## CSEE W3827

# Fundamentals of Computer Systems <br> Homework Assignment 1 

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Due September 20th, 2011 at 10:35 AM
Show your work for each problem; we are more interested in how you get the answer than whether you get the right answer.

This document is formatted for on-screen viewing.

1. What are the values, in decimal, of the bytes

$$
10011100
$$

and
01111000,
if they are interpreted as 8 -bit
(a) Binary numbers?
(b) One's complement numbers?
(c) Two's complement numbers?
2. The DEC PDP-8 used 12-bit words.
(a) What were the most negative and most positive decimal numbers one of its words could represent using two's complement?
(b) Assuming a word represented an address in memory, how many different locations could the PDP-8 address?

3. Convert the hexadecimal number "DEAD" into
(a) Binary
(b) Octal
(c) Decimal
(d) Binary-Coded Decimal
4. Show that $2+-7=-5$ is also true when done in binary using
(a) Signed-magnitude numbers
(b) One's complement numbers
(c) Two's complement numbers
5. Show $42+49=91$ in BCD. Make sure you show when corrections are necessary to normal binary addition.
6. Complete the truth table for the following Boolean functions:

| (a) | $X Y \bar{Z}+X \bar{Y} Z+\bar{X} Y Z$ |  |  |
| :--- | :--- | :--- | :--- |
| (b) | $(X+Y)(Y+Z)(X+\bar{Z})$ |  |  |
| $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ | $\mathbf{a}$ |
| 0 | $\mathbf{b}$ |  |  |
| 0 | 0 | 0 |  |
| 0 | 0 | 1 |  |
| 0 | 1 | 0 |  |
| 0 | 1 | 1 |  |
| 1 | 0 | 0 |  |
| 1 | 0 | 1 |  |
| 1 | 1 | 0 |  |
| 1 | 1 | 1 |  |

7. Consider the function $F$, whose truth table is below.

| $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ | $\mathbf{F}$ |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

(a) Write $F$ as a sum of minterms and draw the corresponding circuit.
(b) Write $F$ as a product of maxterms and draw the corresponding circuit.
(c) Complete the Karnaugh map for $F$ as shown below.

8. Consider the function $F=\bar{X} \bar{Y} \bar{Z}+\bar{X} Y \bar{Z}+X \bar{Y} \bar{Z}+X Y \bar{Z}$
(a) Simplify the function using a Karnaugh map: draw the map F, circle implicants, and write the simplified function in algebraic form.

(b) Show how applying the axioms of Boolean algebra can produce the same result.

## Axioms of Boolean Algebra

| $a \vee b=b \vee a$ | $a \wedge b=b \wedge a$ |
| :---: | :---: |
| $a \vee(b \vee c)=(a \vee b) \vee c$ | $a \wedge(b \wedge c)=(a \wedge b) \wedge c$ |
| $a \vee(a \wedge b)=a$ | $a \wedge(a \vee b)=a$ |
| $a \wedge(b \vee c)=(a \wedge b) \vee(a \wedge c)$ $a \vee \neg a=1$ | $\begin{aligned} a \vee(b \wedge c) & =(a \vee b) \wedge(a \vee c) \\ a \wedge \neg a & =0 \end{aligned}$ |

9. Design a circuit that takes two two-bit binary numbers ( $A_{1}$ and $A_{0}$, $B_{1}$ and $B_{0}$ ) and produces a true output when, in binary, $A$ is strictly greater than $B$.
(a) Fill in the truth table
(b) Fill in the Karnaugh map and use it to minimize

(c) Draw the circuit you derived from the map in part (b).

| $A_{1}$ | $A_{0}$ | $B_{1}$ | $B_{0}$ | $A>B$ |
| :---: | :---: | :---: | :---: | :--- |
| 0 | 0 | 0 | 0 |  |
| 0 | 0 | 0 | 1 |  |
| 0 | 0 | 1 | 0 |  |
| 0 | 0 | 1 | 1 |  |
| 0 | 1 | 0 | 0 |  |
| 0 | 1 | 0 | 1 |  |
| 0 | 1 | 1 | 0 |  |
| 0 | 1 | 1 | 1 |  |
| 1 | 0 | 0 | 0 |  |
| 1 | 0 | 0 | 1 |  |
| 1 | 0 | 1 | 0 |  |
| 1 | 0 | 1 | 1 |  |
| 1 | 1 | 0 | 0 |  |
| 1 | 1 | 0 | 1 |  |
| 1 | 1 | 1 | 0 |  |
| 1 | 1 | 1 | 1 |  |

