Fundamentals of Computer Systems

Thinking Digitally

Stephen A. Edwards

Columbia University

Summer 2016

The Subject of this Class

0

The Subjects of this Class

0 1

But let your communication be, Yea, yea; Nay, nay: for whatsoever is more than these cometh of evil.

— Matthew 5:37


SUPERCODER 2000
Air cooled coding keyboard for professional use.
Done
0
1

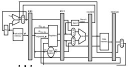
Engineering Works Because of Abstraction

Application Software

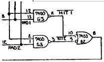
Operating Systems

```
;; voice 1 wave select
ld      a,(#CH1_W_NUM)
and     a
ld      a,(#CH1_W_SEL)
jr      nz,#00b4
ld      a,(#CH1_E_TABLE0)
```

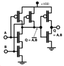
Architecture



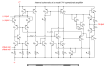
Micro-Architecture



Logic

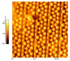


Digital Circuits



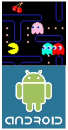
Analog Circuits

Devices

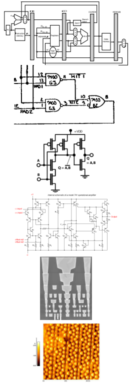


Physics

Engineering Works Because of Abstraction



```
;; voice 1 wave select
ld      a,(#CH1_W_NUM)
and     a
ld      a,(#CH1_W_SEL)
jr      nz,#00b4
ld      a,(#CH1_E_TABLE0)
```



Application Software	COMS 3157, 4156, et al.
Operating Systems	COMS W4118
Architecture	Second Half of 3827
Micro-Architecture	Second Half of 3827
Logic	First Half of 3827
Digital Circuits	First Half of 3827
Analog Circuits	ELEN 3331
Devices	ELEN 3106
Physics	ELEN 3106 et al.

Boring Stuff

`http://www.cs.columbia.edu/~sedwards/classes/2016/3827-summer/`

Prof. Stephen A. Edwards

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462 Computer Science Building

Lectures 5:30–8:40 PM, Mondays and Wednesdays
627 Mudd
May 23–June 29

Weight	What	When
40%	Homeworks	See Webpage
60%	Final exam	June 29th

Homework is due at the beginning of lecture.

Rules and Regulations

You may collaborate with classmates on homework.

Each assignment turned in must be unique; work must ultimately be your own.

List your collaborators on your homework.

Don’t cheat: Columbia Students Aren’t Cheaters

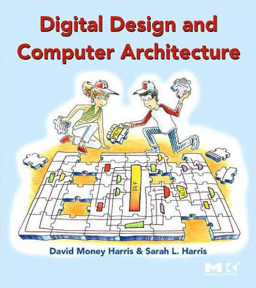
Tests will be closed-book; you may bring a single sheet of your own notes

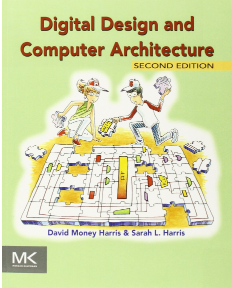
Optional Texts: Alternative 1

No required text. One option:

- David Harris and Sarah Harris. *Digital Design and Computer Architecture*. Either 1st or 2nd ed.

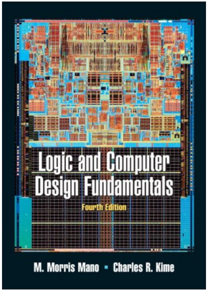
  Almost precisely right for the scope of this class: digital logic and computer architecture.



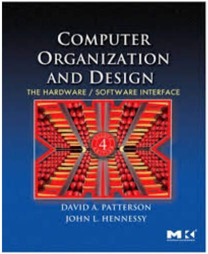


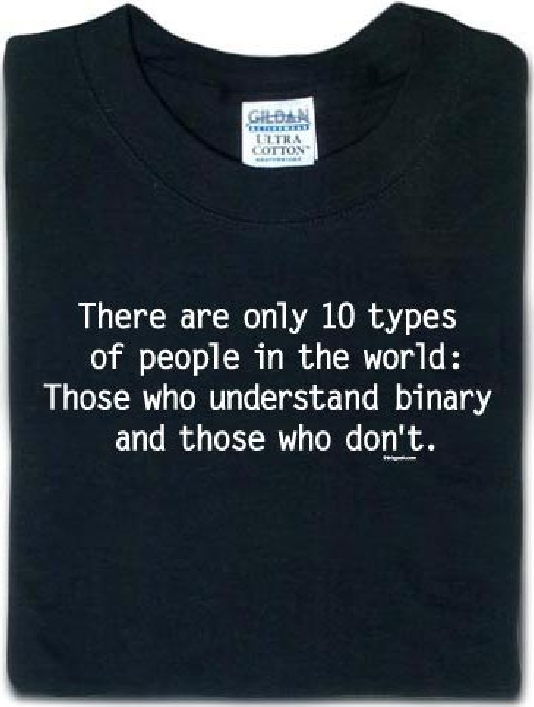
Optional Texts: Alternative 2

- M. Morris Mano and Charles Kime. *Logic and Computer Design Fundamentals.* 4th ed.



- David A. Patterson and John L. Hennessy. *Computer Organization and Design, The Hardware/Software Interface.* 4th ed.





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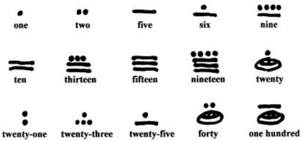
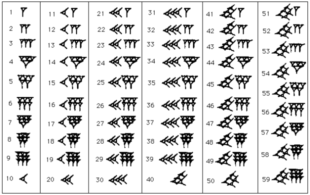
Which Numbering System Should We Use? Some Older Choices:

Roman: I II III IV V VI VII VIII IX X



Mayan: base 20, Shell = 0



Babylonian: base 60

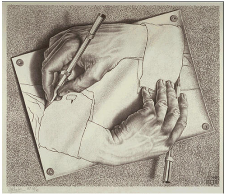
The Decimal Positional Numbering System

Ten figures: 0 1 2 3 4 5 6 7 8 9

$7 \times 10^2 + 3 \times 10^1 + 0 \times 10^0 = 730_{10}$

$9 \times 10^2 + 9 \times 10^1 + 0 \times 10^0 = 990_{10}$

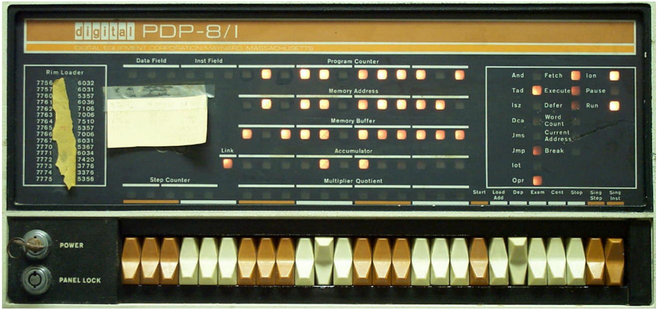
Why base ten?



Hexadecimal, Decimal, Octal, and Binary

Hex	Dec	Oct	Bin
0	0	0	0
1	1	1	1
2	2	2	10
3	3	3	11
4	4	4	100
5	5	5	101
6	6	6	110
7	7	7	111
8	8	10	1000
9	9	11	1001
A	10	12	1010
B	11	13	1011
C	12	14	1100
D	13	15	1101
E	14	16	1110
F	15	17	1111

Binary and Octal



DEC PDP-8/I, c. 1968

Oct	Bin
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111

$$
\begin{aligned}
\mathrm{PC} &= 0\times 2^{11} + 1\times 2^{10} + 0\times 2^{9} + 1\times 2^{8} + 1\times 2^{7} + 0\times 2^{6} + \\
&\quad 1\times 2^{5} + 1\times 2^{4} + 1\times 2^{3} + 1\times 2^{2} + 0\times 2^{1} + 1\times 2^{0} \\
&= 2\times 8^{3} + 6\times 8^{2} + 7\times 8^{1} + 5\times 8^{0} \\
&= 1469_{10}
\end{aligned}
$$

Hexadecimal Numbers

Base 16: 0 1 2 3 4 5 6 7 8 9 A B C D E F

Instead of groups of 3 bits (octal), Hex uses groups of 4.

$$
\begin{aligned}
\mathrm{CAFEF00D}_{16} &= 12 \times 16^7 + 10 \times 16^6 + 15 \times 16^5 + 14 \times 16^4 + \\
&\quad 15 \times 16^3 + 0 \times 16^2 + 0 \times 16^1 + 13 \times 16^0 \\
&= 3{,}405{,}705{,}229_{10}
\end{aligned}
$$

C	A	F	E	F	0	0	D				Hex
11001010111111101111000000001101											Binary
3	1	2	7	7	5	7	0	0	1	5	Octal

Computers Rarely Manipulate True Numbers

Infinite memory still very expensive

Finite-precision numbers typical

32-bit processor: naturally manipulates 32-bit numbers

64-bit processor: naturally manipulates 64-bit numbers

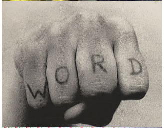
How many different numbers can you

represent with 5 {binary / octal / decimal / hexadecimal} digits?

Jargon

Bit — Binary digit: 0 or 1

Byte — Eight bits



Word — Natural number of bits for the processor, e.g., 16, 32, 64

LSB — Least Significant Bit (“rightmost”)

MSB — Most Significant Bit (“leftmost”)

Decimal Addition Algorithm

$$\begin{array}{r} 434 \\ +628 \\ \hline \end{array}$$

$$4 + 8 \quad = \quad 12$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} 1 \\ 434 \\ +628 \\ \hline 2 \end{array}$$

$$4+8 = 12$$

$$1+3+2 = 6$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} 1 \\ 434 \\ +628 \\ \hline 62 \end{array}$$

$$4 + 8 = 12$$

$$1 + 3 + 2 = 6$$

$$4 + 6 = 10$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$
\begin{array}{r}
\color{red}{1\;\;1} \\
434 \\
+628 \\
\hline
\color{blue}{062}
\end{array}
$$

$$4+8 \;=\; \color{red}{1}\color{blue}{2}$$

$$1+3+2 \;=\; \color{blue}{6}$$

$$4+6 \;=\; \color{red}{10}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} \color{red}{1}\;\;\color{red}{1}\;\;\; \\ 434 \\ +628 \\ \hline \color{blue}{1062} \end{array}$$

$$\begin{aligned} 4+8 &= \color{red}{1}\color{blue}{2} \\ 1+3+2 &= \color{blue}{6} \\ 4+6 &= \color{red}{10} \end{aligned}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Binary Addition Algorithm

$$\begin{array}{r} 10011 \\ +11001 \\ \hline \end{array}$$

$$1 + 1 \quad = \quad \color{red}{1}\color{blue}{0}$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} {\color{red}1} \\ 10011 \\ +11001 \\ \hline {\color{blue}0} \end{array}$$

$$1 + 1 = {\color{red}1}{\color{blue}0}$$

$$1 + 1 + 0 = {\color{red}1}{\color{blue}0}$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} \color{red}{11} \\ 10011 \\ +11001 \\ \hline \color{blue}{00} \end{array}$$

$$1 + 1 \quad = \quad \color{red}{1}\color{blue}{0}$$

$$1 + 1 + 0 \quad = \quad \color{red}{1}\color{blue}{0}$$

$$1 + 0 + 0 \quad = \quad \color{red}{01}$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$
\begin{array}{r}
\color{red}{011} \\
10011 \\
+11001 \\
\hline
\color{blue}{100}
\end{array}
$$

$$1+1 \quad = \quad \color{red}{1}\color{blue}{0}$$

$$1+1+0 \quad = \quad \color{red}{1}\color{blue}{0}$$

$$1+0+0 \quad = \quad \color{red}{0}\color{blue}{1}$$

$$0+0+1 \quad = \quad \color{red}{0}\color{blue}{1}$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} 0011 \\ 10011 \\ +11001 \\ \hline 1100 \end{array}$$

$$\begin{array}{rcl} 1+1 & = & 10 \\ 1+1+0 & = & 10 \\ 1+0+0 & = & 01 \\ 0+0+1 & = & 01 \\ 0+1+1 & = & 10 \end{array}$$

+	0	1
0	00	01
1	01	10
10	10	11

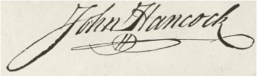
Binary Addition Algorithm

$$
\begin{array}{r}
10011 \\
10011 \\
+11001 \\
\hline
101100
\end{array}
$$

+	0	1
0	00	01
1	01	10
10	10	11

$$
\begin{aligned}
1+1 &= 10 \\
1+1+0 &= 10 \\
1+0+0 &= 01 \\
0+0+1 &= 01 \\
0+1+1 &= 10
\end{aligned}
$$

Signed Numbers: Dealing with Negativity



How should both positive and negative numbers be represented?

Signed Magnitude Numbers

You are most familiar with this:
negative numbers have a leading –

In binary, a leading 1 means negative:

$0000_2 = 0$

$0010_2 = 2$

$1010_2 = -2$

$1111_2 = -7$

$1000_2 = -0?$

Can be made to work, but addition is annoying:

If the signs match, add the magnitudes and use the same sign.

If the signs differ, subtract the smaller number from the larger; return the sign of the larger.

One's Complement Numbers

Like Signed Magnitude, a leading 1 indicates a negative One's Complement number.

To negate a number, complement (flip) each bit.

$0000_2 = 0$

$0010_2 = 2$

$1101_2 = -2$

$1000_2 = -7$

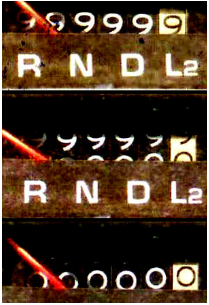
$1111_2 = -0?$

Addition is nicer: just add the one's complement numbers as if they were normal binary.

Really annoying having a -0: two numbers are equal if their bits are the same or if one is 0 and the other is -0.

0 CALORIE COLA
pepsi MAX
zero calories
maximum taste
NOT ALL
ZEROS
ARE CREATED
EQUAL
ZERO CALORIES. MAXIMUM PEPSI® TASTE.
pepsi MAX
zero calories

Two's Complement Numbers



Really neat trick: make the most significant bit represent a *negative* number instead of positive:

$$1101_2 = -8 + 4 + 1 = -3$$

$$1111_2 = -8 + 4 + 2 + 1 = -1$$

$$0111_2 = 4 + 2 + 1 = 7$$

$$1000_2 = -8$$

Easy addition: just add in binary and discard any carry.

Negation: complement each bit (as in one's complement) then add 1.

Very good property: no -0

Two's complement numbers are equal if all their bits are the same.

Number Representations Compared

Bits	Binary	Signed Mag.	One's Comp.	Two's Comp.
0000	0	0	0	0
0001	1	1	1	1
⋮				
0111	7	7	7	7
1000	8	−0	−7	−8
1001	9	−1	−6	−7
⋮				
1110	14	−6	−1	−2
1111	15	−7	−0	−1

Smallest number

Largest number

Fixed-point Numbers

How to represent fractional numbers? In decimal, we continue with negative powers of 10:

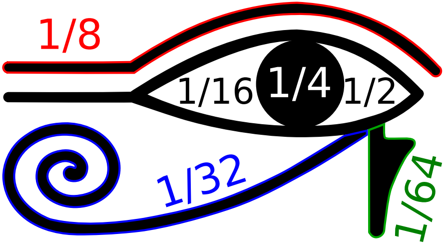
$$
\begin{aligned}
31.4159 &= 3 \times 10^1 + 1 \times 10^0 + \\
&\quad 4 \times 10^{-1} + 1 \times 10^{-2} + 5 \times 10^{-3} + 9 \times 10^{-4}
\end{aligned}
$$

The same trick works in binary:

$$
\begin{aligned}
1011.0110_2 &= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + \\
&\quad 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} \\
&= 8 + 2 + 1 + 0.25 + 0.125 \\
&= 11.375
\end{aligned}
$$

F
F a
u c
Interesting

The ancient Egyptians used binary fractions:



The Eye of Horus

Binary-Coded Decimal

thinkgeek.com

Humans prefer reading decimal numbers; computers prefer binary.

BCD is a compromise: every four bits represents a decimal digit.

Dec	BCD
0	0000 0000
1	0000 0001
2	0000 0010
⋮	⋮
8	0000 1000
9	0000 1001
10	0001 0000
11	0001 0001
⋮	⋮
18	0001 1000
19	0001 1001
20	0010 0000
⋮	⋮

BCD Addition

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

$$\begin{array}{r} 158 \\ +242 \\ \hline \end{array}$$

$$\begin{array}{rl} 0001\,0101\,1000 & \\ +0010\,0100\,0010 & \\ \hline 1010 & \text{First group} \end{array}$$

BCD Addition

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

0001 0101 1000	
+0010 0100 0010	
1010	First group
+ 0110	Correction

158

+242

BCD Addition

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

$$
\begin{array}{r}
1 \\
158 \\
+242 \\
\hline
0
\end{array}
$$

$$
\begin{array}{rl}
1 & \\
0001\,0101\,1000 & \\
+0010\,0100\,0010 & \\
\hline
1010 & \text{First group} \\
+\ 0110 & \text{Correction} \\
\hline
1\,0000 & \text{Second group}
\end{array}
$$

BCD Addition

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

```
  1
 158
+242
----
   0
```

```
         1
  0001 0101 1000
+ 0010 0100 0010
----------------
            1010   First group
          + 0110   Correction
       ---------
       1010 0000   Second group
     + 0110        Correction
----------------

----------------
```

BCD Addition

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

```
   11
   158
 + 242
  ----
    00
```

```
      1    1
   0001 0101 1000
 + 0010 0100 0010
 ----------------
             1010   First group
           + 0110   Correction
        ---------
        1010 0000   Second group
      + 0110        Correction
 ------------
   0100 0000        Third group

 ----------------
```

BCD Addition

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

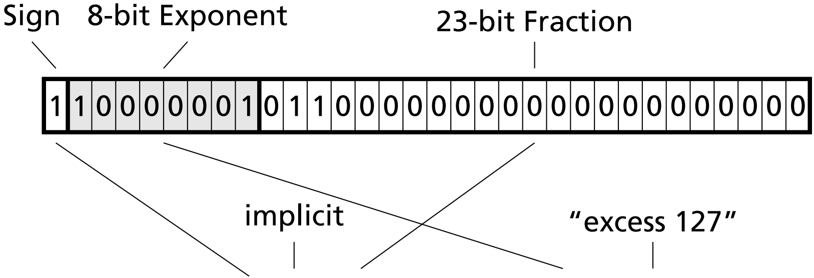
$$
\begin{array}{r}
{\color{red}{11}} \\
158 \\
+242 \\
\hline
400
\end{array}
$$

$$
\begin{array}{rrrl}
 & {\color{red}{1}} & {\color{red}{1}} & \\
0001 & 0101 & 1000 & \\
+0010 & 0100 & 0010 & \\
\hline
 & & 1010 & \text{First group} \\
 & & +\ 0110 & \text{Correction} \\
\cline{2-3}
 & 1010 & 0000 & \text{Second group} \\
 & +\ 0110 & & \text{Correction} \\
\cline{1-2}
0100 & 0000 & & \text{Third group} \\
 & & & \text{(No correction)} \\
\hline
0100 & 0000 & 0000 & \text{Result}
\end{array}
$$

Floating-Point Numbers: “Scientific Notation”

Greater dynamic range at the expense of precision
Excellent for real-world measurements

IEEE 754 Single-Precision (32-bit)



$$= -\ 1.0110000_2 \times 2^{10000001_2 - 127}$$

$$= -1.375 \times 2^2$$

$$= -5.5$$

ASCII For Representing Characters and Strings

	0	1	2	3	4	5	6	7
0	NUL	DLE	SP	0	@	P	‘	p
1	SOH	DC1	!	1	A	Q	a	q
2	STX	DC2	"	2	B	R	b	r
3	ETX	DC3	#	3	C	S	c	s
4	EOT	DC4	$	4	D	T	d	t
5	ENQ	NAK	%	5	E	U	e	u
6	ACK	SYN	&	6	F	V	f	v
7	BEL	ETB	’	7	G	W	g	w
8	BS	CAN	(	8	H	X	h	x
9	HT	EM	)	9	I	Y	i	y
A	LF	SUB	*	:	J	Z	j	z
B	VT	ESC	+	;	K	[	k	{
C	FF	FS	,	<	L	\	l	|
D	CR	GS	–	=	M	]	m	}
E	SO	RS	.	>	N	^	n	~
F	SI	US	/	?	O	_	o	DEL