - 1)]]
    (topLeft, newSeed1) = generateWalls (deep - 1) tr tc
randomRow randomCol seed
    (topRight, newSeed2) = generateWalls (deep - 1) tr randomCol
randomRow bc seed
    (bottomLeft, newSeed3) = generateWalls (deep - 1) randomRow tc br randomCol
seed
    (bottomRight, newSeed4) = generateWalls (deep - 1) randomRow
randomCol br bc seed
    (holes, newSeed5) = getHoles tr tc br bc randomRow randomCol
seed

```

```

generateWallsOriginal tr tc br bc seed

```

```

-- | trace ("generateHWalls for the space: " ++ show (tx, ty,
bx, by)) False = undefined

```

```

-- | trace ("holes: " ++ show holes) False = undefined

```

```

| bc - tc < 4 || br - tr < 4 = (empty, seed)

```

```

| otherwise = (walls

```

```

`union` runEval (rpar (topLeft `union` topRight))

```

```

`union` runEval (rpar (bottomLeft `union` bottomRight))

```

```

, newSeed5)

```

```

where

```

```

    (randomRow, newRowSeed) = pickRandom [(tr + 2), (tr + 4)..(br
- 2)] seed

```

```

    (randomCol, newColSeed) = pickRandom [(tc + 2), (tc + 4)..(bc
- 2)] newRowSeed

```

```

    walls = verticalWalls `union` horizontalWalls `difference`

```

```

holes

```

```

    verticalWalls = fromList [W.Wall {W.r = r, W.c = randomCol} |
r <- [(tr + 1)..(br - 1)]]

```

```

    horizontalWalls = fromList [W.Wall {W.r = randomRow, W.c = c}
| c <- [(tc + 1)..(bc - 1)]]

```

```

    (topLeft, newSeed1) = generateWallsOriginal tr tc randomRow
randomCol seed

```

```

    (topRight, newSeed2) = generateWallsOriginal tr randomCol
randomRow bc seed

```

```

    (bottomLeft, newSeed3) = generateWallsOriginal randomRow tc
br randomCol seed

```

```

(bottomRight, newSeed4) = generateWallsOriginal randomRow
randomCol br bc seed
(holes, newSeed5) = getHoles tr tc br bc randomRow randomCol
seed

```

```

getHoles :: RandomGen g => Int -> Int -> Int -> Int -> Int ->
Int -> g -> (Set W.Wall, g)
getHoles tr tc br bc rr rc seed =
  (fromList $ map (choices !!) $ take 3 $ shuffle seed [0..3],
  bottomSeed)
  where
    randomPick l seed = l !! fst (randomR (0, length l - 1) seed)
    choices = [top, left, right, bottom]
    (top, topSeed) = pickRandom [W.Wall {W.r = r, W.c = rc} | r <-
[(tr + 1), (tr + 3)..(rr - 1)]] seed
    (left, leftSeed) = pickRandom [W.Wall {W.r = rr, W.c = c} | c
<- [(tc + 1), (tc + 3)..(rc - 1)]] topSeed
    (right, rightSeed) = pickRandom [W.Wall {W.r = rr, W.c = c} |
c <- [(rc + 1), (rc + 3)..(bc - 1)]] leftSeed
    (bottom, bottomSeed) = pickRandom [W.Wall {W.r = r, W.c =
rc} | r <- [(rr + 1), (rr + 3)..(br - 1)]] rightSeed

```

```

pickRandom :: RandomGen g => [a] -> g -> (a, g)
pickRandom l seed = Data.Bifunctor.first (l !!) (randomR (0,
length l - 1) seed)

```

```

shuffle :: RandomGen g => g -> [a] -> [a]
shuffle gen [] = []
shuffle gen list = randomElem : shuffle newGen newList
  where
    randomTuple = randomR (0, length list - 1) gen
    randomIndex = fst randomTuple
    newGen      = snd randomTuple
    randomElem  = list !! randomIndex
    newList     = take randomIndex list ++ drop (randomIndex+1)
list

```

## Generator.hs (Sequential)



```

module Generator where

import qualified Maze as M
import qualified Wall as W
import Debug.Trace
import Data.Bifunctor
import Control.Monad
import Data.Array.IO
import Data.Set (Set, fromList, union, empty, difference)
import System.Random

mazeGenerator :: RandomGen g => Int -> Int -> g -> M.Maze
mazeGenerator width height seed =
  M.Maze {
    M.m = m,
    M.n = n,
    M.walls = initializeFrame m n `union` fst (generateWalls 0 0
(m - 1) (n - 1) seed)
    -- M.walls = initializeFrame m n
  }
  where
    m = width * 2 + 1 -- number of matrix rows
    n = height * 2 + 1 -- number of matrix cols

initializeFrame :: Int -> Int -> Set W.Wall
initializeFrame m n = fromList frame
  where
    frame = [ W.Wall {W.r = r, W.c = c} |
      r <- [0..m - 1],
      c <- [0..n - 1],
      r == 0 || r == m - 1 || c == 0 || c == n - 1,
      (r, c) /= (0, 1), -- entrance
      (r, c) /= (m - 1, n - 2)] -- exit

-- Given an frame, generate walls inside the frame
-- top left wall cell: (tr, tc)
-- bottom right wall cell: (br, bc)
generateWalls :: RandomGen g => Int -> Int -> Int -> Int -> g ->
(Set W.Wall, g)
generateWalls tr tc br bc seed
  -- | trace ("generateHWalls for the space: " ++ show (tx, ty,

```

```

bx, by)) False = undefined
-- | trace ("holes: " ++ show holes) False = undefined
| bc - tc < 4 || br - tr < 4 = (empty, seed)
| otherwise = (walls
  `union` topLeft
  `union` topRight
  `union` bottomLeft
  `union` bottomRight, newSeed5)
where
  (randomRow, newRowSeed) = pickRandom [(tr + 2), (tr + 4)..(br
- 2)] seed
  (randomCol, newColSeed) = pickRandom [(tc + 2), (tc + 4)..(bc
- 2)] newRowSeed
  walls = verticalWalls `union` horizontalWalls `difference`
holes
  verticalWalls = fromList [W.Wall {W.r = r, W.c = randomCol} |
r <- [(tr + 1)..(br - 1)]]
  horizontalWalls = fromList [W.Wall {W.r = randomRow, W.c = c}
| c <- [(tc + 1)..(bc - 1)]]
  (topLeft, newSeed1) = generateWalls tr tc randomRow randomCol
seed
  (topRight, newSeed2) = generateWalls tr randomCol randomRow
bc seed
  (bottomLeft, newSeed3) = generateWalls randomRow tc br
randomCol seed
  (bottomRight, newSeed4) = generateWalls randomRow randomCol
br bc seed
  (holes, newSeed5) = getHoles tr tc br bc randomRow randomCol
seed

```

```

getHoles :: RandomGen g => Int -> Int -> Int -> Int -> Int ->
Int -> g -> (Set W.Wall, g)

```

```

getHoles tr tc br bc rr rc seed =
  (fromList $ map (choices !!) $ take 3 $ shuffle seed [0..3],
bottomSeed)

```

where

```

  randomPick l seed = l !! fst (randomR (0, length l - 1) seed)
  choices = [top, left, right, bottom]
  (top, topSeed) = pickRandom [W.Wall {W.r = r, W.c = rc} | r <-
[(tr + 1), (tr + 3)..(rr - 1)]] seed
  (left, leftSeed) = pickRandom [W.Wall {W.r = rr, W.c = c} | c

```

```

<- [(tc + 1), (tc + 3)..(rc - 1)] topSeed
    (right, rightSeed) = pickRandom [W.Wall {W.r = rr, W.c = c}]
c <- [(rc + 1), (rc + 3)..(bc - 1)] leftSeed
    (bottom, bottomSeed) = pickRandom [W.Wall {W.r = r, W.c =
rc}] r <- [(rr + 1), (rr + 3)..(br - 1)] rightSeed

```

```

pickRandom :: RandomGen g => [a] -> g -> (a, g)
pickRandom l seed = Data.Bifunctor.first (l !!) (randomR (0,
length l - 1) seed)

```

```

shuffle :: RandomGen g => g -> [a] -> [a]
shuffle gen [] = []
shuffle gen list = randomElem : shuffle newGen newList
    where
        randomTuple = randomR (0, length list - 1) gen
        randomIndex = fst randomTuple
        newGen       = snd randomTuple
        randomElem   = list !! randomIndex
        newList      = take randomIndex list ++ drop (randomIndex+1)
list

```

## Maze.hs

```

module Maze where
import qualified Wall as W
import Data.Set (Set)
import Control.DeepSeq

data Maze = Maze {
    m :: Int, -- number of matrix rows
    n :: Int, -- number of matrix cols
    walls :: Set W.Wall
}

instance Show Maze where
    show (Maze m' n' walls') =
        unlines maze
    where
        maze = [rowToStrng r | r <- [0 .. m'-1]]
        rowToStrng r = [if isWall r c then '#' else ' ' | c <- [0 ..

```

```
n'-1]]  
    isWall r c = W.Wall r c `elem` walls'
```

```
instance NFData Maze where  
    rnf (Maze m' n' walls') = rnf m' `seq` rnf n' `seq` rnf walls'
```

## Wall.hs

```
module Wall where  
import Control.DeepSeq  
  
data Wall = Wall {  
    r :: Int,  
    c :: Int  
} deriving (Show, Eq, Ord)  
  
instance NFData Wall where  
    rnf (Wall r c) = rnf r `seq` rnf c
```