# Parallel KenKen Solver Report 

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## 1 How it Works

### 1.1 The KenKen Puzzle

My project solves KenKen puzzles, which are Sudoku-like number puzzles played on a square $N \times N$ grid for any size $N$. Each row and each column must contain exactly one instance of each number between 1 and $N$. Additionally, there are units partitioning the game board made of contiguous cells. Each of these units indicate other mathematical constraints that the values entered in the constituent cells must satisfy. In particular, these constraints are adding up to a target sum, subtracting (largest to smallest) to a value, multiplying to a target sum, or dividing (largest to smallest) to a particular quotient. A unit may also contain just one cell, in which case its associated target value is automatically the value that the cell will take in a correct solution. Subject to the constraint of no repeats within any row or column, the same number may appear more than one time in the same unit.

I obtained a corpus of KenKen puzzles from/http://www.mlsite.net/neknek/ play.php. Each puzzle has a single solution. Below, I present an example puzzle.

```
# 7
* 336 A1 A2 A3 A4
+ 8 A5 A6 B6
+ 10 A7 B7 C7
* 210 B1 C1 D1
+ 4 B2 B3
* 20 B4 C3 C4
! 6 B5
! 7 C2
+ 13 C5 D4 D5 E5
+ 13 C6 D6 D7
* 30 D2 E1 E2
+ 7 D3 E3 E4
* 210 E6 E7 F6 F7
! 4 F1
+ 8 F2 G1 G2
+ 8 F3 F4
* 168 F5 G5 G6 G7
-2 G3 G4
```

Observe that each row specifies a unit with a constraining operation, a target value, and the addresses of the constituent cells listed in row major order (i.e. 'B4' refers to row 2 , column 4 ). The ! operator indicates a unit of size 1 , which has as its target value
the correct assignment for the only cell in this unit. Additionally, the first line of the specification indicates the size of the puzzle (in this case, 7).

### 1.2 The Code

The majority of the logic for the game of KenKen is contained within the file src/Kenken. hs. The code which is responsible for parallelizing the execution of the solver on the list of input puzzles is located in app/Main.hs. Each puzzle is individually contained in a file puzzles/<SIZE>_<INDEX>.txt with a corresponding solution in solutions/<SIZE>_<INDEX>.txt.

### 1.2.1 Kenken.hs

This module specifies a handful of datatypes used in the puzzle logic, the most interesting of which are Constraint and Partial. A Constraint is a representation of one row of the puzzle. Specifically, it represents an operation which accumulates its members to reach a target value. Although the uniqueness of elements in rows and columns could be represented as a Constraint, I chose to represent these restrictions implicitly to maintain that each cell has precisely one Constraint.

```
data Constraint = Constraint {members :: [Address]
    , op :: Operation
    , target :: Int
    } deriving (Show, Eq)
```

Additionally, we have the data type Partial. This represents a partial solution to the KenKen puzzle, while also keeping track of the relationships among cells implied by the underlying constraints. In particular, the state field tracks all candidate values for each cell. The field pPeers maps each cell to the set of other cells in the same constraining unit (I call them the cells 'peers') and pUnit maps each address to its own constraining unit.

```
data Partial = Partial {state :: M.Map Address [Int]
    , pPeers :: M.Map Address (S.Set Address)
    , pUnits :: M.Map Address Constraint
    } deriving (Show, Eq)
```

Lines 23 through 80 of Kenken. hs provide the framework for how to parse a puzzle from its String representation into an initial Partial, which represents a puzzle with all constraints formalized but without any steps taken to solve it. I leave these functions to be explored in the source code by the reader, since I believe that the underlying logic used to write them is somewhat easily intuitied from the design of the Constraint and Partial datatypes discussed above.

At this point, I will introduce the basic pipeline of how I solve a KenKen puzzle. The algorithm is broadly a depth-first search, where at each call to the function search, the computer attempts to assign a value to a cell chosen from its current available values. Whenever a cell has a value assigned to it or has a potential value eliminated, three basic steps occur. First, the actual change to the state of the Partial is made, resulting in a new Partial. Second, constraints are propagated from the changed cell to eliminate or assign potential values in its peers, resulting in a new Partial
generated for each deductive change $\downarrow^{1}$ Finally, inconsistent puzzles result in terminated execution paths.

Progress is made in this search by doing either an assignment or an elimination; however, I have chosen to implement assignment of a chosen value (in assign) as repeated elimination of values in this cell that are not the chosen value.

```
assign :: Address -> Int -> Partial -> Maybe Partial
assign a v p =
    do assigned <- foldM f p toRemove
        propagateSet assigned a
    where toRemove = filter (/= v) $ (state p) M.! a
        f = \partial value -> eliminate a value partial
```

The elementary operation of this algorithm is thus the elimination, presented here:

```
eliminate :: Address -> Int -> Partial -> Maybe Partial
eliminate a v p =
    do removed <- remove a v p
        unitPropagated <- propagateUnit removed a
        return unitPropagated
```

Elimination is responsible for removing a single value as a possibility at a particular address and then propagating the constraint of that address' unit forward. The propagation step here is most robust when handling cells with addition or multiplication as their operations. In these cases, it is quite simple to determine whether any of the remaining value combinations lead to a consistent assignment. When there is no such combination, the search down this game state ceases. This pruning is quite imperative for runtime efficiency. For a time, I attempted to carry the effects of the new assignment forward and proactively remove values that would now be inconsistent. While the process of finding newly inconsistent values was relatively simple and efficient, the extra propagation steps resulted in a significant slowdown due to expensive further eliminations down paths that ended up becoming inconsistent anyway.

The search is terminated in a successful state when every list contained in state has exactly one member.

### 1.2.2 Main.hs

I include in full here a description of Main.hs, which provides the logic for solving multiple puzzles from a manifest provided as a command line argument.

```
main :: IO ()
main = do puzzles <- getArgs >>= \[f] -> lines <$> readFile f
    let solutions = parMap rpar solve puzzles
    print $ length $ filter isJust $ solutions
```

Of course, I provide heavy credit to the course notes for providing essentially this exact code. Much like the example provided in class, the file that Main expects contains one puzzle description per line. To make this possible, I converted the puzzle format from a line-separated format to a semicolon-separated format. From there, the use of parMap with the associated strategy rpar results in a dynamically partition set of problems where each call to solve is handled by a separate spark.

[^0]
## 2 Effects of Parallelism

I'll begin by presenting the raw statistics on how my program performs with increasing numbers of cores. Broadly, then, parallelizing this solver is a success. The main

Table 1:

| \# cores | Time elapsed (s) | Speedup | Sparks Created | Conversions | GCs | Fizzles |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 5.026 | 1 x | 1362 | 0 | 258 | 1104 |
| 2 | 2.856 | 1.76 x | 1362 | 1350 | 2 | 10 |
| 3 | 2.054 | 2.45 x | 1362 | 1361 | 1 | 0 |
| 4 | 1.820 | 2.76 x | 1362 | 1246 | 13 | 103 |

shortcoming becomes apparent when looking at a Threadscope trace of an execution on multiple cores. Presented below in Figure 1 the visualization for the execution on four cores. Immediately obvious is a load balancing problem, despite the strategy of


Figure 1: Execution Viz for Four Cores on Puzzles Sizes 3-7
dynamic balancing that I attempted. This is a result of puzzles of size 7 being relatively slow for my solver to handle and the fact that these puzzles begin to have more variance in how long they take to solve. Among the first ten puzzles of size 7, there are some that take less than aa tenth of a second and others that take over half of a second. So while this is obviously detrimental to runtime, this is a shortcoming of the underlying sequential code rather than the parallelization strategy taken. If we compare with Figure 2, an execution on puzzles of up to size 6 which are much quicker to solve, we see that the load balancing issue is much less significant. Indeed, on these smaller puzzles, we get a speedup of $3.2 \times$ when running the solver on four cores $(0.69 s)$ over a single core $(2.21 s)$. To improve this going forward, it would be helpful to parallelize the search step within the solving algorithm as well. This would allow the machine to follow along multiple independent execution paths at once, perhaps speeding up execution even further.

## 3 Code Listing

Kenken.hs


Figure 2: Execution Viz for Four Cores on Puzzles Sizes 3-6

```
{-# LANGUAGE NamedFieldPuns #-}
module Kenken where
import Data.Char (ord)
import qualified Data.Map as M
import qualified Data.Set as S
import qualified Data.List as L
import Data.List.Split (splitOn)
import Control.Monad (guard, foldM, msum, fmap)
import System.IO (IOMode (ReadMode), hGetContents, openFile
    )
type Address = (Int, Int)
data Constraint = Constraint {members :: [Address], op :: Operation,
    target :: Int} deriving (Show, Eq)
data Operation = Add | Sub | Mul | Div | Asrt deriving (Show, Eq)
data Partial = Partial {state :: M.Map Address [Int]
    , pPeers :: M.Map Address (S.Set Address)
    , pUnits :: M.Map Address Constraint
    } deriving (Show, Eq)
crossProd :: [a] -> [b] -> [(a, b)]
crossProd as bs = (,) <$> as <*> bs
translate :: String -> Address
translate (r : c : []) = (ord r - 64, read $ c : "")
translate _ = undefined
symToOp :: String -> Operation
symToOp "+" = Add
symToOp "-" = Sub
symToOp "*" = Mul
symToOp "\\/" = Div
symToOp "!" = Asrt
symToOp _ = undefined
readKenkenFile :: String -> IO (String)
readKenkenFile fname = do
                                    h <- openFile fname ReadMode
                                    hGetContents h
```

showUnits :: Address -> IO ([Constraint], Int) -> IO (Maybe Constraint)
showUnits addr gameIO = do game <- gameIO
let us = units game
return \$ M.lookup addr us
showPeers :: Address -> IO ([Constraint], Int) -> IO (Maybe (S.Set
Address))
showPeers addr gameIO = do game <- gameIO
let ps = peers . units \$ game
return \$ M.lookup addr ps
readKenken :: String -> ([Constraint], Int)
readKenken kkStr = let spec : groups = splitOn ";" kkStr
translated = do
(op : target : group) <- map words
groups
return \$ Constraint $\{o p=$ symToOp
op,
target $=$ read target,
members = map translate
group \}
[_, size] = words spec in
(translated, read size)
units :: ([Constraint], Int) -> M.Map Address Constraint
units (constraints, _) = M.fromList associations
where associations $=[(a, c)$ |
c <- constraints,
a <- members c
]
peers :: M.Map Address Constraint -> M.Map Address (S.Set Address)
peers $m=$ M.fromList [(addr, neighbors) | (addr, c) <- M.toList m,
let duplicateSet = S.fromList \$
members c
neighbors = duplicateSet 'S.
difference' S.singleton addr]
parsePuzzle :: ([Constraint], Int) -> Partial
parsePuzzle cs@(constraints, size) = Partial \{state, pPeers, pUnits\}
where
pUnits = units cs
pPeers = peers punits
state $=$ M.fromList [(a, [1..size]) | C
<- constraints, $a<-$ members c]
eliminate :: Address -> Int -> Partial -> Maybe Partial
eliminate $a \operatorname{v} p=$
do removed <- remove a v p
unitPropagated <- propagateUnit removed a
return unitPropagated
assign :: Address -> Int -> Partial -> Maybe Partial
assign a v p =
do assigned <- foldM (\partial value -> eliminate a value partial) $p$
toRemove
propagateSet assigned a

```
    where toRemove = filter (/= v) $ (state p) M.! a
remove :: Address -> Int -> Partial -> Maybe Partial
remove adr val p =
    do candidates <- M.lookup adr $ state p
    let reducedCands = L.delete val candidates
                len = L.length reducedCands
    guard (len /= 0)
    return p{state=M.insert adr reducedCands $ state p}
propagateSet :: Partial -> Address -> Maybe Partial
propagateSet p@Partial{state} adr@(row, col) =
    let [x] = state M.! adr
    size = fst $ fst $ M.findMax state
    setPeers = [(row, c) | c <- [1..size], c /= col] ++
                                [(r, col) | r <- [1..size], r /= row]
            in foldM (\partial peer -> eliminate peer x partial) p setPeers
propagateUnit :: Partial -> Address -> Maybe Partial
propagateUnit p@Partial{pUnits} adr =
    let constraint = pUnits M.! adr in
    case op $ constraint of
        Add -> propagateAdd p constraint
        Mul -> propagateMul p constraint
        Sub -> propagateSub p constraint
        Div -> propagateDiv p constraint
        _ -> Just p
existsSum :: Int -> [[Int]] -> Bool
existsSum target [] = target == 0
existsSum target (hd:rest) = or [existsSum (target - choice) $ rest |
    choice <- L.reverse hd, choice <= target]
existsProd :: Int -> [[Int]] -> Bool
existsProd target [] = target == 1
existsProd target (hd:rest) = or [existsProd (target 'div' choice) $
    rest | choice <- L.reverse hd, target 'mod' choice == 0]
propagateAdd :: Partial -> Constraint -> Maybe Partial
propagateAdd p@Partial{state} constraint =
    let prs = members constraint
        t = target constraint
        possibilities = [(peer, state M.! peer) | peer <- prs]
        fixed = L.filter (\(_,l) -> L.length l == 1) $ possibilities
        free = L.filter (\(_,l) -> L.length l > 1) $ possibilities
        freeVals = map snd free
        fixedVals = map snd fixed
        newTarget = t - (sum $ msum $ fixedVals) in
    if not $ existsSum newTarget $ L.reverse $ L.sort $ freeVals
    then Nothing
    else return p
propagateMul :: Partial -> Constraint -> Maybe Partial
propagateMul p@Partial{state} constraint =
    let prs = members constraint
        t = target constraint
        possibilities = [(peer, state M.! peer) | peer <- prs]
        fixed = L.filter (\(_,l) -> L.length l == 1) $ possibilities
        free = L.filter (\(_,l) -> L.length l > 1) $ possibilities
```

```
    fixedVals = map snd fixed
    freeVals = map snd free
    newTarget = t 'div' (product $ msum $ fixedVals) in
    if not $ existsProd newTarget $ freeVals
    then Nothing
    else return p
propagateSub:: Partial -> Constraint -> Maybe Partial
propagateSub p@Partial{state} constraint =
    let prs = members constraint
        t = target constraint
        possibilities = [(peer, state M.! peer) | peer <- prs]
        fixed = L.filter (\(_,l) -> L.length l == 1) $ possibilities
        free = L.filter (\(_,l) -> L.length l > 1) $ possibilities
        largest:rest = L.reverse $ L.sort $ msum $ map snd fixed
        result = largest - (sum rest) in
    case free of
    [] -> if result == t then return p else Nothing
    - -> return p
propagateDiv:: Partial -> Constraint -> Maybe Partial
propagateDiv p@Partial{state} constraint =
    let prs = members constraint
        t = target constraint
        possibilities = [(peer, state M.! peer) | peer <- prs]
        fixed = L.filter (\(_,l) -> L.length l == 1) $ possibilities
        free = L.filter (\(_,l) -> L.length l > 1) $ possibilities
        largest:rest = L.reverse $ L.sort $ msum $ map snd fixed
        result = largest 'div' (sum rest) in
    case free of
        [] -> if result == t then return p else Nothing
        _ -> return p
reduce :: Partial -> Address -> Int -> Maybe Partial
reduce p@Partial{state} (row, col) value =
    let size = fst $ fst $ M.findMax state
        rowUnit = [(row, c) | c <- [1..size]]
        colUnit = [(r, col) | r <- [1..size]] in
    do let rowCandidates = [a | a <- rowUnit, value 'elem' state M.! a]
    rowReduced <- case rowCandidates of
                                    [] -> Nothing
                            [adr] -> assign adr value p
                    _ -> return p
    let colCandidates = [a | a <- colUnit, value `elem' state M.! a]
    case colCandidates of
                [] -> Nothing
                [adr] -> assign adr value rowReduced
            _ -> return rowReduced
applyAssertions :: Partial -> Maybe Partial
applyAssertions p@Partial{pUnits} =
    let asserts = M.filter (\c -> op c == Asrt) pUnits
```

        assocList \(=\) L.map ( \(\backslash(\) adr, c) \(\rightarrow\) (adr, target c)) \$ M.toList asserts
        in
    foldM (\partial (a, val) -> assign a val partial) p assocList
    solve :: String -> Maybe Partial
solve $s=$ do $p<-$ (applyAssertions . parsePuzzle . readKenken) $s$
search p
search :: Partial -> Maybe Partial
search p@Partial\{state\} =
let remaining $=$ M.filter ( $\backslash 1 \rightarrow$ (tail l) /= [] state in
if remaining $==$ M.empty then return $p$
else let addresses $=$ (M.toList remaining) : : [(Address, [Int])]
assocComp $=\backslash\left(\_, c 1\right)(\ldots, c 2) \rightarrow$ (L.length c1) 'compare' (L.
length c2)
(nextAdr, candidateVals) = L.minimumBy assocComp addresses
searchAssign $=\backslash v->$ assign nextAdr $v p \gg=\backslash p 2->$ search $p 2$
assigned $=$ map searchAssign candidateVals in
do result <- msum assigned
checkSolution result
checkSolution : : Partial -> Maybe Partial
checkSolution p@Partial\{state, pUnits\} =
if allCorrect then return $p$ else Nothing
where allCorrect $=$ and $\$$ map satisfied $\$$ map snd $\$$ M.toList pUnits
satisfied Constraint\{members, op, target\} =
case op of
Add $\rightarrow$ (sum [head \$ state M.! member | member <- members]) ==
target
Mul -> (product [head \$ state M.! member | member <- members
]) $==$ target
_ -> True
showSolution : : Maybe Partial -> IO ()
showSolution (Just sol) = do putStrLn \$ show [v | (_, [v]) <- M.toAscList
\$ state sol]
showSolution (Nothing) = do putStrLn \$ "No Solution Found"
parseSolutionString : S String $->$ [Int]
parseSolutionString "" = []
parseSolutionString (',' :rest) = parseSolutionString rest
parseSolutionString $(x: r e s t)=($ read $(x: "))$ (parseSolutionString rest)
readSolutionFile : : String -> IO [Int]
readSolutionFile $s$ File $=$ do sol <- readFile sFile
return \$ parseSolutionString sol

## Main.hs

```
module Main where
import Kenken (solve)
import Control.Parallel.Strategies (parMap, rpar)
import Data.Maybe (isJust)
import System.IO (readFile)
import System.Environment (getArgs)
main :: IO ()
main = do puzzles <- getArgs >>= \[f] -> lines <$> readFile f
```


## KenkenTest.hs

```
import Test.Hspec
import Test.QuickCheck
import Control.Exception (evaluate)
import qualified Kenken as K
import qualified Data.Map as M
import qualified Data.Set as S
import Data.Maybe (fromMaybe, isJust)
import Control.Monad (msum)
main :: IO ()
main = hspec $ do
    describe "Kenken" $ do
    describe "translate" $ do
        it "A1 is top left" $ do
            (K.translate "A1") 'shouldBe' ((1, 1) :: K.Address)
        it "translate puts letter as col" $ do
            (K.translate "E9") 'shouldBe' ((5, 9) :: K.Address)
    describe "readKenken" $ do
        it "simple example" $ do
            (K.readKenken "#\t2;+\t3\tA1 B1;-\t1\tA2 B2") 'shouldBe'
                ([K.Constraint {K.members = [ (1,1), (2,1)], K.op = K.Add, K.
                    target = 3},
            K.Constraint {K.members = [ (1, 2), (2,2)], K.op = K.Sub, K.
                    target = 1}],2)
    describe "units" $ do
        it "simple example units of (1,1)" $ do
            let simplePuzzle = K.readKenken "#\t2;+\t3\tA1 B1;-\t1\tA2 B2"
                constraintMap = K.units simplePuzzle
                actualUnits = M.lookup (1,1) $ constraintMap in
                actualUnits 'shouldBe'
                    Just K.Constraint {K.members = [ (1,1), (2,1)], K.op = K.Add
                        , K.target = 3}
        it "simple example units of (1,2)" $ do
            let simplePuzzle = K.readKenken "#\t2;+\t3\tA1 B1;-\t1\tA2 B2"
                constraintMap = K.units simplePuzzle
                actualUnits = M.lookup (1,2) $ constraintMap in
                actualUnits 'shouldBe'
                    Just K.Constraint {K.members = [(1,2), (2, 2)], K.op = K. Sub
                        , K.target = 1}
        it "simple example units of (2,1)" $ do
            let simplePuzzle = K.readKenken "#\t2;+\t3\tA1 B1;-\t1\tA2 B2"
                constraintMap = K.units simplePuzzle
                actualUnits = M.lookup (2,1) $ constraintMap in
                actualUnits 'shouldBe'
                Just K.Constraint {K.members = [ (1, 1), (2,1)], K.op = K.Add
                    , K.target = 3}
    it "simple example units of (2,2)" $ do
            let simplePuzzle = K.readKenken "#\t2;+\t3\tA1 B1;-\t1\tA2 B2"
```

```
    constraintMap = K.units simplePuzzle
    actualUnits = M.lookup (2,2) $ constraintMap in
    actualUnits 'shouldBe'
                Just K.Constraint {K.members = [ (1,2), (2, 2)], K.op = K.Sub
                            , K.target = 1}
    it "address not found" $ do
    let simplePuzzle = K.readKenken "#\t2;+\t3\tA1 B1;-\t1\tA2 B2"
            constraintMap = K.units simplePuzzle
            actualUnits = M.lookup (3, 4) $ constraintMap in
            actualUnits 'shouldBe' Nothing
describe "trying solver" $ do
    it "some 3s" $ do
            let names = ["puzzles/3_" ++ show num ++ ".txt" | num <- [0..10]]
            puzzles = map readFile names
            solved = sequence $ map (\puzzIO -> fmap K.solve $ puzzIO)
                    puzzles
            res = fmap (all isJust) solved in
            res 'shouldReturn' True
    it "some 4s" $ do
            let names = ["puzzles/4_" ++ show num ++ ".txt" | num <- [0..10]]
            puzzles = map readFile names
            solved = sequence $ map (\puzzIO -> fmap K.solve $ puzzIO)
                puzzles
            res = fmap (all isJust) solved in
            res 'shouldReturn' True
    it "some 5s" $ do
        let names = ["puzzles/5_" ++ show num ++ ".txt" | num <- [0..10]]
            puzzles = map readFile names
            solved = sequence $ map (\puzzIO -> fmap K.solve $ puzzIO)
                puzzles
            res = fmap (all isJust) solved in
            res 'shouldReturn' True
    it "some 6s" $ do
            let names = ["puzzles/6_" ++ show num ++ ".txt" | num <- [0..10]]
            puzzles = map readFile names
            solved = sequence $ map (\puzzIO -> fmap K.solve $ puzzIO)
                    puzzles
            res = fmap (all isJust) solved in
            res 'shouldReturn' True
    it "some 7s" $ do
    let names = ["puzzles/7_" ++ show num ++ ".txt" | num <- [0..10]]
            puzzles = map readFile names
            solved = sequence $ map (\puzzIO -> fmap K.solve $ puzzIO)
                puzzles
            res = fmap (all isJust) solved in
            res 'shouldReturn' True
describe "correctness" $ do
    it "a 3" $ do
            let puzzle = "puzzles/3_58.txt"
            solution = "solutions/3_58.txt"
            actualM = fmap K.solve $ readFile $ puzzle
            expectedM = K.readSolutionFile solution in
            do expected <- expectedM
                actual <- actualM
                    let res = msum $ map snd $ M.toList $ K.state $ fromMaybe
                    undefined actual
```

102
103
104
105
106
107
108
109
110

```
        res 'shouldBe' expected
it "a 4" $ do
    let puzzle = "puzzles/4_58.txt"
        solution = "solutions/4_58.txt"
        actualM = fmap K.solve $ readFile $ puzzle
        expectedM = K.readSolutionFile solution in
        do expected <- expectedM
            actual <- actualM
        let res = msum $ map snd $ M.toList $ K.state $ fromMaybe
            undefined actual
            res 'shouldBe' expected
it "a 5" $ do
    let puzzle = "puzzles/5_58.txt"
        solution = "solutions/5_58.txt"
        actualM = fmap K.solve $ readFile $ puzzle
        expectedM = K.readSolutionFile solution in
        do expected <- expectedM
            actual <- actualM
        let res = msum $ map snd $ M.toList $ K.state $ fromMaybe
                undefined actual
            res 'shouldBe' expected
        it "a 6" $ do
    let puzzle = "puzzles/6_58.txt"
        solution = "solutions/6_58.txt"
        actualM = fmap K.solve $ readFile $ puzzle
        expectedM = K.readSolutionFile solution in
        do expected <- expectedM
            actual <- actualM
        let res = msum $ map snd $ M.toList $ K.state $ fromMaybe
            undefined actual
            res 'shouldBe' expected
        it "a 7" $ do
    let puzzle = "puzzles/7_58.txt"
        solution = "solutions/7_58.txt"
        actualM = fmap K.solve $ readFile $ puzzle
        expectedM = K.readSolutionFile solution in
        do expected <- expectedM
            actual <- actualM
        let res = msum $ map snd $ M.toList $ K.state $ fromMaybe
            undefined actual
            res 'shouldBe' expected
```


[^0]:    ${ }^{1}$ As an example of this second step, consider the 'trick' of assigning the value 1 to the only empty cell in a row which has all other values $2 \ldots N$ assigned.

