Scanning and Parsing

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How do you represent one of many things?

Compilers should accept many programs; how do we describe which one we want?

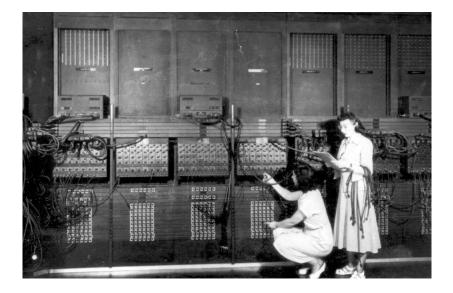
Use continuously varying values?



Very efficient, but has serious noise issues

Edison Model B Home Cylinder phonograph, 1906

The ENIAC: Programming with Spaghetti



Have one symbol per thing?



Works nicely when there are only a few things

Sholes and Glidden Typewriter, E. Remington and Sons, 1874

Have one symbol per thing?





Not so good when there are many, many things

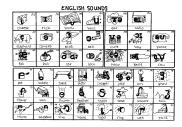
Nippon Typewriter SH-280, 2268 keys

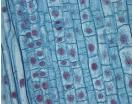
Solution: Use a Discrete Combinatorial System

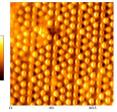
Use combinations of a small number of things to represent (exponentially) many different things.











Every Human Writing System Does This



Hieroglyphics (24+)



Cuneiform (1000 – 300)



Sanskrit (36)



Chinese (214 - 4000)



IBM Selectric (88-96)



Mayan (100)

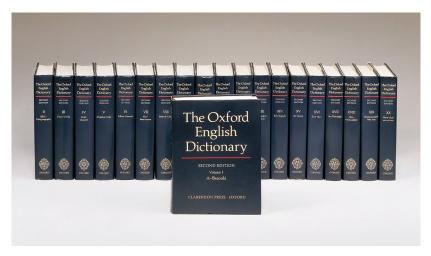


Roman (21-26)

How do you describe only certain combinations?

Compilers should only accept correct programs; how should a compiler check that its input is correct?

Just List Them?



Gets annoying for large numbers of combinations

Just List Them?

3 AA-AAAAAAAAAAAAAAA

A A A A A Budget Moving

A A A A A Budget Moving	
AAAAA sugget moving	WilbyCr. 241-5468
10	WIIDYLT _ 241-3400
A A A A A Canadian Mini-War	ehouse
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1001 ArrowRd. 24 JeffersonAv 4120 FinchE. A A A A A Critter Control	747-0228
Tool Anomio	533 7533
24 JeffersonAv	
4120 FinchE	
A A A A A Critter Control	201-4711
A A A A A Critter Control	
A A A A A Chitter Control	nionville , 410-8727
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A A A A A Devco Glass	
A A A A A Drainworks Ltd	
The second second second	433.0501
Toronto East. A A A A A Eevening Rendezve	422-0501
A A A A A Eevening Rendezve	ous 929-6848
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	eweyDr : 247-6294
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A A A A A Limousine Connect	
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	106 301-2400
A A A A A Mature Escorts	
A A A A A Move Master	588-4656
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A A A A A Prince Claude Mov AAAAA Silk Stockings. A A A A A Woodbine Moving	ing 287-6701
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65 Ci	rockford, 751-4900
A A A A A A A Alert Glass&Mirr	or 638-1989
A A A A A A A All Star Movers	
60	3 Evans _ 259-1578
A A A A A A A Armstrong Movi	nga Storana 233,2477
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AAAAAAAAAAAAAAAAAAAA Towing 18 Canso . 245-7676 Robertson Moving&Storage 236 NorthOuren 620-1212 Rezz 652.5252 AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA Law: 784-2020 Accompanying Injuries&Criminal Practice 1000 FinchW, 663-2211 Claims 2 StClairW 944-2313 Ability 2 SheppardAvE . 224-0750 Edge Door Systems _ 222-8322 Executive's Choice , 929-9390 AAAAAAAAAAAAAAAAA Automatic Garage Doors 64 Clarkson , 785-7820 Etobicoke 252-5686 A A A A A A A A A A A A A A A A Cross Alarms 280 Consumers . 494-9777 A A A A A A A A A A A A A A A A Elegant Mature Escorts 923-3333 ****** Professional Express System 425 AdelaideW . 504-9111 A A A A A A A A A A A A A A A Sweet Escorts&You, 259-3940 A AAAA AAAA AAAA AAAA Anthony De Marco 1205 StClairW, 651-2295 A AAAA AAAA AAAA AAAA Domenic Available 465,9191 Class Escort Service . 461-8110 Apple Auto Glass No Charge-Dial 1 800 50 1 800 506-5665 Cardinal Custom Building 2 BloorW . 966-4728 A A A A A A A A L U Student Movers... 693-2403 A A A AAABCO Door Co 1860 BonhillRid Mississauga Toronto 748-3667 A A A A A A B S Movers 643 LansdowneAv, 588-1499 A AA AABBCCDEF Locksmith 80 StClairE . 922-2255 A A A A A B C Movers Inc 6 Columbus : 535-3413 A A A A G B Best Movers 503-9321 AAAAA M O I Moving Systems 955 Middlefield, 299-4239 A A A ABC Glass Supply 11 Concord. 531-1548 AAAABCO Door&Window Co 1860 BonhillRd Mississauga Toronto 748-3667

Can be really redundant

Choices: CS Research Jargon Generator

Pick one from each column

an integrated	mobile	network
a parallel	functional	preprocessor
a virtual	programmable	compiler
an interactive	distributed	system
a responsive	logical	interface
a synchronized	digital	protocol
a balanced	concurrent	architecture
a virtual	knowledge-based	database
a meta-level	multimedia	algorithm

E.g., "a responsive knowledge-based preprocessor."

http://www.cs.purdue.edu/homes/dec/essay.topic.generator.html

SCIgen: An Automatic CS Paper Generator Rooter: A Methodology for the Typical Unif of Access Points and Redundancy

Jeremy Stribling, Daniel Aguayo and Maxwell Krohn

ABSTRACT

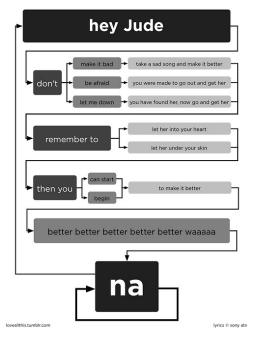
Many physicists would agree that, had it not been for congestion control, the evaluation of web browsers might never have occurred. In fact, few hackers worldwide would disagree with the essential unification of voice-over-IP and publicprivate key pair. In order to solve this riddle, we confirm that SMPs can be made stochastic, cacheable, and interposable.

I. INTRODUCTION

Many scholars would agree that, had it not been for active networks, the simulation of Lamport clocks might never have occurred. The notion that end-users synchronize with the investigation of Markov models is rarely outdated. A theoretical grand challenge in theory is the important unification The rest of this paper is organized as followe motivate the need for fiber-optic cable work in context with the prior work in the dress this obstacle, we disprove that even the tauted autonomous algorithm for the construction oriented languages can be made signed, do signed. Along these same lines, to accomplish concentrate our efforts on showing that the far algorithm for the exploration of robots by S $\Omega((n + \log n))$ time [22]. In the end, we con

II. ARCHITECTURE

Our research is principled. Consider the earby Martin and Smith; our model is similar,



http://loveallthis.tumblr.com/post/506873221

How about more structured collections of things?

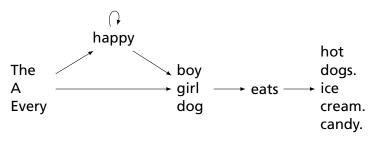
The boy eats hot dogs.

The dog eats ice cream.

Every happy girl eats candy.

A dog eats candy.

The happy happy dog eats hot dogs.



Pinker, The Language Instinct

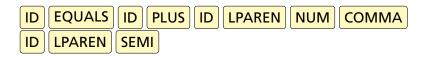
Lexical Analysis

Lexical Analysis (Scanning)

Translate a stream of characters to a stream of tokens



f o o
$$_$$
 = $_$ a + $_$ bar (0 , $_$ 42 , $_$ q) ;



Token	Lexemes	Pattern
EQUALS	=	an equals sign
PLUS	+	a plus sign
ID	a foo bar	letter followed by letters or digits
NUM	0 42	one or more digits

Lexical Analysis

Goal: simplify the job of the parser and reject some wrong programs, e.g.,

%#@\$^#!@#%#\$

is not a C program[†]

Scanners are usually much faster than parsers.

Discard as many irrelevant details as possible (e.g., whitespace, comments).

Parser does not care that the the identifer is "supercalifragilisticexpialidocious."

Parser rules are only concerned with tokens.

[†] It is what you type when your head hits the keyboard

Describing Tokens

Alphabet: A finite set of symbols Examples: { 0, 1 }, { A, B, C, ..., Z }, ASCII, Unicode

String: A finite sequence of symbols from an alphabet Examples: ϵ (the empty string), Stephen, $\alpha\beta\gamma$

Language: A set of strings over an alphabet

Examples: \emptyset (the empty language), { 1, 11, 111, 1111 }, all English words, strings that start with a letter followed by any sequence of letters and digits

Operations on Languages

Let $L = \{ \epsilon, wo \}, M = \{ man, men \}$

Concatenation: Strings from one followed by the other

 $LM = \{ man, men, woman, women \}$

Union: All strings from each language $L \cup M = \{\epsilon, wo, man, men\}$

Kleene Closure: Zero or more concatenations

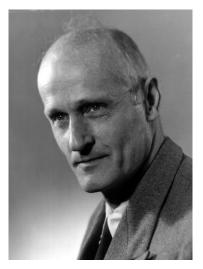
 $M^* = \{\epsilon\} \cup M \cup MM \cup MMM \cdots =$

 $\{\epsilon, man, men, manman, manmen, menman, menmen, manmanman, manmanmen, manmenman, ... \}$

Kleene Closure

"*" is named after Stephen Cole Kleene, the inventor of regular expressions, who pronounced his last name "clay-nee."

His son Ken writes "As far as I am aware this pronunciation is incorrect in all known languages. I believe that this novel pronunciation was invented by my father."



Regular Expressions over an Alphabet Σ

A standard way to express languages for tokens.

- 1. ϵ is a regular expression that denotes $\{\epsilon\}$
- **2.** If $a \in \Sigma$, *a* is an RE that denotes $\{a\}$
- 3. If r and s denote languages L(r) and L(s),

(r) | (s) denotes $L(r) \cup L(s)$ (r)(s) $\{tu: t \in L(r), u \in L(s)\}$ (r)* $\cup_{i=0}^{\infty} L(r)^{i}$ where $L(r)^{0} = \{c\}$ and $L(r)^{i} = L(r)L(r)^{i-1}$

Regular Expression Examples

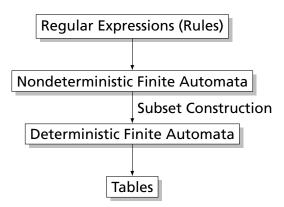
 $\Sigma = \{a, b\}$

Regexp.	Language
$a \mid b$	$\{a,b\}$
$(a \mid b)(a \mid b)$	$\{aa, ab, ba, bb\}$
a^*	$\{\epsilon, a, aa, aaa, aaaa, \ldots\}$
$(a b)^*$	$\{\epsilon, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, \ldots\}$
$a \mid a^* b$	$\{a, b, ab, aab, aaab, aaaab, \ldots\}$

Specifying Tokens with REs

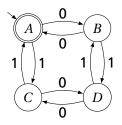
```
Typical choice: \Sigma = ASCII characters, i.e.,
{_,!,",#,$,...,0,1,...,9,...,A,...,Z,...,~}
letters: A | B | \cdots | Z | a | \cdots | z
digits: 0 | 1 | \cdots | 9
identifier: letter(letter | digit)*
```

Implementing Scanners Automatically



Nondeterministic Finite Automata

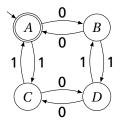
"All strings containing an even number of 0's and 1's"



1. Set of states $S:\left\{ \begin{array}{c} A \\ B \end{array} \begin{array}{c} C \\ D \end{array} \right\}$					
2. Set of input symbols Σ : {0,1}					
3. Transiti	on	funct	ion a	$\sigma: S \times \Sigma_{\epsilon} \to 2^S$	
state	е	0	1	_	
A	Ø	$\{B\}$	{ <i>C</i> }	-	
B	Ø	$\{A\}$	$\{D\}$		
С	Ø	$\{D\}$	$\{A\}$		
D	Ø	$\{A\}$ $\{D\}$ $\{C\}$	$\{B\}$		
4. Start state s_0 : A 5. Set of accepting states $F: \{A\}$					

The Language induced by an NFA

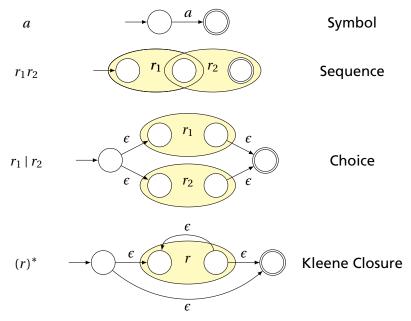
An NFA accepts an input string x iff there is a path from the start state to an accepting state that "spells out" x.



Show that the string "010010" is accepted.

$$(A \xrightarrow{0} B \xrightarrow{1} D \xrightarrow{0} C \xrightarrow{0} D \xrightarrow{1} B \xrightarrow{0} A$$

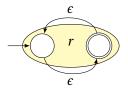
Translating REs into NFAs (Thompson's algorithm)



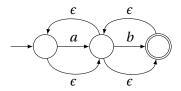
Why So Many Extra States and Transitions?

Invariant: Single start state; single end state; at most two outgoing arcs from any state: helpful for simulation.

What if we used this simpler rule for Kleene Closure?



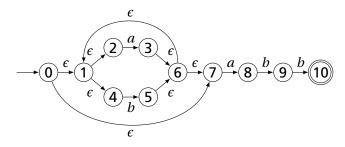
Now consider a^*b^* with this rule:



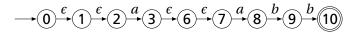
Is this right?

Translating REs into NFAs

Example: Translate $(a | b)^* abb$ into an NFA. Answer:



Show that the string "*aabb*" is accepted. Answer:



Simulating NFAs

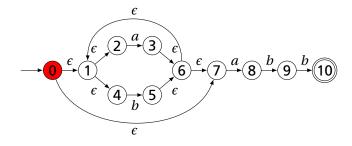
Problem: you must follow the "right" arcs to show that a string is accepted. How do you know which arc is right?

Solution: follow them all and sort it out later.

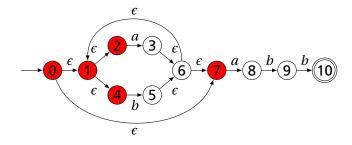
"Two-stack" NFA simulation algorithm:

- 1. Initial states: the *c*-closure of the start state
- 2. For each character *c*,
 - New states: follow all transitions labeled c
 - ► Form the *c*-closure of the current states
- 3. Accept if any final state is accepting

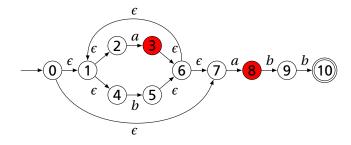
Simulating an NFA: *·aabb*, Start



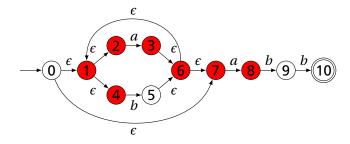
Simulating an NFA: *·aabb*, *c*-closure



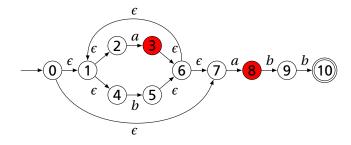
Simulating an NFA: *a*·*abb*



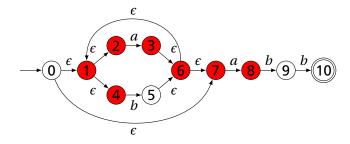
Simulating an NFA: *a*·*abb*, *c*-closure



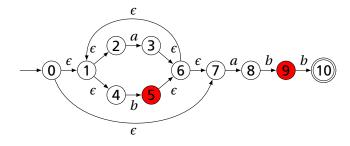
Simulating an NFA: *aa*·*bb*



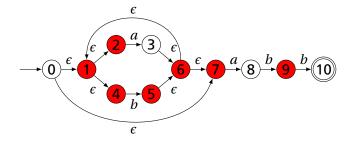
Simulating an NFA: *aa*·*bb*, *c*-closure



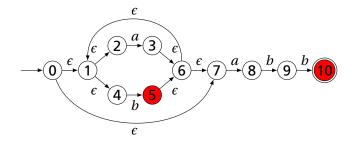
Simulating an NFA: *aab*·*b*



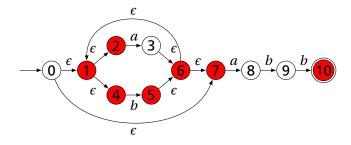
Simulating an NFA: *aab*·*b*, *c*-closure



Simulating an NFA: *aabb*·



Simulating an NFA: *aabb*, Done



Deterministic Finite Automata

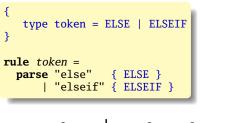
Restricted form of NFAs:

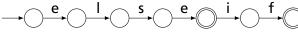
- No state has a transition on ϵ
- ► For each state *s* and symbol *a*, there is at most one edge labeled *a* leaving *s*.

Differs subtly from the definition used in COMS W3261 (Sipser, Introduction to the Theory of Computation)

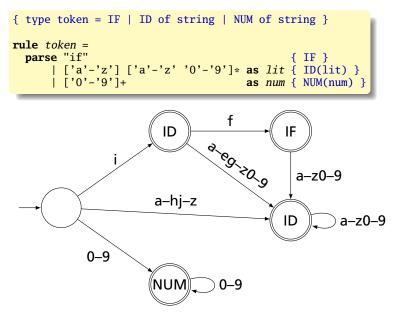
Very easy to check acceptance: simulate by maintaining current state. Accept if you end up on an accepting state. Reject if you end on a non-accepting state or if there is no transition from the current state for the next symbol.

Deterministic Finite Automata





Deterministic Finite Automata

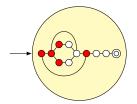


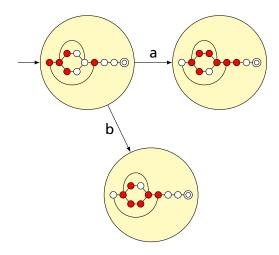
Building a DFA from an NFA

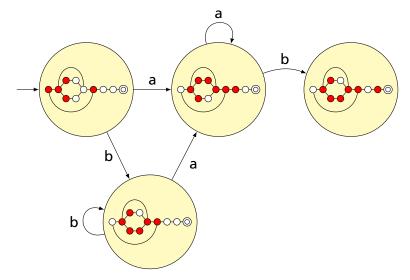
Subset construction algorithm

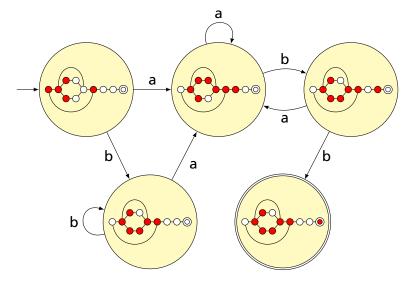
Simulate the NFA for all possible inputs and track the states that appear.

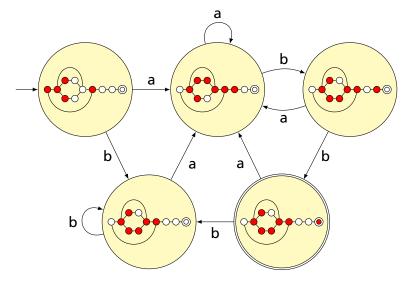
Each unique state during simulation becomes a state in the DFA.



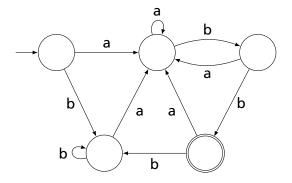






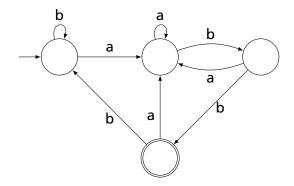


Result of subset construction for $(a | b)^* abb$



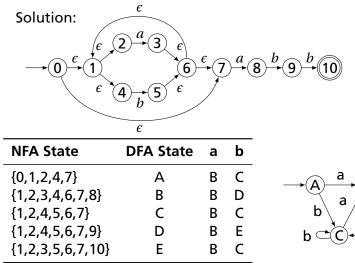
Is this minimal?

Minimized result for $(a | b)^* abb$



Transition Table Used In the Dragon Book

Problem: Translate $(a | b)^* abb$ into an NFA and perform subset construction to produce a DFA.



h

а

h

R

а

h

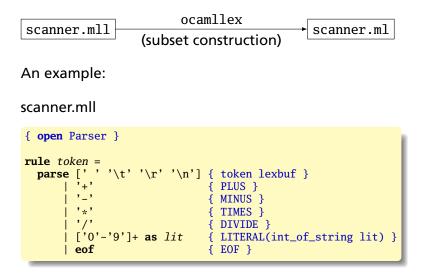
An DFA can be exponentially larger than the corresponding NFA.

n states versus 2^n

Tools often try to strike a balance between the two representations.

Lexical Analysis with Ocamllex

Constructing Scanners with Ocamllex



Ocamllex Specifications

```
{
  (* Header: verbatim OCaml code; mandatory *)
}
(* Definitions: optional *)
let ident = regexp
let ...
(* Rules: mandatory *)
rule entrypoint1 [arg1 ... argn] =
 parse pattern1 { action (* OCaml code *) }
       patternn { action }
and entrypoint2 [arg1 ... argn]} =
  . . .
and ...
{
  (* Trailer: verbatim OCaml code; optional *)
}
```

Pattern	Meaning
'c'	A single character
_	Any character (underline)
eof	The end-of-file
"foo"	A literal string
['1' '5' 'a'-'z']	"1," "5," or any lowercase letter
[^ '0'-'9']	Any character except a digit
(pattern)	Grouping
identifier	A pattern defined in the let section
pattern *	Zero or more patterns
pattern +	One or more <i>patterns</i>
pattern ?	Zero or one patterns
$pattern_1 pattern_2$	$pattern_1$ followed by $pattern_2$
pattern ₁ pattern ₂	Either <i>pattern</i> ^{1} or <i>pattern</i> ^{2}
pattern as id	Bind the matched pattern to variable id

Patterns (In Order of Decreasing Precedence)

An Example

```
{ type token = PLUS | IF | ID of string | NUM of int }
let letter = ['a'-'z' 'A'-'Z']
let digit = ['0'-'9']
rule token =
parse [' ' '\n' '\t'] { token lexbuf } (* Ignore whitespace *)
     | '+' { PLUS }
                                        (* A symbol *)
     | "if" { IF }
                                        (* A keyword *)
                                        (* Identifiers *)
     | letter (letter | digit | '_')* as id { ID(id) }
                                        (* Numeric literals *)
     | digit+ as lit { NUM(int_of_string lit) }
     | "/*" { comment lexbuf } (* C-style comments *)
and comment =
  parse "*/" { token lexbuf } (* Return to normal scanning *)
      [ _ { comment lexbuf } (* Ignore other characters *)
```

Free-Format Languages

Typical style arising from scanner/parser division

Program text is a series of tokens possibly separated by whitespace and comments, which are both ignored.

- keywords (if while)
- punctuation (, (+)
- identifiers (foo bar)
- numbers (10 -3.14159e+32)
- strings ("A String")

Free-Format Languages

Java C C++ C# Algol Pascal Some deviate a little (e.g., C and C++ have a separate preprocessor)

But not all languages are free-format.

FORTRAN 77

FORTRAN 77 is not free-format. 72-character lines:

100 **IF**(**IN** .*EQ*. 'Y' .*OR*. **IN** .*EQ*. 'Y' .*OR*. \$ **IN** .*EQ*. 'T' .*OR*. **IN** .*EQ*. 't') **THEN**



When column 6 is not a space, line is considered part of the previous.

Fixed-length line works well with a one-line buffer.

Makes sense on punch cards.



Python

The Python scripting language groups with indentation

```
i = 0
while i < 10:
    i = i + 1
    print i  # Prints 1, 2, ..., 10

i = 0
while i < 10:
    i = i + 1
print i  # Just prints 10</pre>
```

This is succinct, but can be error-prone.

How do you wrap a conditional around instructions?

Syntax and Language Design

- Does syntax matter? Yes and no
- More important is a language's semantics—its meaning.
- The syntax is aesthetic, but can be a religious issue.
- But aesthetics matter to people, and can be critical.
- Verbosity does matter: smaller is usually better.
- Too small can be problematic: APL is a succinct language with its own character set.
- There are no APL programs, only puzzles.

Syntax and Language Design

Some syntax is error-prone. Classic fortran example:

```
D0 5 I = 1,25 ! Loop header (for i = 1 to 25)
D0 5 I = 1.25 ! Assignment to variable D05I
```

Trying too hard to reuse existing syntax in C++:

vector< vector<int> > foo; vector<vector<int>> foo; // Syntax error

C distinguishes > and >> as different operators.

Bjarne Stroustrup tells me they have finally fixed this.

Modeling Sentences

Simple Sentences Are Easy to Model

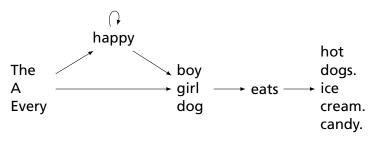
The boy eats hot dogs.

The dog eats ice cream.

Every happy girl eats candy.

A dog eats candy.

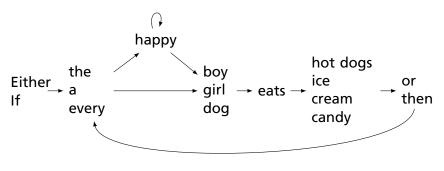
The happy happy dog eats hot dogs.



Pinker, The Language Instinct

Richer Sentences Are Harder

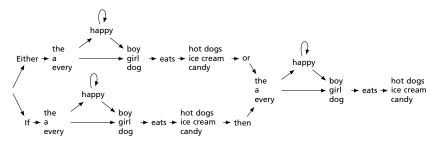
If the boy eats hot dogs, then the girl eats ice cream. Either the boy eats candy, or every dog eats candy.



Does this work?

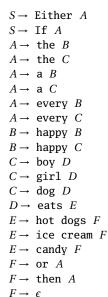
Automata Have Poor Memories

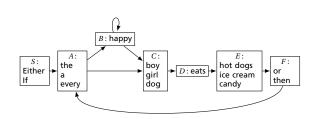
Want to "remember" whether it is an "either-or" or "if-then" sentence. Only solution: duplicate states.



Automata in the form of Production Rules

Problem: automata do not remember where they've been





Solution: Context-Free Grammars

Context-Free Grammars have the ability to "call subroutines:"

```
S \rightarrow Either P, or P. Exactly two Ps
S \rightarrow If P, then P.
P \rightarrow A H N eats O One each of A, H, N, and O
A \rightarrow the
A \rightarrow a
A \rightarrow \text{every}
H \rightarrow happy H
                                      H is "happy" zero or more times
H \rightarrow \epsilon
N \rightarrow boy
N \rightarrow \text{girl}
N \rightarrow \text{dog}
O \rightarrow hot dogs
O \rightarrow ice cream
O \rightarrow candy
```

A Context-Free Grammar for a Simplified C

 $program \rightarrow \epsilon | program vdecl | program fdecl$

fdecl \rightarrow id (formals) { vdecls stmts }

formals $\rightarrow \texttt{id} \,|\, \texttt{formals}$, id

vdecls → vdecl | vdecls vdecl

 $vdecl \rightarrow int id;$

stmts $\rightarrow \epsilon \mid$ stmts stmt

stmt → expr; |return expr; |{ stmts }|if (expr) stmt| if (expr) stmt else stmt| for (expr; expr; expr) stmt|while (expr) stmt

expr → lit|id|id (actuals) | (expr) | expr + expr|expr - expr|expr * expr|expr / expr| expr == expr|expr != expr|expr < expr|expr <= expr| expr > expr|expr >= expr|expr = expr

 $actuals \rightarrow expr | actuals, expr$

Constructing Grammars and Ocamlyacc

Parsing

Objective: build an abstract syntax tree (AST) for the token sequence from the scanner.



Goal: verify the syntax of the program, discard irrelevant information, and "understand" the structure of the program.

Parentheses and most other forms of punctuation removed.

Ambiguity

One morning I shot an elephant in my pajamas.

Ambiguity

One morning I shot an elephant in my pajamas. How he got in my pajamas I don't know. —Groucho Marx

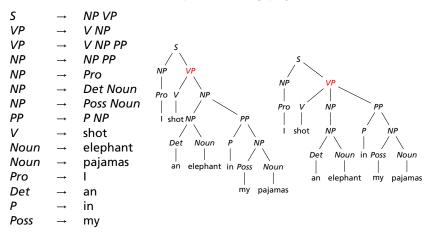






Ambiguity in English

I shot an elephant in my pajamas



Jurafsky and Martin, Speech and Language Processing

The Dangling Else Problem

Who owns the else?

if (a) if (b) c(); else d(); if (a) if (b) c(); else d(); Should this be a if (b) or a if (c) ? b c() d() b c()

Grammars are usually ambiguous; manuals give disambiguating rules such as C's:

As usual the "else" is resolved by connecting an else with the last encountered elseless if.

The Dangling Else Problem

Problem comes after matching the first statement. Question is whether an "else" should be part of the current statement or a surrounding one since the second line tells us "stmt ELSE" is possible.

The Dangling Else Problem

Some languages resolve this problem by insisting on nesting everything.

E.g., Algol 68:

if a < b then a else b fi;</pre>

"fi" is "if" spelled backwards. The language also uses do-od and case-esac.

Another Solution to the Dangling Else Problem

Idea: break into two types of statements: those that have a dangling "then" ("dstmt") and those that do not ("cstmt"). A statement may be either, but the statement just before an "else" must not have a dangling clause because if it did, the "else" would belong to it.

stmt : dstmt cstmt
dstmt : IF expr THEN stmt IF expr THEN cstmt ELSE dstmt
cstmt : IF expr THEN cstmt ELSE cstmt other statements

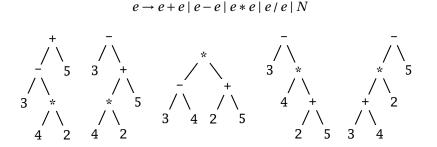
We are effectively carrying an extra bit of information during parsing: whether there is an open "then" clause. Unfortunately, duplicating rules is the only way to do this in a context-free grammar.

Ambiguous Arithmetic

Ambiguity can be a problem in expressions. Consider parsing

3 - 4 * 2 + 5

with the grammar



Operator Precedence and Associativity

Usually resolve ambiguity in arithmetic expressions

Like you were taught in elementary school:

"My Dear Aunt Sally"

Mnemonic for multiplication and division before addition and subtraction.

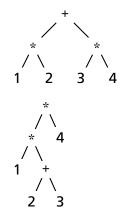
Operator Precedence

Defines how "sticky" an operator is.

1 * 2 + 3 * 4

* at higher precedence than +: (1 * 2) + (3 * 4)

+ at higher precedence than *: 1 * (2 + 3) * 4

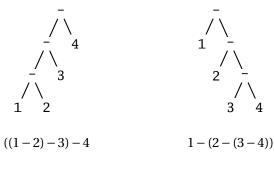


Associativity

Whether to evaluate left-to-right or right-to-left

Most operators are left-associative

1 - 2 - 3 - 4



left associative

right associative

Fixing Ambiguous Grammars

A grammar specification:

expr : expr PLUS expr | expr MINUS expr | expr TIMES expr | expr DIVIDE expr | NUMBER

Ambiguous: no precedence or associativity.

Ocamlyacc's complaint: "16 shift/reduce conflicts."

Assigning Precedence Levels

Split into multiple rules, one per level

expr	: expr PLUS expr expr MINUS expr term
term	: term TIMES term term DIVIDE term atom
atom	: NUMBER

Still ambiguous: associativity not defined

Ocamlyacc's complaint: "8 shift/reduce conflicts."

Assigning Associativity

Make one side the next level of precedence

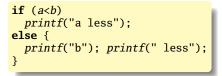
expr	: expr PLUS term expr MINUS term term
term	: term TIMES atom term DIVIDE atom atom
atom	: NUMBER

This is left-associative.

No shift/reduce conflicts.

Statement separators/terminators

C uses ; as a statement terminator.



Pascal uses ; as a statement separator.

```
if a < b then
  writeln('a less')
else begin
  write('a'); writeln(' less')
end</pre>
```

Pascal later made a final ; optional.

Ocamlyacc Specifications

```
%{
    (* Header: verbatim OCaml; optional *)
    /* Declarations: tokens, precedence, etc. */
%%
    /* Rules: context-free rules */
%%
    (* Trailer: verbatim OCaml; optional *)
```

Declarations

- %token symbol ...
 Define symbol names (exported to .mli file)
- %token < type > symbol ...
 Define symbols with attached attribute (also exported)
- %start symbol ...
 Define start symbols (entry points)
- %type < type > symbol ...
 Define the type for a symbol (mandatory for start)
- %left symbol ...
- %right symbol ...
- %nonassoc symbol ...

Define predecence and associtivity for the given symbols, listed in order from lowest to highest precedence

Rules

nonterminal : symbol	symbol {	semantic-action	}
 symbol	symbol {	semantic-action	}

- nonterminal is the name of a rule, e.g., "program," "expr"
- symbol is either a terminal (token) or another rule
- semantic-action is OCaml code evaluated when the rule is matched
- In a semantic-action, \$1, \$2, ... returns the value of the first, second, ... symbol matched
- A rule may include "%prec symbol" to override its default precedence

An Example .mly File

%token <int> TNT **%token** PLUS MINUS TIMES DIV LPAREN RPAREN EOL %left PLUS MINUS /* lowest precedence */ **%left** TIMES DIV %nonassoc UMINUS /* highest precedence */ %start main /* the entry point */ **%type** <int> main %% main: { \$1 } expr EOL expr: TNT { \$1 } LPAREN expr RPAREN { \$2 } expr PLUS expr { \$1 + \$3 } expr MINUS expr { \$1 - \$3 } expr TIMES expr { \$1 * \$3 } expr DIV expr { \$1 / \$3 } MINUS expr %prec UMINUS { - \$2 }

Parsing Algorithms

Parsing Context-Free Grammars

There are $O(n^3)$ algorithms for parsing arbitrary CFGs, but most compilers demand O(n) algorithms.

Fortunately, the LL and LR subclasses of CFGs have O(n) parsing algorithms. People use these in practice.

е

 $1: e \to t + e$ $2: e \to t$ $3: t \to \mathbf{Id} * t$ $4: t \to \mathbf{Id}$

At each step, expand the *rightmost* nonterminal.

nonterminal

"handle": The right side of a production



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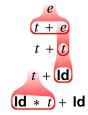


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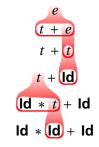


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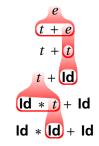


At each step, expand the rightmost nonterminal.

nonterminal

"handle": The right side of a production





At each step, expand the rightmost nonterminal.

nonterminal

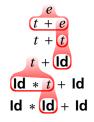
"handle": The right side of a production

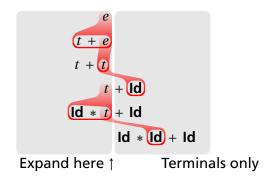
Dragon-book style: underline handles

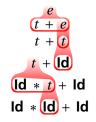
 $e \rightarrow \underline{t + e} \rightarrow t + \underline{t} \rightarrow t + \underline{\mathsf{Id}} \rightarrow \underline{\mathsf{Id}} * \underline{t} + \mathsf{Id} \rightarrow \mathsf{Id} * \underline{\mathsf{Id}} + \mathsf{Id}$

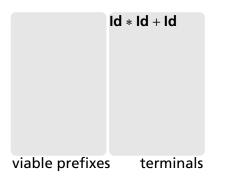
Rightmost Derivation: What to Expand

 $1: e \to t + e$ $2: e \to t$ $3: t \to \mathbf{Id} * t$ $4: t \to \mathbf{Id}$

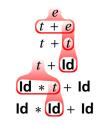


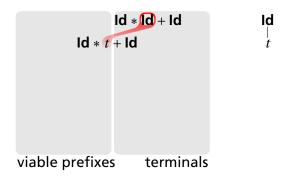


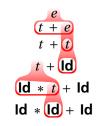


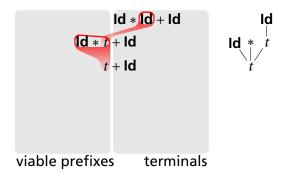


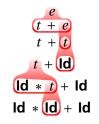
 $1: e \to t + e$ $2: e \to t$ $3: t \to \mathbf{Id} * t$ $4: t \to \mathbf{Id}$

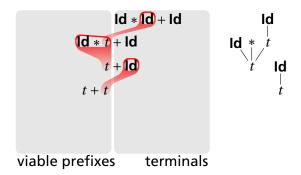


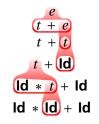


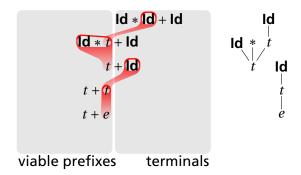




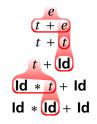


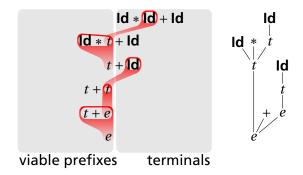


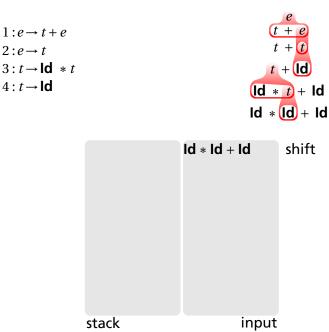


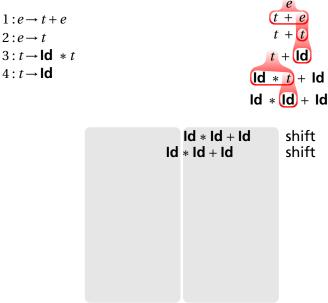


Reverse Rightmost Derivation









stack

input

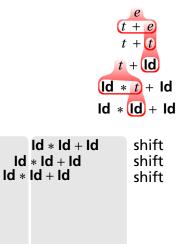
t + e

t + t

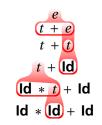
 $t + \mathbf{Id}$

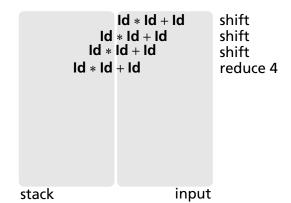
shift

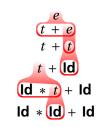
shift

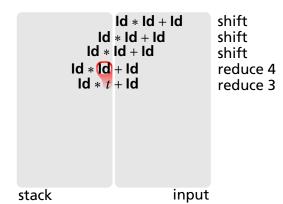


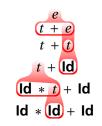


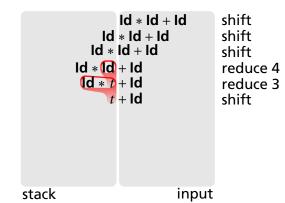


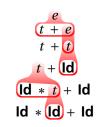


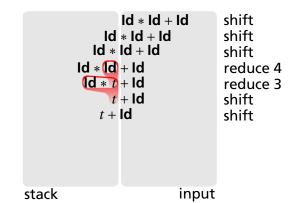


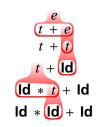


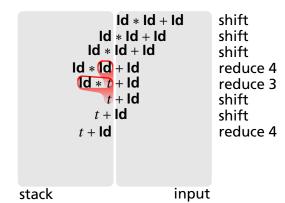


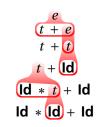


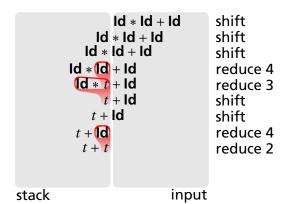


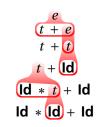


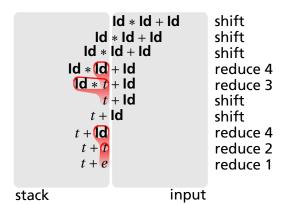


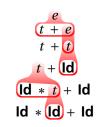


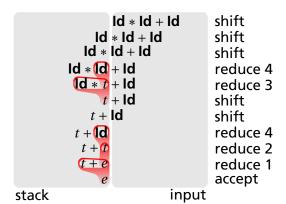












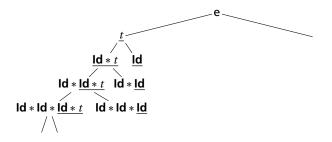
Handle Hunting

Right Sentential Form: any step in a rightmost derivation **Handle:** in a sentential form, a RHS of a rule that, when rewritten, yields the previous step in a rightmost derivation. The big question in shift/reduce parsing:

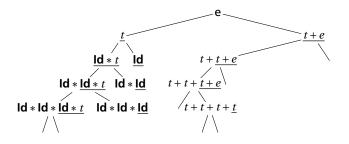
When is there a handle on the top of the stack?

Enumerate all the right-sentential forms and pattern-match against them? Usually infinite in number, but let's try anyway.

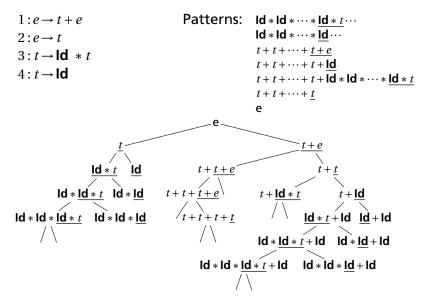
Some Right-Sentential Forms and Their Handles



Some Right-Sentential Forms and Their Handles



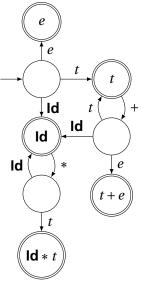
Some Right-Sentential Forms and Their Handles



The Handle-Identifying Automaton

Magical result, due to Knuth: An automaton suffices to locate a handle in a right-sentential form.

$$\mathbf{Id} * \mathbf{Id} * \cdots * \mathbf{Id} * \underline{t} \cdots$$
$$\mathbf{Id} * \mathbf{Id} * \cdots * \mathbf{Id} \cdots$$
$$t + t + \cdots + \underline{t + e}$$
$$t + t + \cdots + t + \mathbf{Id}$$
$$t + t + \cdots + t + \mathbf{Id} * \mathbf{Id} * \cdots * \mathbf{Id} * \underline{t}$$
$$t + t + \cdots + \underline{t}$$
e



Building the Initial State of the LR(0) Automaton

 $e' \rightarrow \mathbb{C}e$

 $1: e \rightarrow t + e$ $2: e \rightarrow t$ $3: t \rightarrow \mathbf{Id} * t$ $4: t \rightarrow \mathbf{Id}$



Key idea: automata identify viable prefixes of right sentential forms. Each state is an equivalence class of possible places in productions.

At the beginning, any viable prefix must be at the beginning of a string expanded from e. We write this condition " $e' \rightarrow \&e''$ "

Building the Initial State of the LR(0) Automaton

 $1: e \rightarrow t + e$ $2: e \rightarrow t$ $3: t \rightarrow \mathbf{Id} * t$ $4: t \rightarrow \mathbf{Id}$ $e' \to \mathfrak{C}e$ $e \to \mathfrak{C}t + e$ $e \to \mathfrak{C}t$

Key idea: automata identify viable prefixes of right sentential forms. Each state is an equivalence class of possible places in productions.

At the beginning, any viable prefix must be at the beginning of a string expanded from e. We write this condition " $e' \rightarrow \&e''$ "

There are two choices for what an *e* may expand to: t + e and *t*. So when $e' \rightarrow \&e, e \rightarrow \&t + e$ and $e \rightarrow \&t$ are also true, i.e., it must start with a string expanded from *t*.

Building the Initial State of the LR(0) Automaton

 $1: e \to t + e$ $2: e \to t$ $3: t \to \mathbf{Id} * t$ $4: t \to \mathbf{Id}$ $e' \rightarrow \& e$ $e \rightarrow \& t + e$ $e \rightarrow \& t$ $t \rightarrow \& \mathsf{Id} * t$ $t \rightarrow \& \mathsf{Id}$

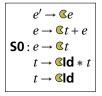
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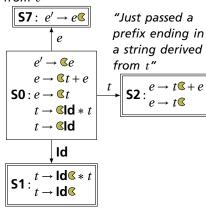
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Similarly, t must be either $\mathbf{Id} * t$ or \mathbf{Id} , so $t \to \mathbf{CId} * t$ and $t \to \mathbf{CId}$

The first state suggests a viable prefix can start as any string derived from *e*, any string derived from *t*, or **Id**.



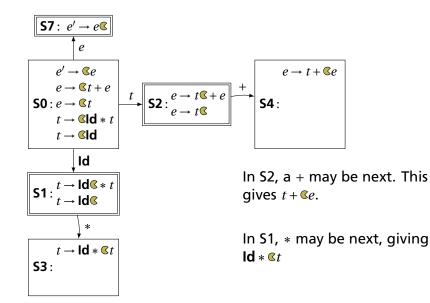
string derived from e"

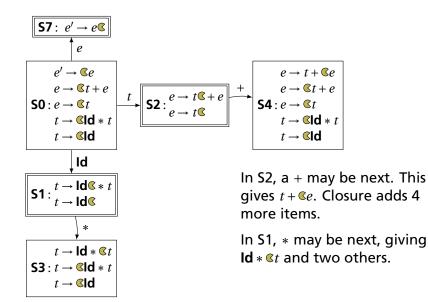


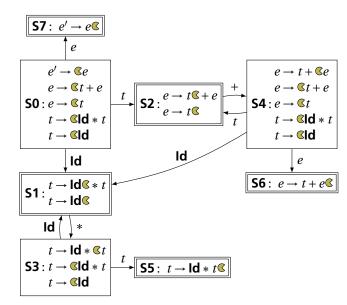
"Just passed a prefix that ended in an **Id**"

The first state suggests a viable prefix can start as any string derived from *e*, any string derived from *t*, or **Id**.

The items for these three states come from advancing the *c* across each thing, then performing the closure operation (vacuous here).







The first function

If you can derive a string that starts with terminal t from some sequence of terminals and nonterminals α , then $t \in \text{first}(\alpha)$.

- 1. Trivially, $first(X) = \{X\}$ if X is a terminal.
- **2**. If $X \rightarrow \epsilon$, then add ϵ to first(*X*).
- 3. For each prod. $X \rightarrow Y \cdots$, add first $(Y) \{c\}$ to first(X). If X can produce something, X can start with whatever that starts with
- 4. For each prod. $X \rightarrow Y_1 \cdots Y_k Z \cdots$ where $\epsilon \in \text{first}(Y_i)$ for i = 1, ..., k, add $\text{first}(Z) \{\epsilon\}$ to first(X). Skip all potential ϵ 's at the beginning of whatever X produces

$1: e \rightarrow t + e$	$first(Id) = \{Id\}$
$2: e \rightarrow t$	first(t) = { Id } because $t \rightarrow $ Id * t and $t \rightarrow $ Id
$3: t \to Id * t 4: t \to Id$	first(e) = { Id } because $e \rightarrow t + e$, $e \rightarrow t$, and first(t) = (Id)
	$first(t) = \{ \mathbf{Id} \}.$

If t is a terminal, A is a nonterminal, and $\cdots At \cdots$ can be derived, then $t \in follow(A)$.

- 1. Add \$ ("end-of-input") to follow(S) (start symbol). End-of-input comes after the start symbol
- 2. For each prod. $\rightarrow \cdots A\alpha$, add first(α) { ϵ } to follow(A). A is followed by the first thing after it
- 3. For each prod. $A \rightarrow \cdots B$ or $a \rightarrow \cdots B\alpha$ where $\epsilon \in \text{first}(\alpha)$, then add everything in follow(A) to follow(B). If B appears at the end of a production, it can be followed by whatever follows that production
- $1: e \rightarrow t + e$ $2: e \rightarrow t$ $3: t \rightarrow Id * t$ $4: t \rightarrow Id$ $first(t) = \{Id\}$ $first(e) = \{Id\}$

```
follow(e) = {$}
follow(t) = {
```

1. Because e is the start symbol

If t is a terminal, A is a nonterminal, and $\cdots At \cdots$ can be derived, then $t \in follow(A)$.

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- 2. For each prod. $\rightarrow \cdots A\alpha$, add first(α) { ϵ } to follow(A). A is followed by the first thing after it
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- $1: e \rightarrow t + e$ $2: e \rightarrow t$ $3: t \rightarrow Id * t$ $4: t \rightarrow Id$ first(t) = {Id} first(e) = {Id}

follow(
$$e$$
) = {\$}
follow(t) = {+ }

2. Because $e \rightarrow \underline{t} + e$ and first(+) = {+}

If t is a terminal, A is a nonterminal, and $\cdots At \cdots$ can be derived, then $t \in follow(A)$.

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- $1: e \rightarrow t + e$ $2: e \rightarrow t$ $3: t \rightarrow Id * t$ $4: t \rightarrow Id$ first(t) = {Id} first(e) = {Id}

```
follow(e) = {$}
```

- $follow(t) = \{+, \$\}$
- 3. Because $e \rightarrow \underline{t}$ and $\$ \in follow(e)$

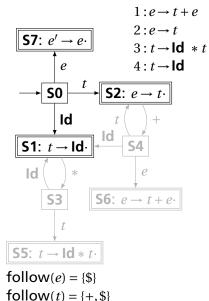
If t is a terminal, A is a nonterminal, and $\cdots At \cdots$ can be derived, then $t \in follow(A)$.

- 1. Add \$ ("end-of-input") to follow(S) (start symbol). End-of-input comes after the start symbol
- 2. For each prod. $\rightarrow \cdots A\alpha$, add first(α) { ϵ } to follow(A). A is followed by the first thing after it
- 3. For each prod. $A \rightarrow \cdots B$ or $a \rightarrow \cdots B\alpha$ where $e \in first(\alpha)$, then add everything in follow(A) to follow(B). If B appears at the end of a production, it can be followed by whatever follows that production
- $1: e \rightarrow t + e$ $2: e \rightarrow t$ $3: t \rightarrow Id * t$ $4: t \rightarrow Id$ first(t) = {Id} first(e) = {Id}

$$follow(e) = \{\}\}$$

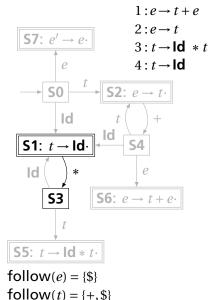
 $follow(t) = \{+, \}\}$

Fixed-point reached: applying any rule does not change any set



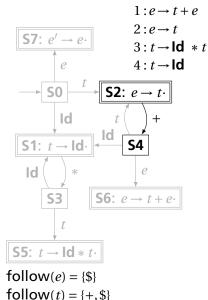
State		Act	Go	oto		
	Id	+	*	\$	e	t
0	s1				7	2

From S0, shift an **Id** and go to S1; or cross a *t* and go to S2; or cross an *e* and go to S7.



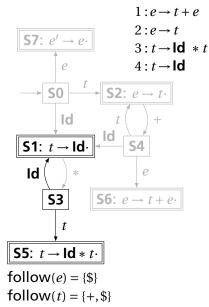
State		Action			Goto		
	Id	+	*	\$	e	t	
0	s1				7	2	
1		r4	s3	r4			

From S1, shift a * and go to S3; or, if the next input \in follow(*t*), reduce by rule 4.



State		Goto				
	Id	+	*	\$	е	t
0	s1				7	2
1		r4	s3	r4		
2		r4 s4		r4 r2		

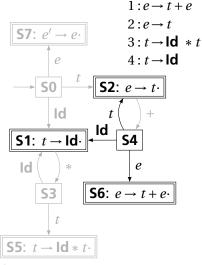
From S2, shift a + and go to S4; or, if the next input \in follow(*e*), reduce by rule 2.



State		Goto				
	Id	+	*	\$	е	t
0	s1				7	2
1		r4 s4	s3	r4		
2		s4		r2		
3	s1					5
	I				I	

From S3, shift an **Id** and go to S1; or cross a *t* and go to S5.

State



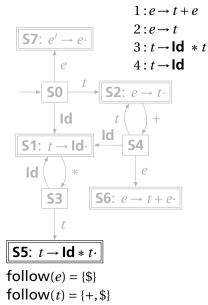
Juic		Action				
	Id	+	*	\$	е	t
0	s1				7	2
1		r4	s3	r4 r2		
2		r4 s4		r2		
2 3 4	s1 s1					5 2
4	s1				6	2
	1				1	

Action

Goto

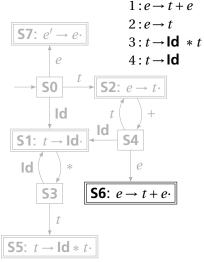
From S4, shift an **Id** and go to S1; or cross an *e* or a *t*.

follow(e) = {\$} follow(t) = {+, \$}



State		Action				Goto	
	Id	+	*	\$	e	t	
0	s1				7	2	
1		r4 s4	s3	r4			
2		s4		r2			
3	s1					5	
4	s1				6	2	
5		r3		r3			

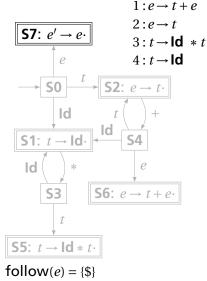
From S5, reduce using rule 3 if the next symbol \in follow(t).



4	SI		6	2
5	r3	r3		
6		r1		
From S6	, reduce u	sing rule	1 if	
the nex	t symbol ∈	follow(e).	

 $follow(e) = \{\$\}$ $follow(t) = \{+, \$\}$

State		Act	Go	oto		
	Id	+	*	\$	e	t
0	s1				7	2
1		r4	s3	r4		
2		s4		r2		
3	s1					5
4	s1				6	2
5		r3		r3		
6				r1		



State		Action			Go	oto
	Id	+	*	\$	e	t
0	s1				7	2
1		r4	s3	r4		
2		s4		r2		
3	s1					5
4	s1				6	5 2
5		r3		r3		
6				r1		
7				\checkmark		

If, in S7, we just crossed an *e*, accept if we are at the end of the input.

 $follow(t) = \{+, \$\}$

_	Stack	Input	Action
$1: e \to t + e$ $2: e \to t$	0	ld * ld + ld \$	Shift, goto 1
$\begin{array}{l} 3:t \to Id & *t \\ 4:t \to Id \end{array}$		the state on d the next in	

State		Act		Go	oto	
	Id	+	*	\$	е	t
0	s1				7	2
1		r4	s3	r4		
2		r4 s4		r2		
2 3	s1					5
4	s1				6	2
5 6		r3		r3		
6				r1		
7				\checkmark		

stack and the next input token. Find the action (shift, reduce, or

error) in the table.

In this case, shift the token onto the stack and mark it with state 1.

	Stack	Input	Action
$1: e \to t + e$ $2: e \to t$ $3: t \to \mathbf{Id} * t$ $4: t \to \mathbf{Id}$	0 0 Id 1	ld * ld + ld \$ * ld + ld \$	Shift, goto 1 Shift, goto 3

State		Action											
	Id	+	*	\$	е	t							
0	s1				7	2							
1		r4	s3	r4									
2 3		r4 s4		r2									
	s1					5							
4	s1				6	2							
5		r3		r3									
6				r1									
7				\checkmark									

Here, the state is 1, the next symbol is *, so shift and mark it with state 3.

$1: e \rightarrow t + e$	
$2: e \rightarrow t$	
$3: t \rightarrow \mathbf{Id} * t$	
$4: t \rightarrow \mathbf{Id}$	

State			Goto			
	Id	+	*	\$	e	t
0	s1				7	2
1		r4	s3	r4		
2		s4		r2		
3	s1					5
4	s1				6	2
5		r3		r3		
6				r1		
7				\checkmark		

Stack	Input	Action			
0	ld * ld + ld \$	Shift, goto 1			
0 <mark>1</mark>	* Id + Id \$	Shift, goto 3			
0 1 3	ld + ld \$	Shift, goto 1			
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	+ Id \$	Reduce 4			

Here, the state is 1, the next symbol is +. The table says reduce using rule 4.

 \checkmark

7

					-		Stack	Input	Action
$1: e \rightarrow 2: e \rightarrow 3: t \rightarrow 4: t \rightarrow 1$	t Id *	t					0 0 1d 0 1 3	Id * Id + Id \$ * Id + Id \$ Id + Id \$	Shift, goto 1 Shift, goto 3 Shift, goto 1
State		Act	tion		Go	oto	0 1 3 1	+ Id \$	Reduce 4
	Id	+	*	\$	е	t	0 1 3	+ Id \$	
0	s1				7	2			
1		r4	s3	r4			Remove	the RHS of th	ne rule (here,
2		s4		r2			just Id),	observe the s	tate on the
3	s1					5	top of t	he stack, and	consult the
4	s1				6	2	"goto"	portion of the	e table.
5		r3		r3					
6				r1					

							Slack
$1:e \rightarrow$							0 Id
$2: e \rightarrow \\ 3: t \rightarrow $		t					0 d 1 *
$4: t \rightarrow$							0 1 3 Id
State		Act	tion		Go	oto	0 1 3 1 + I
	Id	+	*	\$	e	t	$\begin{array}{c c} \mathbf{Id} & * & t \\ 1 & 3 & 5 \\ \end{array} + \mathbf{I}$
0	s1				7	2	
1		r4	s3	r4			Here, we pu
2		s4		r2			This effective

5 2

6

r3

r1

 \checkmark

3

4

5

6

7

s1

s1

r3

Stack	Input	Action		
0	ld * ld + ld \$	Shift, goto 1		
0 <mark>1d</mark>	* Id + Id \$	Shift, goto 3		
0 1 3	Id + Id \$	Shift, goto 1		
0 1 3 1	+ Id \$	Reduce 4		
$ \begin{bmatrix} 1d & * & t \\ 1 & 3 & 5 \end{bmatrix} $	+ Id \$	Reduce 3		

Here, we push a *t* with state 5. This effectively "backs up" the LR(0) automaton and runs it over the newly added nonterminal.

In state 5 with an upcoming +, the action is "reduce 3."

							Stack	Input	Action
$1: e \rightarrow 2: e \rightarrow$							0 0 Id	Id * Id + Id \$	Shift, goto
$3:t \rightarrow$	ld *	t						* Id + Id \$	Shift, goto
$4: t \rightarrow$	ld						0 1 3	ld + ld \$	Shift, goto
State		Act	tion		Go	oto	0 1 3 1	+ Id \$	Reduce 4
	ld	+	*	\$	е	t	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	+ Id \$	Reduce 3
0	s1	_	_	_	7	2	0 ^{<i>t</i>} / ₂	+ Id \$	Shift, goto
1		r4	s3	r4					
2		s4		r2					
3	s1					5		we strip off th	
4	s1				6	2	rule 3, ld *	t, exposing sta	ate 0, so
5		r3		r3			we push a	t with state 2.	
6				r1					
7				\checkmark					

							 S	ta	ck		Input	Action
$1: e \rightarrow 2: e \rightarrow 0$										0	Id * Id + Id \$	Shift, goto 1
$2:e \rightarrow 3:t \rightarrow 1$	-	t							0		* Id + Id \$	Shift, goto 3
$4: t \rightarrow $	ld								Id 1		ld + ld \$	Shift, goto 1
State		Act	tion		Go	oto	0	Id 1	-	ld 1	+ Id \$	Reduce 4
	Id	+	*	\$	е	t	0	ld 1	*3	<i>t</i> 5	+ Id \$	Reduce 3
0	s1		_		7	2			0	<i>t</i> 2	+ Id \$	Shift, goto 4
1 2		r4 s4	s3	r4 r2				0	<i>t</i> 2	<mark>+</mark> 4	ld\$	Shift, goto 1
3 4	s1 s1				6	5 2	0	<i>t</i> 2	+ 4	ld 1	\$	Reduce 4
5	51	r3		r3		2	0	<i>t</i> 2	+4	$\frac{t}{2}$	\$	Reduce 2
6 7				r1 √			0		+4		\$	Reduce 1
	1				1			-	0	е 7	\$	Accept

The Punchline

This is a tricky, but mechanical procedure. The Ocamlyacc parser generator uses a modified version of this technique to generate fast bottom-up parsers.

You need to understand it to comprehend error messages:

Shift/reduce conflicts are caused by a state like

```
t \rightarrow \cdot \mathbf{Else} \, s
```

 $t \rightarrow \cdot$

If the next token is **Else**, do you reduce it since **Else** may follow a *t*, or shift it? Reduce/reduce conflicts are caused by a state like

$$t \rightarrow \mathbf{Id} * t \cdot$$

 $e \to t + e \, \cdot$

Do you reduce by " $t \rightarrow \mathbf{Id} * t$ " or by " $e \rightarrow t + e$ "?