

Robust Performance of Complex Network Infrastructures

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Networks

- Complex networks pervade numerous fields of modern sciences
- Types of networks:
 - Financial: institutions linked by contractual relationships
 - Interbanking liabilities network, centrally cleared network
 - Social: group of interacting people
 - Facebook, friendship networks, crime and education networks
 - Technological: designed for distribution of a commodity or service
 - Internet, power grids, transportation networks (road, rail, airline), supply chain networks

Financial Networks

- Interbanking system consists of financial institutions linked to each other via unsecured debt contracts
- Each institution holds external assets and claims on other institutions in the network
- If an institution cannot meet its liabilities in full, it defaults and repays its creditors on a pro rata basis
- Failure of an institution to repay its debt may impair the ability of its creditors to repay their own creditors (systemic risk)
- **Goal:** Can we construct a framework for measurement and valuation of systemic risk?

Social Networks

- Each player applies a certain action in order to maximize his utility
- The benefit of taking a higher action to a player may increase or decrease the higher the actions of her neighbors are (criminal efforts, education decisions)
- Social welfare measured by aggregating the utility functions of the agents in the network
- **Goal:** How is social welfare affected by the network structure and agents' utilities?

Supply Chain Networks

- Nodes represent retailers. Lateral transshipments occur between different retailers
- Each retailer holds an inventory of commodities to serve its customers
- If the inventory of a retailer is not sufficient to fulfill the demand of its customers, the surplus stock from other retailers will be transshipped to him to cover the excess demand
- **Goal:** How to design or adapt the supply chain network so to reduce the costs of transshipping inventory?

Common Questions

- **Performance:** How does the network structure affect performance?
- **Learning:**
 - What network structures emerge? Core-periphery, small world networks?
 - Can we learn the patterns of interaction in a network?
- **Robustness:** Can we make statements about performance in the absence of perfect information on the network state?

Model for Complex Networks

- The network is a 4-tuple (A, D, S, f) :
 - $A = (a_{ij})_{i,j=1,\dots,n}$: **interaction matrix**. Captures pattern and extent of interaction between the nodes of the network
 - D and S : **demand** and **supply** function. They may depend on the state of the network
 - f : **interaction function**. The algorithm used by each node to compute its state is a function f of its interactions with the other states of the network, its demand and its supply.
- Designed to incorporate the key features of a wide variety of networks, including financial, social, and technological networks.

Network Equilibrium

- An **equilibrium** of the network is a fixed point of the interaction function f
- It is a vector $x = (x_1, x_2, \dots, x_n)$ solving simultaneously the set of equations

$$x_i = f \left(\sum_{j \neq i} a_{ji} x_j, D_i, S_i \right), \quad i = 1, \dots, n$$

Financial Network

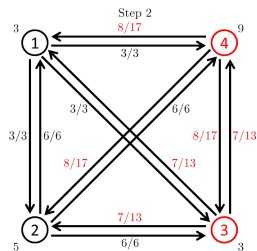
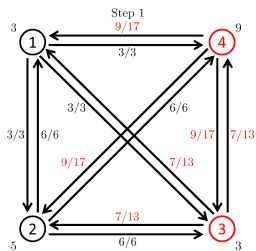
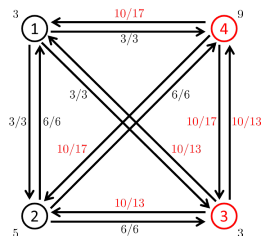
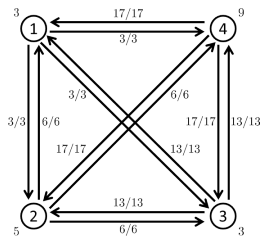
- Demand $D = (\ell_1, \ell_2, \dots, \ell_n)$, where $\ell_i = \sum_{j \neq i} \ell_{ij}$ are the total liabilities of bank i
- Interaction matrix A is the relative liability matrix: $a_{ij} = \frac{\ell_{ij}}{\ell_i}$
- Asset supply $S = (S_1, \dots, S_n)$ is endogeneous, where

$$S_i = \underbrace{\sum_j \ell_j a_{ji}}_{\text{interbank assets}} + \underbrace{c_i}_{\text{external assets}}$$

- The vector of payments solves the fixed point equation (Eisenberg and Noe (2001)):

$$p_i = \min \left(\underbrace{\ell_i}_{\text{full pay}}, \underbrace{\sum_j a_{ji} p_j + c_i}_{\text{recovery at default}} \right)$$

Equilibrium Algorithm



Step 3

Step 4

Games on Social Networks

- Players take actions (criminal effort, education effort) to maximize their utilities
- Player i takes his action x_i to maximize

$$u_i(x_1, x_2, \dots, x_n) = \underbrace{\alpha x_i - \frac{1}{2} x_i^2}_{\text{individual part}} + \underbrace{\alpha \sum_{j \neq i} a_{ji} x_i x_j}_{\text{aggregate effect of peers}}$$

where α measures the intensity of interaction.

- Using the first order condition, the optimal action x_i chosen by player i is

$$x_i = \alpha \left(\sum_{j \neq i} a_{ji} x_j + 1 \right)$$

- Neither demand nor supply appear in the specification of the network interaction function

Multifaceted Performance and Concentration

- Develop a **multifaceted** framework for performance analysis of complex network infrastructures
- Analyze the sensitivity of the network performance to its infrastructure: how “**concentrated**” network interactions affect *multiple* measures of performance **simultaneously**
- Examples of performance measures: aggregate, worst-case, and average performance

Multifaceted Performance

- Performance measures are all increasing Schur-convex functions defined on the network equilibrium state
- A function ϕ is *increasing Schur-convex* if

$$\sum_{i=1}^k x_{[i]} \leq \sum_{i=1}^k y_{[i]} \text{ for } k = 1, \dots, n \text{ implies } \phi(x) \leq \phi(y),$$

where $x_{[k]}$ denotes the k -th largest entry of the vector x .

- “Preference order” between states is preserved:
 - Suppose x and y are two states which quantify costs: shortfalls, criminal effort levels, surplus transhipped stock
 - If largest cost, sum of the two largest costs, ..., and total cost in x are lower than in y , $\phi(x) \leq \phi(y)$ for all increasing Schur-convex functions ϕ

Performance Measures

- The class of increasing Schur-convex functions is a **rich** class:
 - It consists of those which are increasing, symmetric, and convex
 - maximum shortfall in a financial network: $\phi(x) = \max_i x_i$,
 $x_i = \ell_i - p_i$
 - social welfare in social networks: $\phi(x) = \frac{1}{2} \sum_{i=1}^n x_i^2$

Concentration of Network Interactions

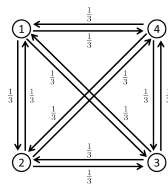
- **Question:** Why focusing on concentration of interactions?
- Concentration plays a key role in shaping performance:
 - Financial Network: is the systemic loss higher when banks only have few creditors in the system or if they have a heterogenous set of creditors?
 - Social Network: is the total crime smaller when criminal i only benefits from the action of criminal j or if he equally benefits from the actions of other players?
 - Supply Chain Network: when is it cost-effective to design a network infrastructure which concentrates transshipment of surplus stock to a limited set of retailers?

Concentration Comparison Measures

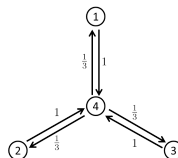
- Use **matrix majorization** to compare concentration of the interaction matrices of two networks
- Let A and B be two matrices. A is *majorized* by B , $A \prec B$, if there exists a doubly stochastic matrix S such that

$$A = BS$$

Network Structure and Performance



(a)



(b)

Asset–Liability	Total Loss (a)	Total Loss (b)
Case 1: (30, 20, -10, -20)	45	70
Case 2: (-20, -10, 20, 30)	45	30

(c)

- How should policies intervene on the network infrastructure to reduce losses?
- Are policies in favour of a centralized network structure necessarily maximizing welfare?
- **Answer:** depending on the network state (balance sheets) and on the banking interaction patterns, some policies may be more effective than others

Highly Capitalized Financial Networks

- Empirical/historical evidence suggests that small banks (with low outstanding liabilities) default
- This leads to define the class of **highly capitalized** financial networks
- The network is highly capitalized if, for $j = 1, \dots, n - 1$,

$$S_{j+1}(A) - D_{j+1} \geq S_j(A) - D_j,$$

where nodes are labeled so that $D_1 \leq D_2 \dots \leq D_n$.

Empirical Analysis

- Consider financial system induced by the banking sectors of eight representative European countries
- These countries account for 80% of the total liabilities of the European banking sector
- Consolidated banking data released from the European Central Bank and foreign claims data from the BIS are used to estimate parameters of the financial system.

Banks' consolidated foreign claims (BIS)

December 2009	(UK)	(Germany)	(France)	(Spain)	(Netherland)	(Ireland)	(Belgium)	(Portugal)
(UK)	0.00	500.62	341.62	409.36	189.95	231.97	36.22	10.43
(Germany)	172.97	0.00	292.94	51.02	176.58	36.35	20.52	4.62
(France)	239.17	195.64	0.00	50.42	92.73	20.60	32.57	8.08
(Spain)	114.14	237.98	219.64	0.00	119.73	30.23	26.56	28.08
(Netherland)	96.69	155.65	150.57	22.82	0.00	15.47	28.11	11.39
(Ireland)	187.51	183.76	60.33	15.66	30.82	0.00	64.50	21.52
(Belgium)	30.72	40.68	301.37	9.42	131.55	6.11	0.00	1.17
(Portugal)	24.26	47.38	44.74	86.08	12.41	5.43	3.14	0.00

June 2010	(UK)	(Germany)	(France)	(Spain)	(Netherland)	(Ireland)	(Belgium)	(Portugal)
(UK)	0.00	462.07	327.72	386.37	135.37	208.97	43.14	7.72
(Germany)	172.18	0.00	255.00	39.08	149.82	32.11	20.93	3.93
(France)	257.11	196.84	0.00	26.26	80.84	18.11	29.70	8.21
(Spain)	110.85	181.65	162.44	0.00	72.67	25.34	18.75	23.09
(Netherland)	141.39	148.62	126.38	20.66	0.00	12.45	23.14	11.11
(Ireland)	148.51	138.57	50.08	13.98	21.20	0.00	53.99	19.38
(Belgium)	29.15	35.14	253.13	5.67	108.68	5.32	0.00	0.39
(Portugal)	22.39	37.24	41.90	78.29	5.13	5.15	2.57	0.00

Table: All values are in USD billion.

Consolidated banking sector data (ECB)

December 2009 (in USD billion)				
Country	Assets	Liabilities	Equity	External assets
UK	13,833	13,204	674	12,849
Germany	12,366	11,901	504	10,557
France	9,053	8,616	472	7,155
Spain	5,350	5,024	374	4,545
Netherlands	3,795	3,632	213	2,747
Ireland	1,919	1,828	149	1,446
Belgium	1,706	1,629	133	1,427
Portugal	732	686	104	627

June 2010 (in USD billion)				
Country	Assets	Liabilities	Equity	External assets
UK	13,956	13,258	736	12,982
Germany	11,533	11,126	443	9,936
France	8,485	8,077	439	6,864
Spain	4,765	4,482	325	4,052
Netherlands	3,506	3,366	184	2,715
Ireland	1,758	1,678	129	1,337
Belgium	1,530	1,464	116	1,276
Portugal	654	615	90	563

- The network induced by the sovereign banking sectors is highly capitalized

Notation and Definitions

- A is **ordered preserving** with respect to x if $A^\top x$ is similarly ordered to x
- A n -dimensional square matrix A is **weak supermajorization preserving** if for x, y ,

$$x <^w y \text{ implies } A^\top x <^w A^\top y,$$

where $x <^w y$ if

$$\sum_{i=1}^k x_{(i)} \geq \sum_{i=1}^k y_{(i)} \text{ for } k = 1, \dots, n,$$

and $x_{(i)}$ is the i -th smallest component of x

Main Result

Theorem

*Let (A, D, S, f) and (B, D, S, f) be two **highly capitalized** networks. If A and B are order-preserving and weak supermajorization preserving, then $A < B$ implies $\phi(p^A) \geq \phi(p^B)$ for any performance function ϕ .*

- Highly capitalized: banks with small liabilities default
- Liabilities order-preserving: banks with lower (higher) liabilities receive smaller (higher) payments
- **Concentration**: in the most concentrated network banks make even lower (higher) payments to smaller (larger) banks
- Weak supermajorization preserving: payments received by small banks are primarily made by small banks
- Since small banks default, their recovery rate is lower and the network suffers higher losses

Policy Implications

- Empirical evidence suggests that financial networks encountered in practice are highly capitalized
- Theoretical results indicate that higher concentration of liabilities induces larger systemic losses in highly capitalized networks
- Desirable for regulatory purposes to prevent high concentration of liabilities in the network
- Support the supervisory framework by the Basel Committee limiting the size of gross exposures to individual counterparties

Conclusions

- Introduced a framework for analyzing network performance **simultaneously** across multiple dimensions
- Network performance is closely linked to the state of the network and to its topological structure
- Identify **concentration** of network interactions as a key driver of performance
- Measure concentration by applying the matrix majorization order to the network interaction matrix

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