COMSW4995 Parallel Functional Programming Proposal
Galaxy Simulator

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1 Introduction

`Galaxy Simulator(GS)` is a Haskell program which simulates celestial movement and visualizes celestial bodies using `Gloss`. The visualization of the galaxy should be dynamic which represent the the whole program assuming the universe is a 2-D plane.

2 Model

2.1 Simplification

1. `GS` assumes an isolated system which is not affected by any other system.
2. Instead of 3-D which is the real world situation, `GS` simulates 2-D world.
3. There are only two kinds of celestial body: star and planet.
4. All celestial bodies are considered as mass points. `GS` doesn't worry about collision between celestial bodies.

2.2 Celestial Body

Celestial body defined as algebraic data type `Planet` in Haskell has the following properties:

1. Coordinate: `float` and `float`
2. Mass: `float`
3. Velocity: `[float]`

2.3 Gravity

The equation of gravity is

$$F = G\frac{m_1 m_2}{r^2}$$

The sign of force F_x and F_y should be the same as $x_2 - x_1$ and $y_2 - y_1$.

2.4 Acceleration

We use Newton's second law to calculate acceleration:

$$F = ma$$

Then we can have acceleration in different dimension:

$$a_x = \frac{F_x}{m}$$
$$a_y = \frac{F_y}{m}$$

Because acceleration is a vector, we define that a_x has the same sign as F_x.

2.5 Velocity

Let Δt denote the smallest time interval defined by user or default. The velocity of body in `galaxy` should change as

$$v'_x = v_x + a_x \Delta t$$
$$v'_y = v_y + a_y \Delta t$$

2.6 In Haskell

In the source code, celestial body is define as algebraic data type `Planet` in the `Planet.hs` module.

3 Algorithm

The algorithm of `GS` is pretty straight forward. Let s_i denote the i-th state of the system. s_i can be determined if we know s_{i-1}. To identify different s_i, we only need status of all bodies.
`GS` computes m states of n celestial bodies. Let p_{ij} denote the j-th body in i-th state. p_{ij} is determined by $p_{(i-1)k}, k \in \{1, \ldots, n\}$. After compute the compound force from other bodies, `GS` accelerates p_{ij} and make an approximate move. After all $p_{ik}, k \in \{1, \ldots, n\}$ are computed, we say `GS` finished computation of s_i. The time complexity of computing each state is $O(n^2)$. The time complexity of the whole program is $O(mn^2)$.
`GS` generate s_0 randomly and starts the simulation. When the s_m is computed, `GS` terminates.

Algorithm 1 GS

n, m, `interval` $\leftarrow$ `input`
$s_0 \leftarrow$ randomly generate n bodies
for i in $[1, m]$ **do**
    $s_i \leftarrow$ `move`(s_{i-1}, `interval`)
end for
return s_m

Algorithm 2 move

$[p_{i1}, p_{i2}, \ldots, p_{in}]$, `interval` $\leftarrow$ `input`
for j in $[1, n]$ **do**
    `force` $\leftarrow [0, 0]$
    for k in $[1, n]$ **do**
        if **then**$j \neq k$
            `force` $\leftarrow$ `force` + `Gravity`(p_{ij}, p_{ik})
        end if
    end for
    $p_{(i+1)j} \leftarrow$ `Adjust`(p_{ij}, `force`,`interval`)
end for
return $[p_{(i+1)1}, p_{(i+1)2}, ..., p_{(i+1)n}]$

4 Strategy and Performance

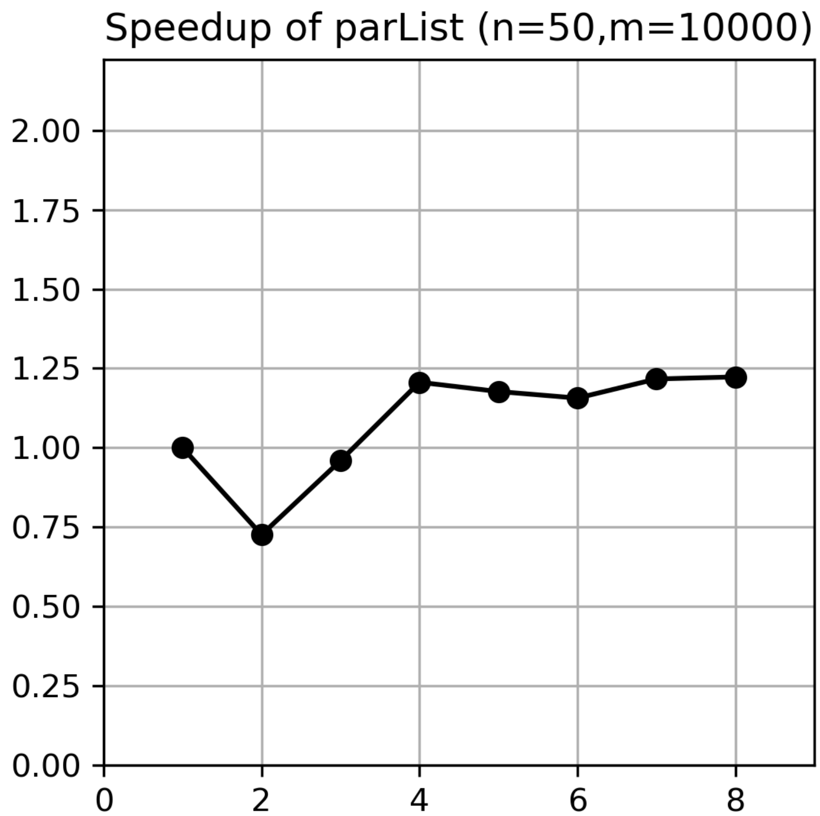
There are several strategies were evaluated and some of which the performance is very inefficient(static partition, for example). In this report, there are three strategies worth mentioning. `parList`, `parListChunk`, and `parListChunk` with `depth` limitation. Also, in this chapter, how m and n changed performance will be discussed.

4.1 parList

`parList` is the first reasonable efficient strategy used in this project. It has the finest granularity to evaluate `[Planet]`. It is applied to the core function `move`. It generates large amount of sparks. Most of these sparks are `overflowed`. When set $n = 50, m = 10000$, it has a poor performance.

4.1.1 Speedup

This is the speedup figure of `parList` strategy with finest granularity.



The speedup is less than 1.25.

4.1.2 Spark Statistic

This is the spark statistic of `parList` strategy with finest granularity. It is also stored in `parList-50-10000.csv`

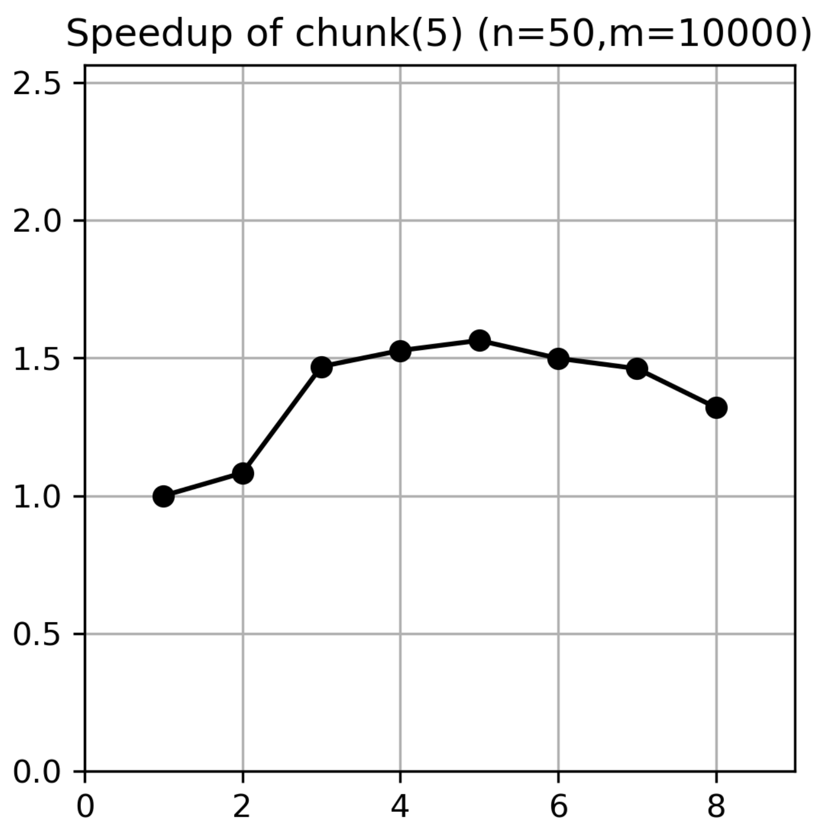
`parList` with n=50, m=10000								
thread	total time	total elapsed	total spark	converted	overflowed	dud	GC	fizzled
`-N1`	4.290	4.811	500100	0	152703	0	58	3275
`-N2`	11.961	6.627	500100	3431	152535	0	0	70
`-N3`	12.664	5.014	500100	3026	152949	0	0	61
`-N4`	12.953	3.988	500100	3040	152935	0	0	61
`-N5`	15.588	4.087	500100	3111	152864	0	0	61
`-N6`	18.152	4.160	500100	3169	169189	0	0	62
`-N7`	19.801	3.954	500100	4559	167772	0	0	89
`-N8`	21.338	3.933	500100	7707	172754	0	0	151

4.2 parListChunk

Rather than sparking every element in list, `parListChunk` sparks chunks of elements in list. The performance of `parListChunk` is not only related to the input, but also related to the size of chunk. `parListChunk` is so far the best strategy in this project.

4.2.1 Speedup

This is the speedup figure of `parListChunk` strategy with coarse granularity.



The speedup starts to exceed 1.5. This is a better result than `parList` but still far from theoretical limitation.

4.2.2 Spark Statistic

This is the spark statistic of `parListChunk` strategy with coarse granularity. It is also stored in `chunk-50-10000-size5.csv`

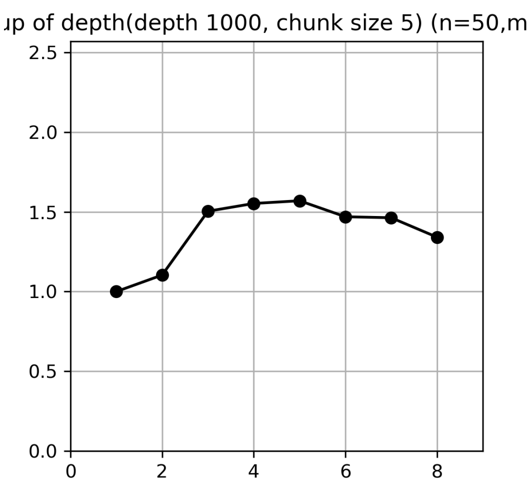
`parListChunk` with chunk_size=5, n=50, m=10000								
thread	total time	total elapsed	total spark	converted	overflowed	dud	GC	fizzled
`-N1`	4.290	4.811	500100	0	152703	0	58	3275
`-N2`	11.961	6.627	500100	3431	152535	0	0	70
`-N3`	12.664	5.014	500100	3026	152949	0	0	61
`-N4`	12.953	3.988	500100	3040	152935	0	0	61
`-N5`	15.588	4.087	500100	3111	152864	0	0	61
`-N6`	18.152	4.160	500100	3169	169189	0	0	62
`-N7`	19.801	3.954	500100	4559	167772	0	0	89
`-N8`	21.338	3.933	500100	7707	172754	0	0	151

4.3 Depth Limitation

After a few attempts, we can conclude that `depth` has nothing to do with speedup and running time.

4.3.1 Speedup

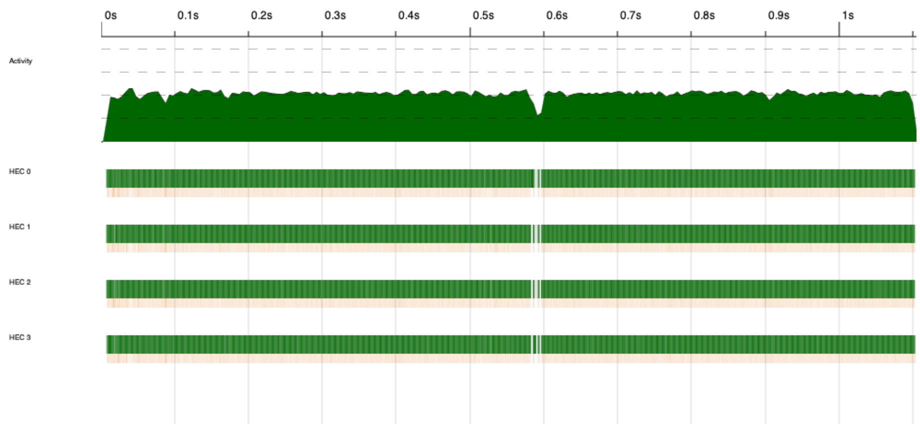
This is the speedup figure of speed up when we simulate 50 bodies in 10000 steps. The strategy it uses is `parListChunk` and the size of chunk is 5. The x axis is the number of threads and the depth is set to 1000.



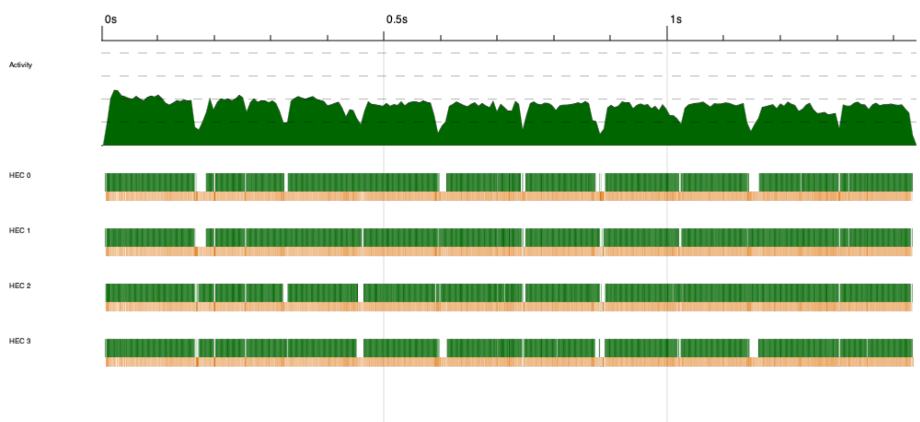
4.4 How m and n changes performance

The speedup relies on the input parameters heavily. These are two diagram from `threadscope`.

4.4.1 n=50, m=10000

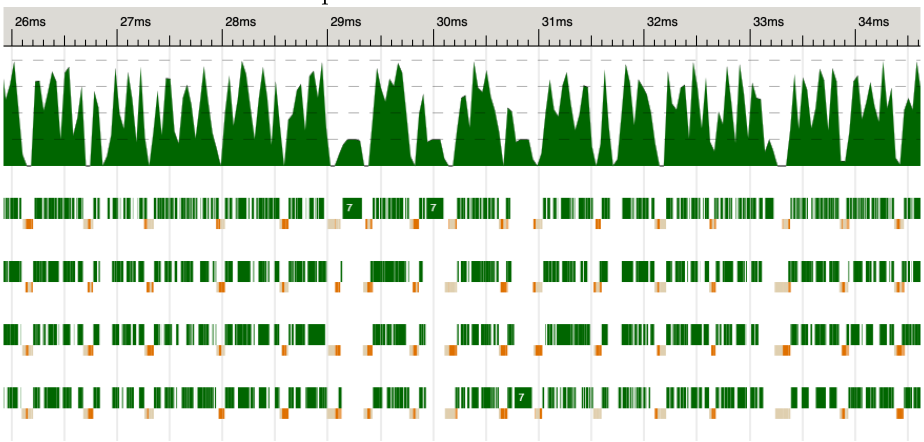


4.4.2 n=1500, m=10

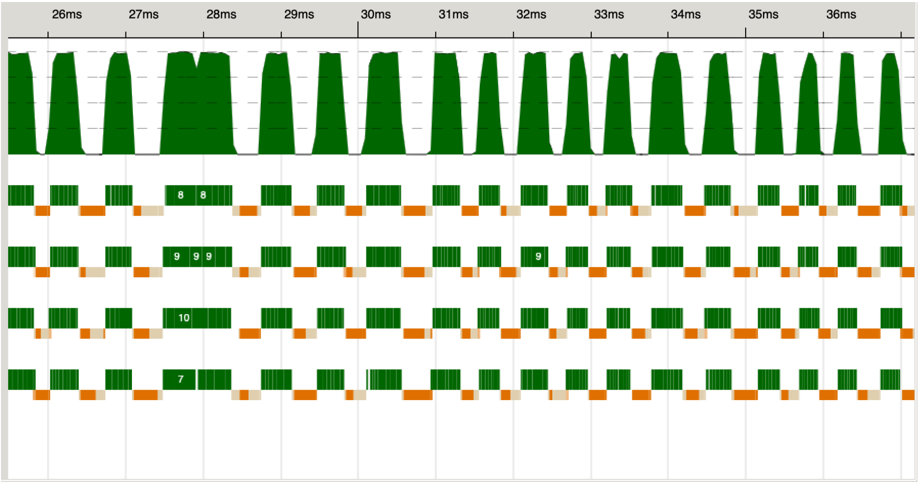


At the first glance, the first diagram is better than the latter one. However, the speedup of the first one is 1.5 the second one is 2.5. This is the detail after zooming in.

50 bodies and 10000 steps:

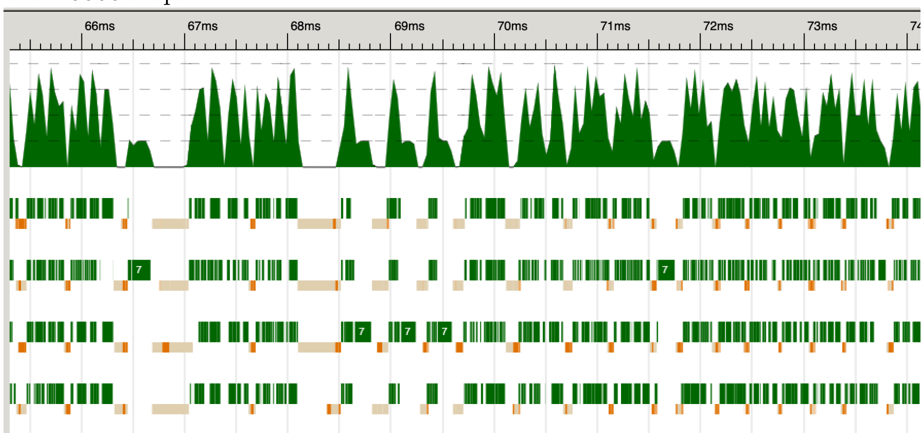


1500 bodies and 10 steps:



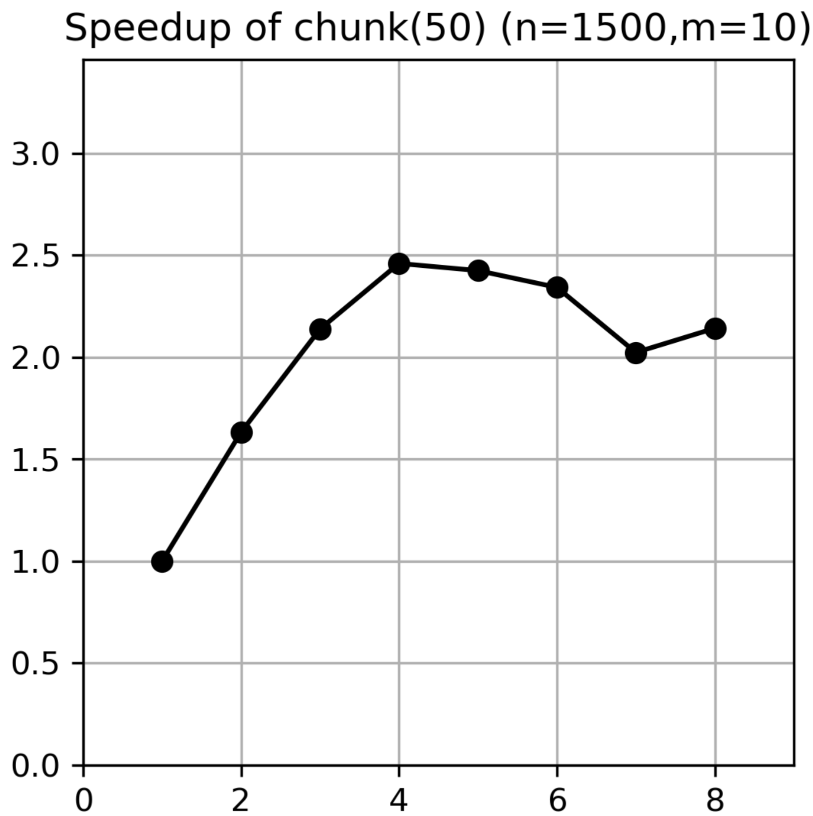
We can tell from the figures above that when the number of body is larger, the better usage will be achieved.

Also, we can take a look at what happened when serial waiting: 50 bodies and 10000 steps:



1500 bodies and 10 steps:

Although it costs more at a single GC or waiting if we set n large, it's less frequent. The parallel proportion is larger when we set n larger. This is the 2.5 speed up when we set $n = 1500$ and $m = 10$:



5 Visualization

The result of visualization should be dynamic. After a certain time interval, the graph of current state of `galaxy` should update. It should show every existing celestial body as dot.

6 What's Next

1. `Barnes-Hut` algorithm. This will improve the time complexity from n^2 to $nlog(n)$.
2. Interactive Simulation.

7 Reference

1. http://www.cs.columbia.edu/ sedwards/classes/2021/4995-fall/proposals/Galaxy.pdf
2. http://www.cs.columbia.edu/ sedwards/classes/2020/4995-fall/reports/NBody.pdf
3. https://hackage.haskell.org/package/gloss-1.13.2.1/docs/Graphics-Gloss-Interface-Pure-Simulate.html
4. http://www.cs.columbia.edu/ sedwards/classes/2021/4995-fall/strategies.pdf
5. http://www.cs.columbia.edu/ sedwards/classes/2021/4995-fall/laziness.pdf
6. https://physics.princeton.edu// fpretori/Nbody/intro.htm

8 Source File

```haskell
{-
  This is the entry file which starts the whole program.
-}

import Control.Parallel(par, pseq)
-- import Control.DeepSeq(deepseq)
import System.Environment
import System.Random
import Control.Parallel.Strategies
import qualified Planet as P
import qualified Laws as L
import qualified Visualize as V


chunkSize :: Int
chunkSize = 5

depth :: Int
depth = 2000

getSeeds :: Int -> IO [Float]
getSeeds n = sequence $ replicate n $ randomRIO (0,1::Float)


genPlanets :: [Float] -> [Float] -> [P.Planet]
genPlanets []   []    = []
genPlanets [s1] [s2]  = [P.genPlanet s1 s2]
genPlanets l1   l2    = (P.genPlanet h1 h2) : genPlanets t1 t2
  where h1 = head l1
        h2 = head l2
```

```haskell
        t1 = tail l1
        t2 = tail l2


-- Trivial approach, doesn't work.
trivial :: Int -> Float -> [P.Planet] -> [P.Planet]
trivial 0 _ ps = ps
trivial 1 i ps = [L.move p ps i| p <- ps]
trivial s i ps = trivial (s - 1) i ps'
  where ps' = [L.move p ps i| p <- ps]


-- Static partitioning
staticPart :: Int -> Float -> [P.Planet] -> [P.Planet]
staticPart 0 _ ps = ps
staticPart 1 i ps = [L.move p ps i| p <- ps]
staticPart s i ps = staticPart (s - 1) i ps'
  where ps1'    = [L.move p ps i| p <- ps1]
        ps2'    = [L.move p ps i| p <- ps2]
        ps'     = ps1' `par` ps2' `pseq` (ps1' ++ ps2')
        (ps1, ps2)  = splitAt (length ps `div` 2) ps


-- Finest granularity
finePart :: Int -> Float -> [P.Planet] -> [P.Planet]
finePart 0 _ ps = ps
finePart 1 i ps = [L.move p ps i| p <- ps] `using` parList rseq
finePart s i ps = finePart (s - 1) i ps'
  where ps'       = [L.move p ps i| p <- ps] `using` parList rseq


-- parListChunk
chunkPart :: Int -> Float -> [P.Planet] -> [P.Planet]
chunkPart 0 _ ps = ps
chunkPart 1 i ps = [L.move p ps i| p <- ps] `using` parListChunk
↪  chunkSize rdeepseq
chunkPart s i ps = chunkPart (s - 1) i ps'
  where ps' = [L.move p ps i| p <- ps] `using` parListChunk
   ↪  chunkSize rdeepseq


-- depth limited
depthPart :: Int -> Int -> Float -> [P.Planet] -> [P.Planet]
depthPart 0 _ _ ps   = ps
depthPart s 0 i ps   = chunkPart s i ps
-- depthPart s 0 i ps   = trivial s i ps
```

```haskell
depthPart s d i ps  = depthPart (s - 1) (d - 1) i ps'
  where ps'         = [L.move p ps i| p <- ps] `using`
   ↪  parListChunk chunkSize rdeepseq


forceEval :: [P.Planet] -> IO ()
forceEval ps = do
  print $ length $ filter ((==) (P.Planet [0, 0] 0 0 0)) ps


main :: IO ()
main = do
  args <- getArgs
  let [n, s, i, mode] = args
  let planetNum = read n :: Int
  let steps     = read s :: Int
  let interval  = read i :: Float
  seedList1 <- getSeeds planetNum
  seedList2 <- getSeeds planetNum
  let planets = (genPlanets (seedList1) (seedList2)) `using`
   ↪  parList rseq
  -- let planets = (genPlanets (seedList1 `using` parList rseq)
   ↪  (seedList2 `using` parList rseq)) `using` parList rseq
  case mode of
    "v"       -> do
      let star = P.Planet [0, 0] (1e5 * 7) 0 0
      V.runSimulation (star : planets)
    "trivial"   -> do
      let state = (trivial steps interval planets)
      forceEval state
    "static"  -> do
      let state = (staticPart steps interval planets)
      forceEval state
    "parList" -> do
      let state = (finePart steps interval planets) `using`
       ↪  parList rseq
      forceEval state
    "chunk"   -> do
      let state = (chunkPart steps interval planets) `using`
       ↪  parList rseq
      forceEval state
    "depth"   -> do
      let state = (depthPart steps depth interval planets)
       ↪  `using` parList rseq
      forceEval state
    _         -> do
```

```
      print $ "Usage: ./Galaxy <Number of Bodies> <Number of
      ↪  Steps> <Time Inteval> <mode> +RTS -N<Number of Threads>
      ↪  -s"

{-
  This module defines the basic laws of physics.
-}

module Laws (
  move,
  nextState
) where

import Planet
import Control.Parallel.Strategies
import Control.Parallel(par, pseq)

type Force = Float
type Acc = Float
type Vel = Float
type Time = Float

{-
  Gravitational constant.
-}
g :: Float
g = 10

{-
  This function takes two Planets as input, computes
  the gravity from p1 to p2, and returns a
  list of float value which represents Fx and Fy.
  Gravity formula F = g * m1 * m2 / r^2
-}
gravity :: Planet -> Planet -> [Force]
gravity p1 p2 = [if x2 > x1 then fx else -fx, if y2 > y1 then fy
↪  else -fy]
  where fx = if x1 == x2 then 0
              else g * m1 * m2 / rSqr
        fy = if y1 == y2 then 0
              else g * m1 * m2 / rSqr
        x1 = posX p1
        x2 = posX p2
        y1 = posY p1
        y2 = posY p2
        m1 = mass p1
```

```haskell
        m2 = mass p2
        rSqr = (fSqr (x1 - x2)) + (fSqr (y1 - y2))

{-
  Takes a planet and a pair of forces.
  Returns a pair of velocities.
-}
acceleration :: Planet -> [Force] -> [Acc]
acceleration p fxy = map acc fxy
  where acc f = f / (mass p)

{-
  Change velocities.
-}
accelerate :: [Vel] -> [Acc] -> Time -> [Vel]
accelerate vs as t = zipWith (\v a -> v + a * t) vs as

{-
  Used for visualization.
-}
nextState :: [Planet] -> Double -> [Planet]
nextState ps _ = [move p ps (1/100) | p <- ps] `using` parList
↪  rseq

move :: Planet -> [Planet] -> Time -> Planet
-- move p ps t = moveHelper 100 p ps (t / 100)
move p ps t = moveHelper 1 p ps t

moveHelper :: Int -> Planet -> [Planet] -> Time -> Planet
moveHelper 0 p ps t = p
moveHelper n p ps t = (moveHelper (n - 1) p' ps t)
  where p'          = Planet vs m x y
        vs          = accelerate (velocity p) as t
        as          = acceleration p fs
        -- fs          = foldr (zipWith (+)) [0, 0] ((map
        ↪  (gravity p) ps) `using` parList rseq)
        fs          = foldr (zipWith (+)) [0, 0] (map (gravity p)
        ↪  ps)
        [x, y]      = zipWith (\pos v -> pos + v * t) [posX p,
        ↪  posY p] vs
        m           = mass p

fSqr :: Float -> Float
fSqr x = x * x

{-
```

```haskell
  This module defines and exports data type Planet.
-}

module Planet (
  Planet(..),
  genPlanet
) where

import Control.DeepSeq

{-
  Units:
    velocity  m * s^-1
    mass      kg^-1
    posX      m
    poxY      m
-}
data Planet = Planet {
  velocity    :: ![Float],
  mass        :: !Float,
  posX        :: !Float,
  posY        :: !Float
} deriving (Show, Eq)

instance NFData Planet where
  rnf (Planet v m x y) = rnf v `deepseq` rnf m `deepseq` rnf x
   ↪  `deepseq` rnf y

{-
  Generate a planet from two seed which are randomly generated.
-}
genPlanet :: Float -> Float -> Planet
genPlanet s1 s2 = Planet [vx, vy] m x y
  where vx   = if s1 > 0.5 then (-absVx) else absVx
        vy   = -(x * vx / y)
        m    = 1e2 + d * 1e2 + s1 * 1e1
        x    = -(720 / 4) + s1 * (720 / 2)
        y    = -(720 / 4) + s2 * (720 / 2)
        -- x    = if s1 > 0.5 then 20 + s1 * 200 else -(20 + s1 *
        ↪  200)
        -- y    = if s2 > 0.5 then 20 + s2 * 200 else -(20 + s2 *
        ↪  200)
        absVx = 1 * 1e2 + s1 * 1e1 + (1 - d) * 1e1
        d = ((abs x) + (abs y)) / 720 / 2

{-
```

```haskell
  This file
-}

module Visualize (
  runSimulation,
  windowSize
) where

import GHC.Float
import Graphics.Gloss
import Control.Parallel.Strategies
import qualified Planet as P
import qualified Laws as L

windowSize :: Int
windowSize = 720

drawPlanet :: P.Planet -> Picture
drawPlanet p = Color white $ Translate x y (circleSolid
↪  (realToFrac $ 0.5 * log (P.mass p)))
  where x = realToFrac $ P.posX p
        y = realToFrac $ P.posY p

drawPlanets :: [P.Planet] -> [Picture]
drawPlanets ps = map drawPlanet ps

runSimulation :: [P.Planet] -> IO ()
runSimulation ps =  simulate (InWindow "Galaxy Simulation"
↪  (windowSize, windowSize) (100, 100))
                    black 60
                    ps
                    (\ps' -> pictures $ drawPlanets ps')
                    (\_ dt ps' -> L.nextState ps' (float2Double
                    ↪  dt))
```