Nested Cages

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**Problem:** Given fine resolution mesh, construct progressive decimations such that each coarser layer contains all finer layers.

**Input:** overlapping decimations  
**Output:** nested cages

**Idea:** Work pairwise. Flow fine layer inside coarse layer, once inside re-inflate and push coarse layer outward. Then coarse layer becomes fine layer for next pair.

**Overlapping pair** $F$ and $\bar{C}$  
**Flow** $F$ to $\bar{F}$ until inside $\bar{C}$  
**Re-inflate** $\bar{F}$ to $F$ pushing $\bar{C}$ to $C$

**Question:** Why flow the fine into the coarse? Why not expand the coarse outside the fine?

**Answer:** The distance field of the coarse mesh is simpler, fewer gradient discontinuities.

**Flow:** Not all flows shrink the fine inside the coarse in a way accommodating re-inflation.

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**Homeomorphic:** Cages should match topology of input, ensuring that solutions to PDEs have same low frequency behavior.

**Our nested layers**

**Applications:** Multiresolution PDE solving, collision detection, cage deformation

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**Homeomorphic**

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<th>Ours</th>
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**Stress tests:** Extremely tight layers, robust to noisy input

**Overlapping cage**  
**Flow**  
**Enclosing cage**  
**Deformation**

**Limitation:** Coarse layer collides with self; "pinch" prevents further cages.

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