Robust Performance Analysis of Complex Network Infrastructures

Agostino Capponi
Industrial Engineering and Operations Research Department
Columbia University
New York, NY 10027
ac3827@columbia.edu

Abstract

We develop a multifaceted framework for performance analysis of complex network infrastructures. In contrast to using traditional one-dimensional aggregate performance measurements on the state of the network, the proposed framework allows comparing networks simultaneously through the rich class of Schur-convex functions (e.g. Arnold et al. (2011)). Such a class includes those which are increasing, symmetric, and jointly convex, such as the maximum, worst-case, average and aggregate performance.

We consider a general network model, consisting of (1) a network interaction matrix capturing the pattern and extent of interaction between the nodes in the network, (2) a demand and supply function, and (3) a network interaction function used by each node in the network to compute its state based on the states of the others.

We study the dependence of the equilibrium network performance, defined as a fixed point of the network interaction function, on the concentration of network interactions. We quantify concentration by applying the matrix majorization order to the network interaction matrix. We analytically characterize how the interplay of demand-supply imbalance, network interaction function and concentration of the network interaction matrix shape the performance of the network. We show that, depending on whether or not the network interaction matrix preserves the order of demand-supply imbalances, higher concentration may improve or deteriorate network performance, simultaneously across multiple dimensions.

We show how the proposed framework can be specialized to capture several types of networks encountered in practice, including power grids (e.g. Blanchet et al. (2011)), social (e.g. Acemoglu et al. (2015b)) and financial networks (e.g. Eisenberg and Noe (2001)). Our analysis allows answering questions of the type: Is it more cost-effective if power generators in a region shed their excess electricity demand to a higher number of their neighbors? Is it more systemically efficient if banks only have few creditors? Is social welfare always higher when players distribute their socialization effort on a higher number of their peers? The key advantage of our framework and accompanying analysis is the flexibility: different scientific fields having different needs may choose the set of performance measures of interest. Economists may be interested in the performance of the four and eight largest firms (two common measures are the four-firm and eight-firm concentration ratio), power engineers may be interested in the worst case performance as well as the performance of the largest regions in the power grid network, while financial regulators are interested in identifying the top five or ten institutions causing the largest losses in the system, when they perform stress test scenarios.

We indicate how network data may be used to both drive model design and to calibrate network parameters. We present an empirical analysis showing how concen-

tration risk affects systemic losses and default cascades for a class of empirically driven financial networks (e.g. Capponi et al. (2015a)). Such a class is constructed using historical balance sheets and interbank liabilities data from the Bank of International Settlement Quarterly Review and European Central Bank.

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