## CSEE W3827

## Fundamentals of Computer Systems Homework Assignment 1

Prof. Stephen A. Edwards<br>Columbia University<br>Due Tuesday, June 9th, 2015 at 5:30 PM

Print this out and turn it in. Enter answers on the computer or manually on the printout.
This homework requires you to use Logisim, which you can download from http://www.cburch.com/logisim/

Name: $\square$
Uni: $\square$

1. (5 pts.) What are the values, in decimal, of the following bytes if they are interpreted as 8-bit numbers in

## 0001001110011010

binary
one's complement
two's complement

$\square$
2. (5 pts.) Complete the truth table for the following Boolean functions:

$$
\begin{aligned}
& a=X \bar{Y}+\bar{X} Y Z+\bar{X} \bar{Z} \\
& b=(X+\bar{Y})(X+Z)(\bar{X}+Z)
\end{aligned}
$$

| $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ | $\mathbf{a}$ | $\mathbf{b}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | $\square$ | $\square$ |
| 0 | 0 | 1 |  | $\square$ |
| 0 | 1 | 0 |  | $\square$ |
| 0 | 1 | 1 |  | $\square$ |
| 1 | 0 | 0 |  | $\square$ |
| 1 | 0 | 1 |  | $\square$ |
| 1 | 1 | 0 |  | $\square$ |
| 1 | 1 | 1 |  | $\square$ |

3. (20 pts.) Consider the function $F$ whose truth table is shown below
(a) Write the function $F$ in
sum-of-minterms form. Two are given.

| $\mathbf{W}$ | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ | $\mathbf{F}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

(b) Fill in this Karnaugh map for $F$

(c) Use your Karnaugh map to write a minimal sum-of-products representation for $F$

In Logisim,
(d) Implement the circuit corresponding to your minimal sum-of-products representation. Verify your circuit using Logisim's Combinational Analysis feature (Project $\rightarrow$ Analyze Circuit).

Print your solution and attach it.
(e) Use your Karnaugh map to write a minimal product-of-sums representation for $F$.

(f) Implement the circuit corresponding to your minimal product-of-sums. Again, verify your circuit.

Print your solution and attach it.
4. (20 pts.) Create a circuit for a 4-to-10 decoder using AND gates and inverters only. Arrange and name the inputs and outputs as shown below. Treat $W$ as the most significant bit. Only one of the outputs should ever be true.

$$
\begin{array}{cl}
W \rightarrow & \rightarrow A 0 \\
X \rightarrow & \rightarrow A 1 \\
Y \rightarrow & \vdots \\
Z \rightarrow & \rightarrow A 9
\end{array}
$$

Implement your circuit in Logisim, verify it, and print and attach it.
5. (15 pts.) In Logisim, implement $F=X Y \bar{Z}+Y Z+\bar{X} Y$ using just constants and
(a) a 3-to-8 decoder (under "Plexers $\rightarrow$ Decoder." Set "include enable" to "No" and note the input wires are a bundle at the bottom) and an OR gate;
(b) an 8 input mux; and
(c) a 4 input mux whose select inputs are $X$ and $Y$, and an inverter.

Implement each of these circuits in Logisim, verify them, and print and attach them.

$$
\begin{aligned}
& X \rightarrow \\
& Y \rightarrow \\
& Z \rightarrow
\end{aligned} \quad \rightarrow F
$$

6. (15 pts.) Implement an eight-input mux using two-input muxes only (constants are OK).
Arrange your inputs and outputs as shown below.

$$
\begin{aligned}
& A 0 \rightarrow \\
& A 1 \rightarrow \\
& \vdots \\
& A 7 \rightarrow \\
& X \rightarrow F \\
& Y \\
& Z \\
&
\end{aligned}
$$

Here, $A 0$ through $A 7$ are the eight inputs, and $X, Y$, and $Z$ are the three selects. $X$ is the most significant bit, selecting between, e.g., A0 and A4.
Implement your circuit in Logisim, verify it, and print and attach it.
7. (20 pts.) Implement the combinational portion of a three-bit binary counter with an enable input. Give it four inputs, $X, Y, Z$, and $E$, and three outputs $A, B$, and $C$.
When $E$ is $0, A, B$, and $C$ should be $X, Y$, and $Z$ respectively.
When $E$ is $1, A, B$, and $C$ should be $X, Y$, and $Z$ plus one, with $A$ and $X$ the MSBs.
Your counter should wrap around, i.e., $7+1=0$.
Implement your circuit in Logisim, verify it, and print and attach it.

