## CSEE W3827

# Fundamentals of Computer Systems Homework Assignment 1 

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Due February 6, 2012 at 1:10 PM

Write your name and UNI on your solutions
Show your work for each problem; we are more interested in how you get the answer than whether you get the right answer.

1. (5 pts.) What are the values, in decimal, of the bytes

$$
10011100
$$

and
01111000,
if they are interpreted as 8 -bit
(a) binary numbers;
(b) one's complement numbers; and
(c) two's complement numbers?
2. (10 pts.) Show how to compute $6+-14$ using 5 -bit
(a) signed-magnitude numbers;
(b) one's complement numbers; and
(c) two's complement numbers.
3. (10 pts.) Show how to compute $45+57$ in BCD.
4. (10 pts.) Complete the truth table for the following Boolean functions:

$$
\begin{aligned}
& a=X \bar{Y} \bar{Z}+\bar{X} \bar{Y} Z+\bar{X} \bar{Z} \\
& b=(X+\bar{Y})(Y+\bar{Z})(X+\bar{Z})
\end{aligned}
$$

| $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ | $\mathbf{a}$ | $\mathbf{b}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 |  |  |
| 0 | 0 | 1 |  |  |
| 0 | 1 | 0 |  |  |
| 0 | 1 | 1 |  |  |
| 1 | 0 | 0 |  |  |
| 1 | 0 | 1 |  |  |
| 1 | 1 | 0 |  |  |
| 1 | 1 | 1 |  |  |

5. (10 pts.) Consider the function $F$, whose truth table is below.

6. (10 pts.) Consider the function $F$ whose truth table is shown below

| $\mathbf{W}$ | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ | $\mathbf{F}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

(a) Write the function $F$ in sum-of-minterms form
(b) Minimize the sum-of-minterms expression, justifying each step
7. (10 pts.) Consider the function $F$ from problem 6.
(a) Fill in and minimize the following Karnaugh map for $F$

(b) Express your minimized Karnaugh map as a Boolean expression
(c) Are your minimized expressions in problem 6 and problem 7 the same? Why or why not?
8. (20 pts.) Design a circuit that takes two two-bit binary numbers ( $A_{1}$ and $A_{0}, B_{1}$ and $B_{0}$ ) and produces a true output when, in binary, $A$ is greater than or equal to $B$.
(a) Fill in the truth table
(b) Fill in the Karnaugh map and use it to minimize

(c) Draw the corresponding circuit.

| $A_{1}$ | $A_{0}$ | $B_{1}$ | $B_{0}$ | $A \geq B$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |  |
| 0 | 0 | 0 | 1 |  |
| 0 | 0 | 1 | 0 |  |
| 0 | 0 | 1 | 1 |  |
| 0 | 1 | 0 | 0 |  |
| 0 | 1 | 0 | 1 |  |
| 0 | 1 | 1 | 0 |  |
| 0 | 1 | 1 | 1 |  |
| 1 | 0 | 0 | 0 |  |
| 1 | 0 | 0 | 1 |  |
| 1 | 0 | 1 | 0 |  |
| 1 | 0 | 1 | 1 |  |
| 1 | 1 | 0 | 0 |  |
| 1 | 1 | 0 | 1 |  |
| 1 | 1 | 1 | 0 |  |
| 1 | 1 | 1 | 1 |  |

9. (15 pts.)
(a) Minimize the Karnaugh map for the complement of the $A \geq B$ function from problem 8.

(b) Use this to draw a circuit for $A \geq B$ (i.e., not the complement).
